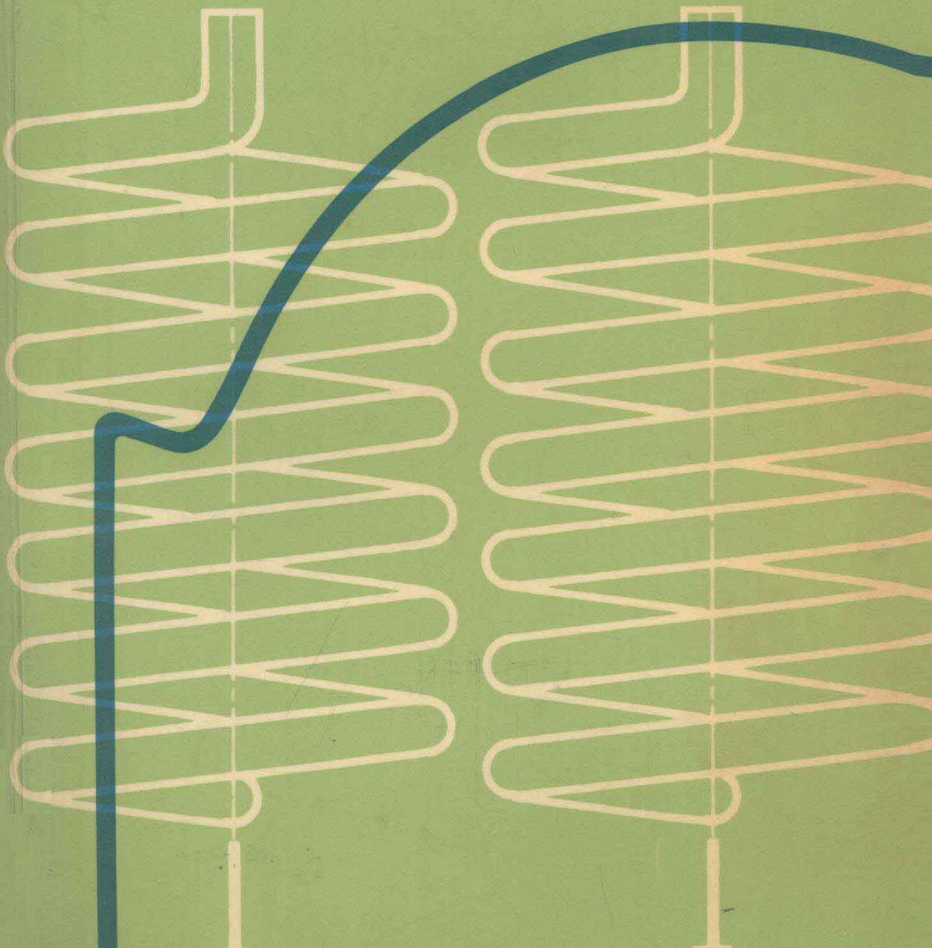
Solution of Problems in Strength of Materials and Mechanics of Solids

A Problem-based Textbook

S A Urry & P J Turner



Solution of Problems in Strength of Materials and Mechanics of Solids

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Preface

This book is intended for engineering students in universities, polytechnics and colleges who are taking courses in Strength of Materials, Stress Analysis and Mechanics of Solids. Its standard is that of University and CNAA degrees and it should also prove suitable for courses leading to Higher National Certificates and Diplomas, and the examinations of the Council of Engineering Institutions. At the same time there is extensive coverage of the more elementary topics and it is hoped that students will find the book useful from the beginning of their courses.

SI units are used exclusively and an explanatory note on the system is given on page ix. Many of the problems first appeared in Imperial units in earlier editions of the book and, in the conversion to SI, numerical values have generally been rounded off.

We have taken the opportunity to make other changes; symbols and abbreviations have been brought into line with current practice and some new topics have been covered by the inclusion of additional examples. The new material includes laterally and eccentrically loaded struts, rotating discs, the bending of circular plates, curved beams, unsymmetrical bending, and plastic deformation.

The following abbreviations are used to indicate the sources of examination questions:

(U.L.) University of London
(I.Mech.E.) Institution of Mechanical Engineers
(I.Struct.E.) Institution of Structural Engineers
(R.Ae.S.) Royal Aeronautical Society

For permission to reproduce these questions we are indebted to the respective authorities but the responsibility for the conversion to SI units and for the solutions is entirely ours. The examinations of the various professional engineering institutions have now been replaced by those of the Council of Engineering Institutions.

Our thanks are due to Mrs Felicity Lefevre for typing the entire manuscript; to the Publishers for their care in the production

of the book; and to our wives for their patience and encouragement.

Despite careful checking some errors may remain and any criticism or correction will be gratefully acknowledged.

S. A. Urry
P. J. Turner

Table of SI Units

Quantity	*Derivation*	*Name of Unit*	*Symbol*
Mass	—	kilogramme	kg
Length	—	metre	m
Time	—	second	s
Area	$(\text{length})^2$	—	m^2
Volume	$(\text{length})^3$	—	m^3
First moment of area	area × length	—	m^3
Second moment of area	area × $(\text{length})^2$	—	m^4
Velocity	length ÷ time	—	m/s
Acceleration	velocity ÷ time	—	m/s^2
Force	mass × acceleration	newton	N (= kg-m/s^2)
Stress and pressure	force ÷ area	—	N/m^2
Torque and moment	force × distance	—	N-m
Work and energy	force × distance	joule	J (=N-m)
Power	work ÷ time	watt	W (=N-m/s)

Multiples and Sub-multiples

Factor	*Unit prefix*	*Symbol*
$1\,000\,000\,000 = 10^9$	giga-	G
$1\,000\,000 = 10^6$	mega-	M
$1\,000 = 10^3$	kilo-	k
$0{\cdot}01 = \frac{1}{10^2} = 10^{-2}$	centi-	c
$0{\cdot}001 = \frac{1}{10^3} = 10^{-3}$	milli-	m
$0{\cdot}000001 = \frac{1}{10^6} = 10^{-6}$	micro-	μ

Note on SI Units

The tables opposite list the units used in this book. They conform to the SI (Système Internationale) scheme which is based on the three fundamental units: *kilogramme* (kg) for mass, *metre* (m) for length and *second* (s) for time. The units of other quantities are derived from these. For example, areas are measured in *square metres,* denoted by m^2, and volumes in *cubic metres* (m^3). Velocity is measured in *metres per second* (m/s) and acceleration in *metres per second per second* (m/s^2).

Force and stress

The units of force may be derived from the relationship force = mass × acceleration and, substituting the SI units of mass and acceleration, we obtain

$$\text{unit force} = 1\ \text{kg} \times 1\ \text{m/s}^2 = 1\ \text{kg-m/s}^2$$

Force is a quantity which occurs so frequently that this unit has been given a separate name, the *newton,* symbol N.

In most questions in this book, loads and forces are given in newtons but there are a few examples in which the load acting is the weight of a body whose mass is given. Suppose, for example, a beam supports a body whose mass is 200 kg, the gravitational acceleration being 9·81 m/s^2. Its weight—and therefore the load on the beam—is the product of its mass and the acceleration due to gravity. Thus:

$$\begin{aligned}\text{Load} &= 200\ \text{kg} \times 9{\cdot}81\ \text{m/s}^2\\ &= 1962\ \text{kg-m/s}^2 \text{ or } 1962\ \text{N}\end{aligned}$$

Stress is the ratio force/area and its units are therefore *newtons per square metre,* abbreviated to N/m^2. The name *pascal* has been adopted for this unit but N/m^2 has the advantage of emphasizing the relationship between stress, force and area. Pressure has the same units as stress (N/m^2); practical values are sometimes given in terms of the unit *bar* which is 100000 N/m^2 and is roughly equal to atmospheric pressure at sea-level.

Work, energy and power

Work and energy have the units of force × distance, i.e. *newton metres* (N-m). This unit has been given the name *joule*, denoted by J. Moment or torque is also the product of force and length with the same units, N-m (but not the joule). Power is the rate of doing work and is therefore expressed in *joules per second* (J/s) a unit which has the name *watt* (W). Thus 1 watt is 1 newton metre per second.

Multiples of units

Practical values of stress and other physical quantities often lead to very large or very small numbers and there are two methods of writing values concisely. One is to express the numbers in terms of powers of 10 such as 10^3 (a thousand) or 10^6 (a million). Alternatively, the units themselves can be modified by similar factors using prefixes. For example 10^3 is denoted by kilo- (k) and 10^6 by mega- (M).

The *preferred* multiples are those in which the power of 10 is a multiple of 3, e.g. 10^6, 10^3, 10^{-3}, etc. The multiples used in this book are listed on page viii.

To illustrate the notation, suppose the stress in a steel bar is 84 500 000 N/m² and the modulus of elasticity for the material is 204 000 000 000 N/m². Then,

$$\text{Stress} = 84\,500\,000 \text{ N/m}^2 = 84{\cdot}5 \times 10^6 \text{ N/m}^2 \text{ or } 84{\cdot}5 \text{ MN/m}^2$$

$$\text{Modulus of elasticity} = 204\,000\,000\,000 \text{ N/m}^2 = 204 \times 10^9 \text{ N/m}^2 \text{ or } 204 \text{ GN/m}^2$$

In some books stress is expressed in *newtons per square millimetre* (N/mm²) and numerical values will then be the same as in MN/m².

A special problem arises with the quantity *second moment of area.* In many examples its value in m^4 leads to very small numbers but in mm^4 it becomes very large. It is often convenient to use cm^4 and it should be noted that $1 \text{ m}^4 = 10^8 \text{ cm}^4$. In the same way, areas are often expressed in *square centimetres* (cm²). Apart from these two quantities the prefix centi- has been avoided in this book.

Numerical examples

It is recommended that, in solving numerical problems, the prefixes to units are replaced by factors which are powers of 10. At the end of the working the prefix form can be used for the answer.

Example. The central deflection δ of a centrally-loaded uniform beam is given by $\delta = WL^3/48\,EI$. Calculate δ when $W = 144$ kN, $L = 4$ m, $E = 200$ GN/m² and $I = 8000$ cm⁴. The work may be set out as follows:

$$\delta = \frac{144\text{ kN} \times (4\text{ m})^3}{48 \times 200\text{ GN/m}^2 \times 8000\text{ cm}^4}$$

$$= \frac{(144 \times 10^3\text{N}) \times (4\text{ m})^3}{48 \times (200 \times 10^9\text{ N/m}^2) \times (8000 \times 10^{-8}\text{ m}^4)}$$

$$= \frac{144 \times 4^3}{48 \times 200 \times 8000} \times \frac{10^3}{10^9 \times 10^{-8}} \frac{\text{N} \times \text{m}^3}{(\text{N/m}^2) \times \text{m}^4}$$

$$= \frac{12}{100 \times 1000} \times \frac{10^3 \times 10^8}{10^9}\text{ m}$$

$$= 12 \times 10^{-3}\text{ m}$$

$$= 12\text{ mm}$$

This layout may seem elaborate but it helps to avoid errors and it can be shortened as confidence in the method is gained.

Contents

Chapter 1

Simple Stress and Strain; Elasticity

The stress, strain, and proportionality between them for direct or for shear forces acting on a body are defined as follows:

For a direct force (tension or compression)

$$\text{Stress } \sigma = \frac{\text{force } P}{\text{area } A}$$

$$\text{Strain } \epsilon = \frac{\text{change in length}}{\text{original length}}$$

$$\text{Young's Modulus of Elasticity } E = \frac{\text{direct stress } \sigma}{\text{direct strain } \epsilon}$$

For a shear force

$$\text{Stress } \tau = \frac{\text{force } F}{\text{area resisting shear } A}$$

and

Shear strain γ = angular displacement (in radians) produced by the shear stress.

$$\text{Shear modulus or modulus of rigidity } G = \frac{\text{shear stress } \tau}{\text{shear strain } \gamma}$$

"Free" temperature expansion of a bar $= \alpha l t$

where

α = coefficient of linear expansion,
l = length of the bar,
t° = rise in temperature.

For a fall in temperature of $t°$ the same expression will give the "free" contraction of the bar.

WORKED EXAMPLES

1.1. Explain the terms "stress" and "strain" as applied to a bar in tension. What is the relation between these two quantities? A steel rod, 25 mm dia. and 6 m long, extends 6 mm under a pull of 100 kN. Calculate the stress and strain in the rod.

Solution. A bar in tension is one which is subjected to a pair of equal and opposite forces which tend to stretch it (Fig. 1.1). At any section, such as XX, there must be internal forces in the material which balance the applied force P.

The *stress* is defined as the total load (P) on the section divided by the cross-sectional area. Stress is usually denoted by σ and, if A is the cross-sectional area

$$\text{Stress } \sigma = \frac{\text{force } P}{\text{area } A}$$

(The same definition applies when the external forces are compressive, i.e. when they tend to shorten the bar.)

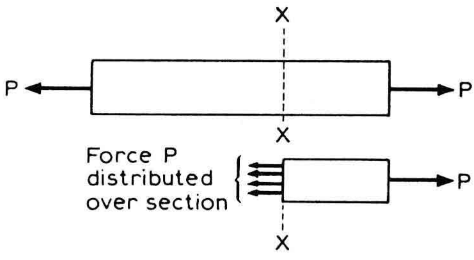

Fig. 1.1

The SI unit of area is the *square metre* (abbreviated to m²) but for many structural and machine components the square centimetre (cm²) or square millimetre (mm²) is more convenient. Note that $1\ \text{m}^2 = 10^4\ \text{cm}^2 = 10^6\ \text{mm}^2$.

The SI unit of force is the *newton* (N) but for many practical purposes kilonewtons (kN) and meganewtons (MN) are used. Hence the units of stress are *newtons per square metre* (N/m²) and practical values are usually expressed in kN/m² or MN/m². (N/mm² is used as an alternative to MN/m².) The name *pascal* (denoted by Pa) has been given to the unit N/m². Stress has the same units as pressure which is sometimes measured in *bars*. A bar is defined as 10^5 N/m².

The *strain* in a bar in tension is defined as the extension of the bar divided by its original length. It is denoted by ϵ and thus

$$\text{Strain } \epsilon = \frac{\text{increase in length}}{\text{original length}}$$

Since strain is a ratio of two lengths, it has no units.

Stress and strain are related by Hooke's Law, which states that *stress is proportional to strain.* This is almost exactly true for many engineering materials within a certain limit of stress called the *limit of proportionality*. Unless otherwise stated, it should be assumed in examples that the limit of proportionality is not exceeded and that Hooke's Law applies.

For the steel rod in the question,

$$\text{Cross-sectional area} = \tfrac{1}{4}\pi \times (\text{diameter})^2$$
$$= \tfrac{1}{4}\pi \times (25\text{ mm})^2 = 491\text{ mm}^2$$

$$\text{Stress} = \frac{\text{force or load}}{\text{cross-sectional area}}$$
$$= \frac{100\text{ kN}}{491\text{ mm}^2} = \frac{100 \times 10^3\text{ N}}{491 \times 10^{-6}\text{ m}^2}$$
$$= 204 \times 10^6\text{ N/m}^2$$
$$= 204\text{ MN/m}^2 \qquad (\textit{Ans})$$

$$\text{Strain} = \frac{\text{increase in length}}{\text{original length}}$$
$$= \frac{6\text{ mm}}{6\text{ m}} = \frac{6 \times 10^{-3}\text{ m}}{6\text{ m}} = 0{\cdot}001 \qquad (\textit{Ans})$$

1.2. What is an "elastic material"? Define Young's modulus of elasticity. A load of 2 kN is to be raised at the end of a steel wire. If the stress in the wire must not exceed 80 MN/m^2 what is the minimum diameter required? What will be the extension of a 3 m length of the wire in this case? (Take $E = 206$ GN/m^2.)

Solution. An *elastic material* may be defined as one which obeys Hooke's Law and in which the strain disappears when the stress causing it is removed (see also page 430). Within the limit of proportionality, many engineering materials exhibit almost perfect elasticity. (The term "elastic limit" is often used to mean "limit of proportionality" though a distinction can be made.) Since stress is proportional to strain (Hooke's Law) then stress divided by strain is a constant. For tensile (and compressive) stresses this constant is called *Young's Modulus of Elasticity* and is denoted by E, that is

$$\text{Young's modulus } E = \frac{\text{stress } \sigma}{\text{strain } \epsilon}$$

Strain has no units and therefore Young's modulus has the same units as stress, namely newtons per square metre. In these units many engineering materials have very high numerical values of Young's modulus and it is convenient to express E in giganewtons per square metre (GN/m²).

Typical values of E are

for steel, $206 \times 10^9\ \text{N/m}^2 = 206\ \text{GN/m}^2$
for copper, $103 \times 10^9\ \text{N/m}^2 = 103\ \text{GN/m}^2$
for timber, $10 \times 10^9\ \text{N/m}^2 = 10\ \text{GN/m}^2$

(In all examples where it is required, the value of E will be given.)

For the wire in question, since stress = load/area, then

$$\begin{aligned}\text{Minimum area required} &= \frac{\text{load}}{\text{stress}} = \frac{2\ \text{kN}}{80\ \text{MN/m}^2} \\ &= \frac{2 \times 10^3\ \text{N}}{80 \times 10^6\ \text{N/m}^2} \\ &= 25 \times 10^{-6}\ \text{m}^2\end{aligned}$$

But the area $= \frac{1}{4}\pi \times (\text{diameter})^2$, or $(\text{diameter})^2 = \frac{4}{\pi} \times \text{area}$.

Therefore

$$\begin{aligned}\text{Minimum diameter} &= \sqrt{\left(\frac{4}{\pi} \times \text{area}\right)} \\ &= \sqrt{\left(\frac{4}{\pi} \times 25 \times 10^{-6}\right)\ \text{m}^2} = 0{\cdot}00565\ \text{m} \\ &= 5{\cdot}65\ \text{mm} \qquad (Ans)\end{aligned}$$

Also

$$\begin{aligned}E = \frac{\text{stress}}{\text{strain}} &= \text{stress} \div \frac{\text{extension}}{\text{original length}} \\ &= \frac{\text{stress} \times \text{original length}}{\text{extension}}\end{aligned}$$

Rearranging,

$$\text{Extension} = \frac{\text{stress} \times \text{original length}}{E}$$

$$= \frac{80 \text{ MN/m}^2 \times 3 \text{ m}}{206 \text{ GN/m}^2}$$

$$= \frac{80 \times 10^6 \text{ N/m}^2 \times 3 \text{ m}}{206 \times 10^9 \text{ N/m}^2}$$

$$= 0{\cdot}001\,17 \text{ m} = 1{\cdot}17 \text{ mm} \qquad (Ans)$$

1.3. The round bar shown in Fig. 1.2 is subjected to a tensile load of 150 kN. What must be the diameter of the middle portion if the stress there is to be 215 MN/m^2?

What must be the length of the middle portion if the total extension of the bar under the given load is to be 0·2 mm? Take $E = 206$ GN/m^2.

Solution

$$\text{Area of middle portion} = \text{load/stress}$$

$$= \frac{150 \text{ kN}}{215 \text{ MN/m}^2} = \frac{150 \times 10^3 \text{ N}}{215 \times 10^6 \text{ N/m}^2}$$

$$= 0{\cdot}698 \times 10^{-3} \text{ m}^2$$

$$\text{Required diameter} = \sqrt{[(4/\pi) \times \text{area}]}$$

$$= \sqrt{\left(\frac{4}{\pi} \times 0{\cdot}698 \times 10^{-3}\right) \text{ m}^2}$$

$$= 0{\cdot}0298 \text{ m} = 29{\cdot}8 \text{ mm} \qquad (Ans)$$

If l m is the length of the middle portion, then $(0{\cdot}25 - l)$ m is the total length of the 50 mm diameter portions.

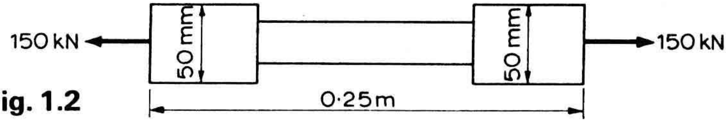

Fig. 1.2

$$\text{Extension of the middle portion} = l \times \text{strain}$$

$$= l \times \frac{\text{stress}}{E} = l \text{ m} \times \frac{215 \text{ MN/m}^2}{206 \text{ GN/m}^2}$$

$$= l \text{ m} \times \frac{215 \times 10^6 \text{ N/m}^2}{206 \times 10^9 \text{ N/m}^2} = 1{\cdot}04\, l \times 10^{-3} \text{ m}$$

Total extension of the two end portions

$$= (0{\cdot}25 - l) \times \frac{\text{stress}}{E} = (0{\cdot}25 - l) \times \frac{\text{load}}{\text{area} \times E}$$

$$= \frac{(0{\cdot}25 - l)\ \text{m} \times 150 \times 10^3\ \text{N}}{\frac{1}{4}\pi \times (5 \times 10^{-2}\ \text{m})^2 \times 206 \times 10^9\ \text{N/m}^2}$$

$$= 0{\cdot}371\ (0{\cdot}25 - l) \times 10^{-3}\ \text{m}$$

Total extension of bar = extension of middle portion + extension of two end portions

or

$$0{\cdot}0002\ \text{m} = 1{\cdot}04l \times 10^{-3}\ \text{m} + 0{\cdot}371\ (0{\cdot}25 - l) \times 10^{-3}\ \text{m}$$

from which

$$0{\cdot}669l = 0{\cdot}2 - 0{\cdot}093 = 0{\cdot}107$$

$$l = 0{\cdot}160\ \text{m}$$

The middle portion of the bar should be 160 mm long. (*Ans*)

1.4. A cast-iron column of the section shown in Fig. 1.3 is 2 m high and supports a load of 20 kN, in addition to its own weight. What is the maximum compressive stress in the column? The density of cast iron is 7200 kg/m³. Take 1 kgf = 9·81 N.

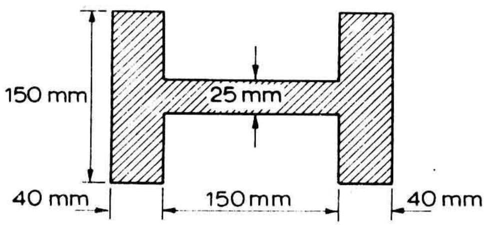

Fig. 1.3

Solution. The maximum stress occurs at the base where the total load of the section is the sum of the external load and the weight of the column.

Cross-sectional area = area of web + area of two flanges

$$= (0{\cdot}15 \times 0{\cdot}025)\ \text{m}^2 + 2(0{\cdot}04 \times 0{\cdot}15)\ \text{m}^2$$

$$= 15{\cdot}75 \times 10^{-3}\ \text{m}^2$$