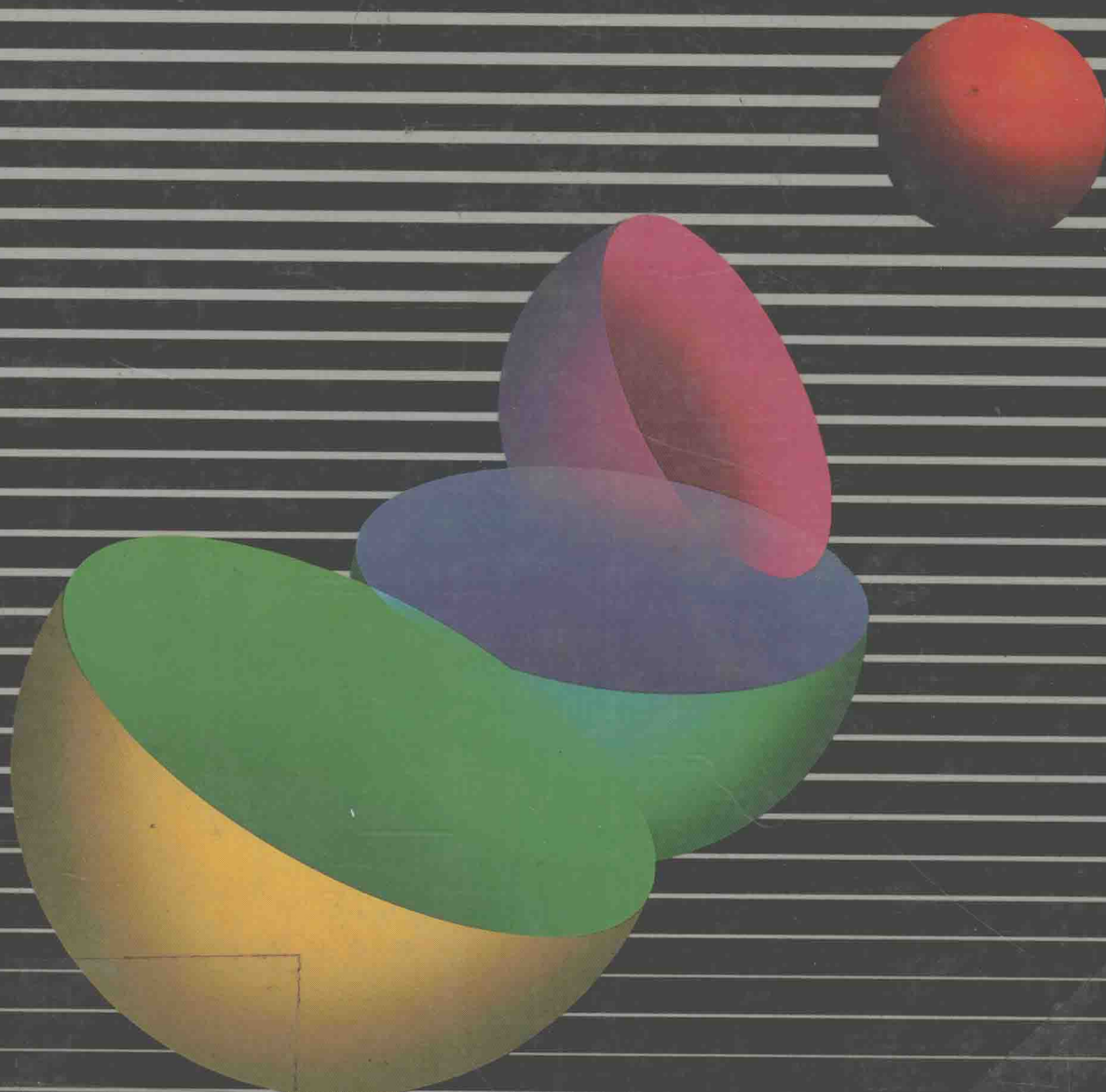


# College Algebra

S E C O N D E D I T I O N

Larson/Hostetler



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# **College Algebra**

**SECOND EDITION**

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## Preface

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Success in college level mathematics courses begins with a good understanding of algebra and the goal of this text is to help students develop this understanding. Although we review some of the basic concepts in algebra, we assume that most students in this course will have completed two years of high school algebra.

### *What's New in the Second Edition*

Many users of the first edition of the text have given us suggestions for improving the text. We appreciate this type of input very much and have incorporated most of the suggestions into the Second Edition. *Every* section in the text has been revised and many sections were completely rewritten. Most sections in the Second Edition have more exercises than were in the First Edition. The major changes are as follows.

In Chapter 1, we combined the first three sections of the First Edition, postponed the discussion of intervals on the real line to Section 2.7, and postponed the discussion of complex numbers until Section 2.5. In Chapter 2, we expanded the section on solving inequalities to two sections (Sections 2.7 and 2.8). Chapter 3 contains earlier coverage of lines in the plane (Section 3.3). The introduction of functions in Section 3.4 was completely rewritten. The discussion of transformations of graphs of functions has been improved (Section 3.5). We expanded the coverage of composition and inverse functions to two sections (Sections 3.6 and 3.7). In Chapter 4, we added a discussion of Descartes's Rule of Signs to Section 4.4. Chapter 6 was extensively reorganized and rewritten. More emphasis is now given to solving exponential and logarithmic equations (Section 6.4), and we included a new section on applications of exponential and logarithmic equations (Section 6.5). In Chapter 7, we expanded the material on systems of inequalities and linear programming to two sections (Sections 7.4 and 7.5). The first three sections of Chapter 8 were extensively revised, and Section 8.6 now contains Cramer's Rule as well as other applications of matrices and determinants.

## Features of the Second Edition

The Second Edition contains many features that we have found help students improve their skills and acquire an understanding of the material.

*Graphics* Skill in visualizing a problem is a critical part of a student's ability to solve the problem. This text contains over 775 figures. Of these, approximately 235 are in the exercise sets and approximately 275 are in the odd-numbered answers in the back of the text. The art package for the Second Edition is completely new. Every graph in the text was computer generated for the greatest possible accuracy.

*Applications* Throughout the Second Edition we have included many applied problems that give students insight about the usefulness of algebra in a wide variety of fields including business, economics, biology, engineering, chemistry, and physics.

*Examples* The Second Edition contains 575 examples, each carefully chosen to illustrate a particular concept or problem-solving technique. Each example is titled for quick reference, and many examples include color side comments to justify or explain the steps in the solution.

*Exercises* Over 4725 exercises are included in the Second Edition. These are designed to build competence, skill, and understanding. Each exercise set is graded in difficulty to allow students to gain confidence as they progress. To help students develop skills in analytic geometry, we stress a graphical approach in many sections and have included numerous graphs in the exercises.

*Warm up Exercises* [New in the Second Edition] We have found that students in algebra can benefit greatly from reinforcement of previously learned concepts. Most sections in the text contain a set of ten Warm up exercises that give students practice in the "old skills" that are necessary to master the "new skills" presented in the section. *All* of the Warm up exercises are answered in the back of the text.

*Calculators* Hints and instructions for working with calculators occur in many places in the Second Edition. Because calculators have become commonplace, we no longer identify exercises that require decimal approximations.

*Algebra of Calculus* Special emphasis has been given to algebraic skills that are needed in calculus. In addition to the material in Section 1.7, many other examples in the Second Edition discuss algebraic techniques that are used in calculus. These examples are clearly identified.

*Remarks* In the Second Edition we include special instructional notes to students called *Remarks*. These appear after definitions, theorems, or examples and are designed to give additional insight, help avoid common errors, and describe generalizations.

## Supplements

- For students, the *Study and Solutions Guide* by Dianna L. Zook contains detailed solutions of approximately 40% of the odd-numbered exercises in the text. Each of these is indicated in the text by a box surrounding the exercise number. This guide also contains summaries of important concepts for each section and self-tests for each chapter.
- For students, *Computer Activities for Precalculus* is an IBM-PC®\* package that offers activities based on programs that enhance the learning of such topics as linear functions, quadratic functions, exponential and logarithmic functions. Exploratory practice and directed tutorials are included as well as a function grapher.
- For instructors, the *Complete Solutions Guide* contains solutions for all of the exercises in the text.
- For instructors, the *Instructor's Guide* by Meredythe M. Burrows contains sample tests for each chapter in the text and suggestions for classroom instruction.
- For instructors, we have prepared test-generating software called *HeathTest* to accompany the text. This software will run on an IBM-PC® that has at least 256K of memory. (It will also run on many IBM compatibles.) To print the tests, the software requires an IBM graphics-compatible dot matrix printer. An Apple II®† version of the software is also available.
- For instructors, we have prepared a package containing 25 two-color transparencies of figures from the text.

## Acknowledgements

We would like to thank the many people who have helped us at various stages of preparing the First and Second Editions of this text. Their encouragement, criticisms, and suggestions have been invaluable to us.

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If you have suggestions for improving the text, please feel free to write to us. Over the past several years we have received many useful comments from both instructors and students and we value this very much.

Roland E. Larson  
Robert P. Hostetler



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# ***The Larson and Hostetler Precalculus Series***

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To accommodate the different methods of teaching college algebra, trigonometry, and analytic geometry, we have prepared four volumes. Each has its own supplement package. These four titles are described below.

## ***College Algebra, Second Edition***

This text is designed for a one-term course covering standard topics such as algebraic functions and their graphs, exponential and logarithmic functions, systems of equations, matrices, determinants, sequences, series, and probability.

## ***Trigonometry, Second Edition***

This text is used in a one-term course covering the trigonometric functions and their graphs, exponential and logarithmic functions, and analytic geometry (including polar coordinates and parametric equations).

## ***Algebra and Trigonometry, Second Edition***

This book combines the content of the two texts mentioned above (with the exception of polar coordinates and parametric equations). It is comprehensive enough for a two-term course, or, with careful selection, may be used in a one-term course.

## ***Precalculus, Second Edition***

With this book, students cover the algebraic, exponential and logarithmic, and trigonometric functions and their graphs, as well as analytic geometry in preparation for a course in calculus. This may be used in a one- or two-term course.

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## Introduction to Calculators

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This text includes several examples and exercises that use a scientific calculator. As we encounter each new calculator application, we will give instructions for using a calculator efficiently. These instructions are somewhat general and may not agree precisely with the steps required by your calculator.

For use with this text, we recommend a calculator with the following features.

1. At least 8-digit display
2. Four arithmetic operations:  $+$ ,  $-$ ,  $\times$ ,  $\div$
3. Change sign key:  $\pm$
4. Memory key and Recall key: **STO**, **RCL**
5. Parentheses:  $($ ,  $)$
6. Exponential key:  $y^x$
7. Natural logarithmic key: **ln**
8. Pi and Degree-Radian conversion:  $\pi$ , **DRG**
9. Inverse, reciprocal, square root: **INV**,  $1/x$ ,  $\sqrt{x}$

One of the basic differences in calculators is their order of operations. Some calculators use an order of operations called RPN (for Reverse Polish Notation). In this text, however, all calculator steps will be given using *algebraic logic*. For example, the calculation

$$4.69[5 + 2(6.87 - 3.042)]$$

can be performed with the following steps.

$$4.69 \times ( 5 + 2 \times ( 6.87 - 3.042 ) ) =$$

This yields the value 59.35664. Without parentheses, we would work from the inside out with the sequence

$$6.87 - 3.042 = \times 2 + 5 = \times 4.69 =$$

to obtain the same result.

## Rounding Numbers

For all their usefulness, calculators do have a problem representing numbers because they are limited to a finite number of digits. For instance, what does your calculator display when you compute  $2 \div 3$ ? Some calculators simply truncate (drop) the digits that exceed their display range and display .66666666. Others will round the number and display .66666667. Although the second display is more accurate, *both* of these decimal representations of  $2/3$  contain a rounding error.

When rounding decimals, we use the following rules.

1. Determine the number of digits of accuracy you wish to keep. The digit in the last position you keep is called the **rounding digit**, and the digit in the first position you discard is called the **decision digit**.
2. If the decision digit is 5 or greater, round up by adding 1 to the rounding digit.
3. If the decision digit is 4 or less, round down by leaving the rounding digit unchanged.

Here are some examples. Note that we round down in the first example because the decision digit is 4 or less, and we round up in the other two examples because the decision digit is 5 or greater.

	<i>Number</i>	<i>Rounded to three decimal places</i>	
(a)	$\sqrt{2} = 1.4142136 \dots$	1.414	<i>Round down</i>
(b)	$\pi = 3.1415927 \dots$	3.142	<i>Round up</i>
(c)	$\frac{7}{9} = 0.7777777 \dots$	0.778	<i>Round up</i>

One of the best ways to minimize error due to rounding is to leave numbers in your calculator until your calculations are complete. If you want to save a number for future use, store it in your calculator's memory.

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- 1.2 Integer Exponents
- 1.3 Radicals and Rational Exponents
- 1.4 Polynomials and Special Products
- 1.5 Factoring
- 1.6 Fractional Expressions
- 1.7 Algebraic Errors and Some Algebra of Calculus
- Review Exercises

## Review of Fundamental Concepts of Algebra

### 1.1 The Real Number System

We begin our study of algebra with a look at the **real number system**. Real numbers are used in everyday life to describe quantities like age, miles per gallon, container size, population, and so on. To represent real numbers we use symbols such as

$$9, -5, 0, \frac{4}{3}, 0.6666\ldots, 28.21, \sqrt{2}, \pi, \text{ and } \sqrt[3]{-32}.$$

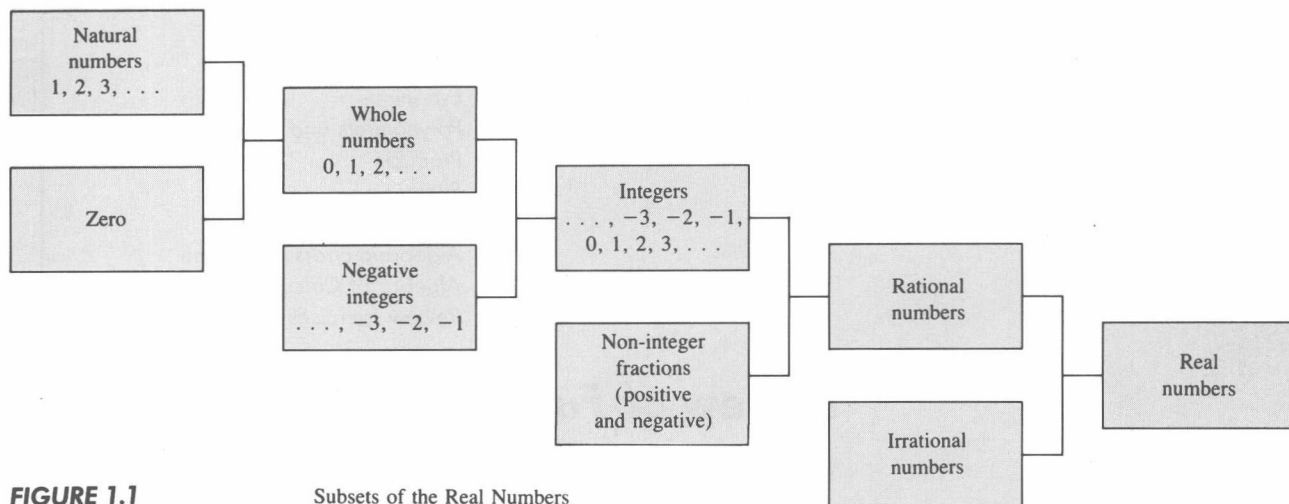
The set of real numbers contains some important subsets with which you need to be familiar. For instance, the numbers

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

are called **integers**. A real number is called **rational** if it can be written as the ratio  $p/q$  of two integers, where  $q \neq 0$ . For instance, the numbers

$$\frac{1}{3} = 0.3333\ldots, \quad \frac{1}{8} = 0.125, \quad \text{and} \quad \frac{125}{111} = 1.126126\ldots$$

are rational. The decimal representation of a rational number either repeats (as in  $3.1454545\ldots$ ) or terminates (as in  $1/2 = 0.5$ ). Real numbers that



**FIGURE 1.1**

Subsets of the Real Numbers

cannot be written as the ratio of two integers are called **irrational**. For instance, the numbers

$$\sqrt{2} \approx 1.4142136 \quad \text{and} \quad \pi \approx 3.1415927$$

are irrational. (The symbol  $\approx$  means “approximately equal to.”) Several subsets of real numbers are shown in Figure 1.1.

## Arithmetic Operations

There are four arithmetic operations with real numbers: **addition**, **multiplication**, **subtraction**, and **division**, denoted by the symbols  $+$ ,  $\times$  (or  $\cdot$ ),  $-$ , and  $\div$ . Of these, addition and multiplication are considered to be the two primary arithmetic operations. We summarize their properties as follows.

### Properties of Addition and Multiplication

Let  $a$ ,  $b$ , and  $c$  be real numbers. Then the following properties are true.

- |                  |   |  |
|------------------|---|--|
| 1. Closure:      | $a + b$ is a real number.   | Addition   |
|                  | $a \cdot b$ is a real number.   | Multiplication                                       |
| 2. Commutative:  | $a + b = b + a$   | Addition   |
|                  | $a \cdot b = b \cdot a$   | Multiplication                                       |
| 3. Associative:  | $(a + b) + c = a + (b + c)$   | Addition   |
|                  | $(a \cdot b) \cdot c = a \cdot (b \cdot c)$                           | Multiplication                                       |
| 4. Identity:     | $a + 0 = a = 0 + a$   | 0 is the additive identity.                          |
|                  | $a \cdot 1 = a = 1 \cdot a$   | 1 is the multiplicative identity.                    |
| 5. Inverse:      | $a + (-a) = 0 = (-a) + a$   | $-a$ is the additive inverse of $a$ .                |
|                  | $a\left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right)a, a \neq 0$ | $\frac{1}{a}$ is the multiplicative inverse of $a$ . |
| 6. Distributive: | $a(b + c) = ab + ac$  | Left Distributive Property                           |
|                  | $(a + b)c = ac + bc$  | Right Distributive Property                          |



**Remark:** Multiplication is implied when no symbol is used between two letters or groups of letters. For instance, we can write  $ab$  instead of  $a \cdot b$ , and  $a(b + c)$  instead of  $a \cdot (b + c)$ .

### EXAMPLE 1 Properties of Addition and Multiplication

(a) The statement

$$(3 + 6) + 8 = 3 + (6 + 8)$$

is justified by the Associative Property of Addition. Roughly speaking, this property tells us that parentheses are not needed when writing the sum of several real numbers. In other words, we can write this sum as  $3 + 6 + 8$  without ambiguity because we obtain the same sum whether we first add 3 and 6, or 6 and 8.

(b) The statement

$$2(5 + 3) = 2 \cdot 5 + 2 \cdot 3$$

is justified by the Distributive Property, or more formally by the *left distributive property of multiplication over addition*.

Subtraction and division are defined as the inverse operations of addition and multiplication, respectively.

**Subtraction:**  $a - b = a + (-b)$

**Division:** If  $b \neq 0$ , then  $a \div b = a\left(\frac{1}{b}\right) = \frac{a}{b}$ .

In these definitions,  $-b$  is called the **negative** (or additive inverse) of  $b$ , and  $1/b$  is called the **reciprocal** (or multiplicative inverse) of  $b$ . In place of  $a \div b$ , we often use the fraction symbol  $a/b$ . In this fractional form,  $a$  is called the **numerator** of the fraction and  $b$  is called the **denominator**.

**Remark:** Be sure you see the difference between the *negative of a number* and a *negative number*. If  $b$  is already negative, then its additive inverse,  $-b$ , is positive. For instance, if  $b = -5$ , then  $-b = -(-5) = 5$ .

The following three lists summarize the basic properties of negation, zero, and fractions. When you encounter such lists, we suggest that you not only *memorize* each property, but also try to gain an *intuitive feeling* for the validity of each.