Applied Combinatorial Mathematics

Edited by Edwin F. Beckenbach

APPLIED COMBINATORIAL MATHEMATICS

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To Professor Clifford Bell, Head
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in recognition of his imaginative and tireless devotion
to the Engineering and Physical Sciences Lecture Series

this book is affectionately dedicated
at the time of his retirement

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FOREWORD

Engineering achievement depends on the extent to which knowledge generated through research, in universities, in industry, and in government, knowledge expanded through the use of knowledge in industry, and knowledge handed to us through the ages is utilized effectively and at the proper time.

Modern studies in biological, social, physical, and mathematical sciences are uncovering exciting problems in combinatorial mathematics, a subject that is concerned with arrangements, operations, and selections within a finite or discrete system. It includes problems of systems analysis, information transmission, behavior of neural networks, and many others. These problems are yielding to new attacks, based in part on the availability of high-speed automatic computers. To keep pace with this progress, University Extension, Engineering and Physical Sciences Divisions, offered a Statewide Lecture Series on Applied Combinatorial Mathematics in the spring of 1962. This book is an outgrowth of the lecture series; it presents valuable aspects of the underlying theory and also some significant applications of this increasingly important and vital subject.

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PREFACE

. . . We will therefore refer to this group of problems as those of organized complexity.

Warren Weaver, "Science and Complexity," *American Scientist* **36** (1948), 538

In the article quoted above, Warren Weaver first points out that the physical sciences of the sixteenth, seventeenth, and eighteenth centuries were largely concerned with the analysis of two-variable, or few-variable, problems: The relation between distance and gravitational force, between voltage and electric current, between pressure and the volume of a gas, and so on. These great problems were those of *simplicity*. The life and social sciences were still largely in the preliminary, observational stages of the scientific method.

At about the beginning of this century, however, the pendulum swung far in the other direction, and much scientific progress was made through statistical techniques in the analysis of problems of *disorganized complexity*. The exact solution of a ten-body problem, say the problem of the motion of ten pool balls on a pool table, can be quite complicated; but statistical mechanics can give good answers for average behavior when we are dealing with huge numbers of molecules or of subatomic particles. Statistical methods also are quite effective in some aspects of the life sciences, as exemplified by the general reliability of mortality tables.

This leaves the middle ground of *organized complexity*, which is largely in the scientific foreground today. The operation of a petroleum-processing plant or of a military organization might involve hundreds or even thousands of variables, but such a problem is tractable by modern mathematical techniques through the use of high-speed computing machines. In the same way, complex mathematical models of subsystems of human physiology, essential to space-age technology, are showing promise of far-reaching results in the diagnosis and prevention of disease.

For a simple example involving some typical combinatorial problems, let us consider a round-robin tennis tournament with a given number of players and a given number of courts. Is it possible to arrange the schedule so that no player participates twice consecutively? This is an *existence* problem. If there is such a schedule, how do we go about determining it? This is a *construction* or *evaluation* problem. For variety in subsequent tournaments, it might be desirable to list all the different possible schedules. This is an *enumeration* problem. Of all schedules, it might be desirable to hit on the most enjoyable one, as measured, for example, by sustaining interest through having the best players meet each other last. This is an *extremization* problem. Most combinatorial problems are of one or another of these types, although, of course, the distinction is not always precise.

Analytic problems, involving continuous variables, are often solved approximately through the use of digital computing machines. Thus these problems are of concern to the combinatorial mathematician. The first two chapters of this book are definitely machine-oriented; the rest are not. It is only this point that separates these two chapters from the other four chapters of Part 1, for all six chapters are concerned with computation and evaluation, just as all six are concerned with counting and enumeration.

The six chapters of Part 2 are also concerned with computational problems, but now the emphasis has turned toward the determination of the solution that, in some sense, is *best*. Similarly, the six chapters of Part 3 are concerned with these same problems, but with greater emphasis on the construction of examples of which the existence initially was more in doubt. The last portion of Part 3 deals also with problems of physical existence and finally with more philosophical considerations.

Unfortunately, two of the lecturers in the Statewide Lecture Series were unable to spare the time required for the preparation of manuscripts. These would have been concerned with error-correcting codes and network-flow problems, respectively. The former subject, however, is treated in part in the chapter on block designs; for the latter subject, the chapter contributed by the editor is a partial substitute. On the other hand, the material in the Lecture Series has been augmented by the addition of Chapters 5, 6, and 18, for the subjects treated in these chapters were not included in the Series.

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EDWIN F. BECKENBACH

August, 1964

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