FOUNDATIONS
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ENGINEERING
Series

Dynamics

G. E. Drabble

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DYNAMICS

Titles in the *Foundation of Engineering* Series

J. A. Cain and R. Hulse, *Structural Mechanics*
G. E. Drabble, *Dynamics*
R. G. Powell, *Electromagnetism*
P. Silvester, *Electric Circuits*

SERIES EDITOR'S FOREWORD

This series of programmed texts has been written specifically for first year students on degree courses in engineering. Each book covers one of the core subjects required by electrical, mechanical, civil or general engineering students, and the contents have been designed to match the first year requirements of most universities and polytechnics.

The layout of the texts is based on that of the well-known text, *Engineering Mathematics* by K. Stroud (first published by Macmillan in 1970, and now in its third edition). The remarkable success of this book owes much to the skill of its author, but it also shows that students greatly appreciate a book which aims primarily to help them to learn their chosen subjects at their own pace. The authors of this present series acknowledge their debt to Mr Stroud, and hope that by adapting his style and methods to their own subjects they have produced equally helpful and popular texts.

Before publication of each text the comments of a class of first year students, of some recent engineering graduates and of some lecturers in the field have been obtained. These helped to identify any points which were particularly difficult or obscure to the average reader or which were technically inaccurate or misleading. Subsequent revisions have eliminated the difficulties which were highlighted at this stage, but it is likely that, despite these efforts, a few may have passed unnoticed. For this the authors and publishers apologise, and would welcome criticisms and suggestions from readers.

Readers should bear in mind that mastering any engineering subject requires considerable effort. The aim of these texts is to present the material as simply as possible and in a way which enables students to learn at their own pace, to gain confidence and to check their understanding. The responsibility for learning is, however, still very much their own.

G.E. Drabble

HOW TO USE THIS BOOK

This book is one of a series which has been designed to help you in learning the basic subjects of a first-year course in Engineering. You may have seen similar texts before, but if not, then you need to understand that they are all written in short sections, called frames. Each frame normally contains just one or two facts that you need to understand or to apply, before going further. So the texts are designed to be read one frame at a time, in order, without skipping. Frequently, a frame will finish with a question, or a short exercise, to test that you have understood the work up to that point. You should always attempt the appropriate response to such endings: answer the question asked; undertake the exercise; write down your version of the formula or theory asked for, and so on. When your response is incorrect, find out why as soon as you can, either by re-reading the previous work, or by getting help from someone else. If you cannot do this immediately, make a note, and do so as soon as you have the opportunity.

A note on accuracy. The margin of error of most engineering calculations is of the order of 1 %, and some calculations may have a margin of error nearer to 5 %. One reason for this is that the data available are often not known to greater accuracy, and it is impossible for the accuracy of a calculation to be higher than that of the data used. Normally, three-figure accuracy is sufficient for most engineering calculations. But the purpose of this text is to help you to learn engineering theory. If, in the text, a wheel reaction force on a car is given as 2127.5 N, this does not mean that the answer is accurate to that extent. But if you check the calculations yourself (as you should frequently do), you will know you are doing them correctly if you arrive at the same answer. Of course, you cannot always expect to reach exactly the same answer; the order in which the operations are performed, and even the make of calculator, may result in small variations.

CONTENTS

Programme 1

REVISION

1

In this first programme, we deal with the simple treatment of **vectors**, the basic laws of **statics**, elementary **kinematics**, and elementary **kinetics**. You may have covered this work before, but look through the contents of it quickly, even if you do not work through all the examples and problems, so that if there are any points about which you are not sure, you have a chance to brush them up a little. Begin by writing down the definitions of vector, statics, kinematics and kinetics. Remember: always try to answer the questions asked before referring to the answers given, which are usually in the following frame.

2

A **vector**	quantity is one which possesses both magnitude and direction.
Statics	is the general study of force systems.
Kinematics	is the study of motion without regard to the forces which cause it.
Kinetics	is the study of forces, and their relation to motion.

You may have defined Statics as 'The study of bodies at rest' or something similar. You were not far wrong, but as you will see, we often have to use the techniques of Statics when dealing with moving bodies as well as stationary ones. More of this later. We shall now look at Vectors. Write down four physical quantities which must be expressed as vectors. If you can't think of four, write down as many as you can.

3

Displacement is a vector.
Velocity is a vector.
Acceleration is a vector.
Force is a vector.

You may have thought of several not on the list, particularly if you are keener on electrical theory than on mechanical. But these are the four that we shall be most concerned with in this book. We shall eventually need to add vectors together—forces, velocities, and accelerations. This will not be difficult, but we have to stick to the rules. What is the principle to be observed when we add vectors?

4

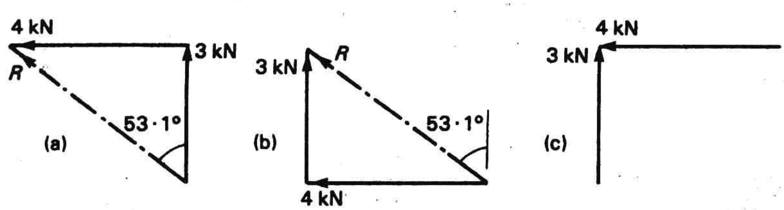
Vectors must be added graphically

The vectors must be drawn, end-to-end, correct as to length and direction, the arrows on them all running the same way. This, of course, is to take account of the *direction* of the vector as well as its magnitude. What would be the result of adding a displacement of 3 km north to a displacement of 4 km west?

5

5 km in direction 53.1 deg. west of north

The solution is shown below; both diagrams (a) and (b) are correct; the vectors may be taken in either order. The resultant vector is shown as a chain-line, to distinguish it from the component vectors, and the arrow on this vector opposes the direction of those on the components. Diagram (c) shows how *not* to add the two vectors; the arrows are not running the same way.



When the diagram is very simple, as here, we obtain the result by calculation. We need only to sketch the diagram, and not to draw it accurately. We used the words 'resultant' and 'component' above. When a number of vectors are added together, the answer is called the **resultant**: each of the vectors is called a **component** of the result.

6

We now turn to **Statics** and recall how to find the resultant of forces acting at a point. Here is a simple problem. Try to solve it before turning over.

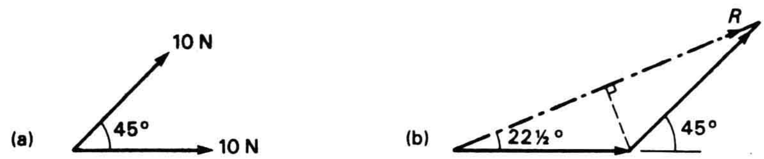
Example. A body is subjected to two forces, both of magnitude 10 N and the second at 45 deg. to the first. What is the magnitude and direction of the resultant force?

The answer to the first part (the magnitude) can be calculated. Just sketch the vector diagram, and do not try to draw it accurately.

7

18.48 N midway between the two forces

Here is the sketch. (a) shows the two forces, and (b) is the vector diagram.



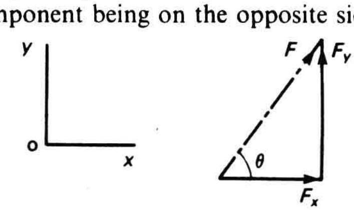
If we draw the perpendicular from the apex of the isosceles triangle, it is then easy to see that $R = 2 \times 10 \cos 22\frac{1}{2}° = 18.48$ N. (Or you can use the Cosine formula. We shall look at this in Frame 14.)

8

It is always more accurate to calculate than to draw and measure. This brings us to force **resolution**. This consists of replacing a single force by two components, acting in two directions mutually at right-angles. In the figure below, the single force F is expressed as the two components F_x and F_y in the x and y directions shown. The magnitudes of the two components are:

$$F_x = F \cos\theta; \qquad F_y = F \sin\theta$$

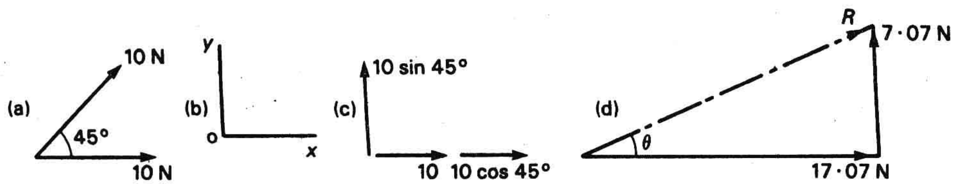
Some people find this difficult to remember; they become confused as to which component is which. It might help to remember that the **COS** component is **ClOSe** to the angle (the sine component being on the opposite side of the triangle).



So, to find the resultant of a number of forces acting at a point, we choose an arbitrary set of x- and y-axes, calculate the x and y components of every force, add all the x components, add all the y components, and finally, calculate the resultant of the two final components. For practice, solve the problem of Frame 6 again, using this technique of resolution. The answer is, of course, the same as before: 18.48 N at $22\frac{1}{2}°$. You can choose any set of x- and y-axes you want, of course, but it is clearly simpler to choose directions which will make for the easiest calculation. This means that you choose either horizontal and vertical axes or axes at 45° to the horizontal and vertical. In the solution following, we have chosen horizontal and vertical axes.

9

Here is the diagram: (a) shows the two forces; (b) shows x- and y-axes, arbitrarily horizontal and vertical; (c) shows the x and y components; and (d) the vector diagram.



The resultant x and y components, F_x and F_y are given by:

$$F_x = 10 + 10\cos 45° = 17.07\text{ N}; \qquad F_y = 10\sin 45° = 7.07\text{ N}$$

The resultant, R, of these components is calculated from the triangle in (d):

$$R = \sqrt{\{(17.07)^2 + (7.07)^2\}} = \mathbf{18.48\text{ N}}$$

and angle θ from:

$$\theta = \tan^{-1}(7.07/17.07) = \mathbf{22.50°}$$

10

Here are three statements concerning the **Polygon of Forces.**

1. If a number of co-planar forces acting at a point are in equilibrium, they may be represented, in magnitude and direction, by the sides of a closed polygon.
2. If a number of co-planar forces acting at a point are not in equilibrium, the single extra force required to produce equilibrium (the **equilibrant**) may be found from the single vector needed to close the polygon.
3. If a number of co-planar forces acting at a point are not in equilibrium, the **resultant** force is the force which is equal in magnitude to the equilibrant but opposite in direction.

The example solved in Frame 7 makes use of the third statement. Any system of co-planar forces acting at a point may be solved by means of a force polygon. But drawing is time-consuming, and can be inaccurate, and it is usually preferable to use the technique of force resolution. There are some occasions, however, where a force polygon solution is preferable. You will find one in the problems which follow in Frame 16.

11

The principle of the **Parallelogram of Forces** establishes that the resultant of two forces acting at a point may be represented vectorially by the diagonal of the parallelogram of which the two forces are represented by two adjacent sides.

The principle of the **Triangle of Forces** established that when three co-planar forces acting at a point are in equilibrium, they can be represented vectorially by the three sides of a triangle.

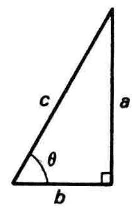
But you don't need to bother about these two principles, because the Polygon of Forces is a more general statement which includes both of them, and which covers any number of co-planar forces acting at point, whether in equilibrium or not. Force resolution is a method of mathematically analysing the geometry of a force polygon. Sometimes, although rarely, it may be desirable to draw the polygon accurately. and solve by actual measurement.

12

Solution of Statics problems, and indeed, of all problems in engineering theory, requires a certain fluency in mathematics. You have probably discovered yourself that if you fail to get the 'book' answer to a problem, the reason is often a mathematical slip. The only final answer to this is, practice, and more practice, but the following suggestions may be of some help.

1. Keep work *neat*, even if working a problem in 'rough'. Experience shows a high correlation between untidy scribbling and mathematical slips. Perform your procedures steadily and carefully, and set down your work neatly and clearly.
2. If you fail to obtain the correct answer, begin the solution again, on a fresh sheet, not referring to your first attempt at all. If possible, look for an alternative way of solving. (If you come up again with the same answer, there is always the possibility that the 'book' answer may be wrong.).
3. Fractions are traps for the careless worker. Example: A car travels at 15.6 m s^{-1}. To travel 12 m takes $(15.6/12) = 1.3$ seconds! If you used this calculation to determine the time to travel 1 metre, the incorrect answer of 15.6 seconds would most probably be obvious, but in this case, it could easily be overlooked.

We mentioned the Cosine formula in Frame 7. This, with the Sine formula, is valuable in solving triangles, that is, determining the lengths of all sides and values of all angles. Frames 13 and 14 are devoted to a statement of the trigonometric formulae you are most likely to encounter in this book.



$$\sin\theta = a/c;\ \cos\theta = b/c;\ \tan\theta = a/b$$

This may seem rather basic, but many students get them wrong. It may help to think that **COS** θ is calculated by taking side b of the triangle ClOSe to the angle, while $\sin\theta$ uses the side which iS IN front of θ.

It is easily seen from the triangle that:

$$\sin\theta = \cos(90° - \theta);\ \cos\theta = \sin(90° - \theta);\ \tan\theta = \sin\theta/\cos\theta$$

Some special values (found by analysing 30–60–90 and 45–45–90 triangles):

$$\sin 30° = \cos 60° = \tfrac{1}{2};\ \sin 60° = \cos 30° = \tfrac{1}{2}\sqrt{3} = 0.866$$
$$\tan 30° = \tfrac{1}{3}\sqrt{3} = 0.5774;\ \tan 60° = \sqrt{3} = 1.732$$
$$\sin 45° = \cos 45° = \tfrac{1}{2}\sqrt{2} = 0.707;\ \tan 45° = 1$$

Other special values:

$$\sin 0° = 0;\quad \cos 0° = 1;\quad \tan 0° = 0$$
$$\sin 90° = 1;\quad \cos 90° = 0;\ \tan 90° = \infty \text{ (infinity)}$$
$$\sin 180° = 0;\ \cos 180° = -1$$

$$\sin(180° - \theta) = \sin\theta;\ \cos(180° - \theta) = -\cos\theta$$

The angle-summation identities:

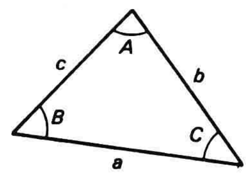
$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B};\qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

The sine and cosine formulae are given in the following frame, but try and write them down yourself first.

14



We use a special notation of the triangle for the sine and cosine formulae. The sine formula:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The cosine formula:

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = c^2 + a^2 - 2ca \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

15

Here is a summary of the work so far.

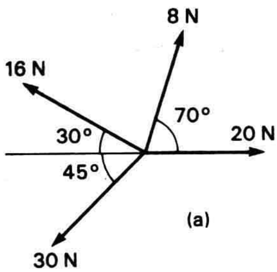
1. Force is a vector quantity, having magnitude and direction.
2. Most systems of co-planar forces acting at a point may be analysed by the method of resolution:

   (a) Choose a set of x- and y-axes at right-angles.
   (b) Calculate the x and y components of all forces.
   (c) Calculate the algebraic sum of the x and y components, F_x and F_y.
   (d) For a system in equilibrium, both these will be zero.
   (e) For a non-equilibrium system, the sums of (c) will be the components of the resultant, R, directed at an angle θ to the x-axis. Then:

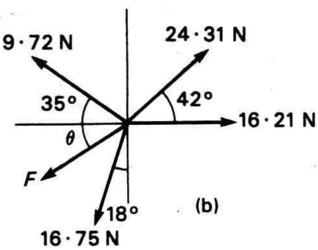
$$R = \sqrt{(F_x^2 + F_y^2)}; \qquad \theta = \tan^{-1}(F_y/F_x)$$

3. When resolution does not give a ready solution, the geometry of the polygon may provide an alternative analytical solution. The Sine and Cosine formulae may be found helpful.

PROBLEMS

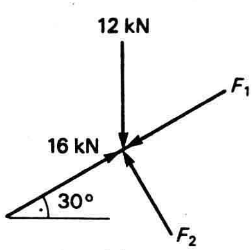


(a)

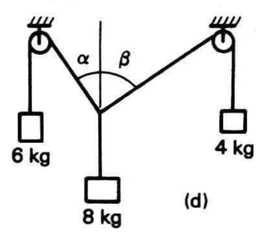


(b)

1. Diagram (a) shows forces of 20 N, 8 N, 16 N and 30 N acting at a single point in relative directions of 0°, 70°, 150° and 225°. Calculate the magnitude and direction of the resultant of the four forces. [*Ans. 13.59 N at 204.8°.*]
2. Five forces acting at the same point are shown in diagram (b) above. Given that the forces are in equilibrium, determine the magnitude of the force F, and the value of the angle θ. [*Ans. 21.95 N; 15.63°.*]
   Hint: write equilibrium equations along horizontal and vertical, giving $F \cos\theta$ and $F \sin\theta$. Divide one by the other to get $\tan\theta$ and hence θ. Substitute in either equation to find F.



(c)



(d)

3. Diagram (c) shows a joint of a roof member, with a vertical load of 12 kN acting on it. The force in the lower inclined member is 16 kN compressive. Evaluate the magnitudes of the two forces F_1 and F_2. [*Ans. 10.0 kN, 10.39 kN.*]
   Hints: the forces at the joint are in equilibrium. Do *not* resolve along horizontal and vertical directions, but along, and perpendicular to the slope. One equation will give F_1, and the second, F_2.
4. Three hanging weights are connected by strings so that two strings pass over pulleys, while the third hangs free as shown in diagram (d) above. The masses of the weights are shown. Assuming the pulleys frictionless, determine the inclinations, α and β, of the strings to the vertical. [*Ans. 28.96°, 46.57°.*]
   Hint: sketch the polygon of the three forces acting at the string junction, and use the cosine rule to determine the angles. (See Frame 14.)