

THIRD EDITION

# DOE Simplified

Practical Tools for Effective Experimentation

**Mark J. Anderson • Patrick J. Whitcomb**



CRC Press  
Taylor & Francis Group

A PRODUCTIVITY PRESS BOOK

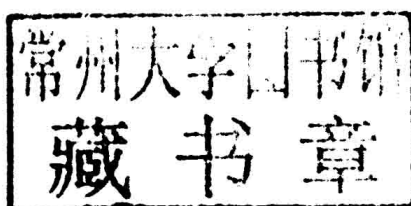


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Boca Raton London New York

CRC Press is an imprint of the  
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CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

© 2015 by Mark J. Anderson, Patrick J. Whitcomb  
CRC Press is an imprint of Taylor & Francis Group, an Informa business

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Printed on acid-free paper  
Version Date: 20141201

International Standard Book Number-13: 978-1-4822-1894-7 (Paperback)

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Library of Congress Cataloging-in-Publication Data

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Anderson, Mark J., 1953-  
DOE simplified : practical tools for effective experimentation / Mark J. Anderson and Patrick J. Whitcomb.  
-- Third edition.  
pages cm  
"A CRC title."  
Includes bibliographical references and index.  
ISBN 978-1-4822-1894-7  
1. Experimental design. I. Whitcomb, Patrick J., 1950- II. Title. III. Title: Design of experiments simplified.

QA279.A5287 2015  
658.4'033--dc23

2014046030

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THIRD EDITION

# DOE Simplified



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# Preface

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Without deviation from the norm, progress is not possible.

**Frank Zappa**

Design of experiments (DOE) is a planned approach for determining cause and effect relationships. It can be applied to any process with measurable inputs and outputs. DOE was developed originally for agricultural purposes, but during World War II and thereafter it became a tool for quality improvement, along with statistical process control (SPC). Until 1980, DOE was mainly used in the process industries (i.e., chemical, food, pharmaceutical) perhaps because of the ease with which engineers could manipulate factors, such as time, temperature, pressure, and flow rate. Then, stimulated by the tremendous success of Japanese electronics and automobiles, SPC and DOE underwent a renaissance. The advent of personal computers further catalyzed the use of these numerically intense methods.

This book is intended primarily for engineers, scientists, quality professionals, Lean Six Sigma practitioners, market researchers, and others who seek breakthroughs in product quality and process efficiency via systematic experimentation. Those of you who are industrial statisticians won't see anything new, but you may pick up ideas on translating the concepts for nonstatisticians. Our goal is to keep DOE simple and make it fun.

By necessity, the examples in this book are generic. We believe that, without much of a stretch, you can extrapolate the basic methods to your particular application. Several dozens of case studies, covering a broad cross section of applications, are cited in the Recommended Readings at the end of the book. We are certain you will find one to which you can relate.

*DOE Simplified: Practical Tools for Effective Experimentation* evolved from over 50 years of combined experience in providing training and computational tools to industrial experimenters. Thanks to the constructive feedback

of our clients, the authors have made many improvements in presenting DOE since our partnership began in the mid-1970s. We have worked hard to ensure the tools are as easy to use as possible for nonstatisticians, without compromising the integrity of the underlying principles. Our background in process development engineering helps us stay focused on the practical aspects. We have gained great benefits from formal training in statistics plus invaluable contributions from professionals in this field.

## What's New in This Edition

A major new revision of the software that accompanies this book (via download from the Internet) sets the stage for introducing experiment designs where the randomization of one or more hard-to-change factors can be restricted. These are called *split plots*—terminology that stems from the field of agriculture, where experiments of this nature go back to the origins of DOE nearly a century ago. Because they make factors such as temperature in an oven so much easier to handle, split-plot designs will be very tempting to many experimenters. However, as we will explain, a price must be paid in the form of losses in statistical power; that is, increasing the likelihood of missing important effects. After studying the new chapter on split plots, you will know the trade-offs for choosing these designs over ones that are completely randomized.

This edition adds a number of other developments in design and analysis of experiments, but, other than the new material on split plots, remains largely intact. The reviews for *DOE Simplified* continue coming in strongly positive, so we do not want to tamper too much with our system. Perhaps the biggest change with this third edition is it being set up in a format amenable to digital publishing. Now web-connected experimenters around the globe can read *DOE Simplified*.

Another resource for those connected to the Internet is the “Launch Pad”—a series of voiced-over PowerPoint® lectures that cover the first several chapters of the book for those who do better with audiovisual presentation. The goal of the Launch Pad is to provide enough momentum to propel readers through the remainder of the *DOE Simplified* text. The reader can contact the authors for more information about the Launch Pad.

After publication of the first edition of this book, the authors wrote a companion volume called *RSM Simplified: Optimizing Processes Using Response Surface Methods for Design of Experiments* (Productivity Press, 2004).

It completes the statistical toolset for achieving the peak of performance via empirical modeling. If *DOE Simplified* leaves you wanting more, we recommend you read *RSM Simplified* next.

We are indebted to the many contributors to development of DOE methods, especially George Box and Douglas Montgomery. We also greatly appreciate the statistical oversight provided by our advisors, University of Minnesota statistics professors Kinley Larntz and Gary Oehlert.

**Mark J. Anderson**  
([mark@statease.com](mailto:mark@statease.com))

**Patrick J. Whitcomb**  
([pat@statease.com](mailto:pat@statease.com))





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# Introduction

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There are many paths to enlightenment. Be sure to take one with a heart.

**Lao Tzu**

This book provides the practical tools needed for performing more effective experimentation. It examines the nuts and bolts of design of experiments (DOE) as simply as possible, primarily by example. We assume that our typical reader has little or no background in statistics. For this reason, we have kept formulas to a minimum, while using figures, charts, graphs, and checklists liberally. New terms are denoted by quotation marks and also are included in a glossary for ready reference. As a spoonful of sugar to make the medicine go down, we have sprinkled the text with (mostly) relevant text boxes. Please enjoy (or forgive) the puns, irreverent humor, and implausible anecdotes.

Furthermore, we assume that readers ultimately will rely upon software to set up experimental designs and do statistical analyses. Many general statistical packages now offer DOE on mainframe or personal computers. Other software has been developed specifically for experimenters. For your convenience, one such program accompanies this book. You will find instructions for downloading the software (and viewing its tutorials) at the back of the book. However, you must decide for yourself how to perform the computations for your own DOE.

Chapter 1 presents the basic statistics that form the foundation for effective DOE. Readers already familiar with this material can save time by skipping ahead to Chapter 2 or Chapter 3. Others will benefit by a careful reading of Chapter 1, which begins with the most basic level of DOE: comparing two things, or two levels of one factor. You will need this knowledge to properly analyze more complex DOEs.

Chapter 2 introduces more powerful tools for statistical analysis. You will learn how to develop experiments comparing many categories, such as various suppliers of a raw material. After completing this section, you will be equipped with tools that have broad application to data analysis.

Chapters 3 through 5 explain how to use the primary tool for DOE: two-level factorials. These designs are excellent for screening many factors to identify the vital few. They often reveal interactions that would never be found through one-factor-at-a-time methods. Furthermore, two-level factorials are incredibly efficient, producing maximum information with a minimum of runs. Most important, these designs often produce breakthrough improvements in product quality and process efficiency.

Chapter 6 introduces more complex tools for two-level factorials. Before you plow ahead, be sure to do some of the simpler factorials described in prior chapters. Practice makes perfect.

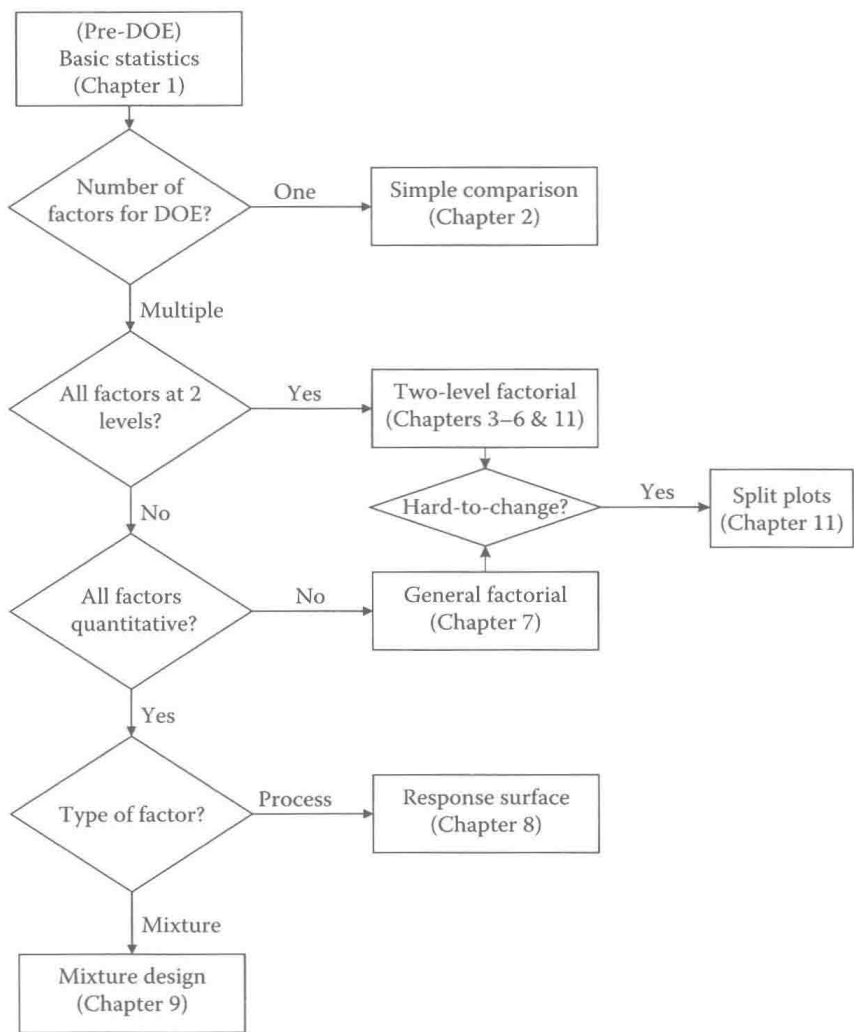
Chapter 7 goes back to the roots of DOE, which originated in agriculture. This chapter provides more general factorial tools, which can accommodate any number of levels or categories. Although these designs are more flexible, they lack the simplicity of focusing on just two levels of every factor.

At this point, the book begins to push the limits of what can be expected from a DOE beginner. Chapters 8 and 9 definitely go beyond the boundary of elementary tools. They offer a peek over the fence at more advanced tools for optimization of processes and mixtures. Because these final chapters exceed the scope of the working knowledge meant to be provided, “DOE Simplified,” we did not include practice problems. However, advanced textbooks, such as the companion volume to this book—*RSM Simplified: Optimizing Processes Using Response Surface Methods for Design of Experiments* (Productivity Press, 2004)—are readily available to those of you who want to expand your DOE horizons.

Chapter 11 brings readers back to the basics with keys to doing good DOE. It also provides a process for planning experiment designs that takes statistical power into account.

Chapter 12 details split plots, which, as explained in the Preface, provide a workaround for factors that experimenters find difficult to change in random fashion. However, the relief from randomization comes at the cost of power. Consider the trade-offs carefully.

The flowchart in Figure I.1 provides a chapter-by-chapter “map.” At the end of Chapters 1 through 7, you will find at least one practice problem. We strongly recommend that readers work through these problems (answers to which are posted on the Internet; see About the Software at



**Figure I.1** Flowchart guide to *DOE Simplified*.

the back of the book for details). As with any new tool, the more you know about it, the more effectively you will use it.

Our hope is that this book inspires you to master DOE. We believe that reading this book, doing the exercises, and following up immediately with your own DOE will give you a starting point, a working knowledge of simple comparative and factorial designs. To foster this “DOE it yourself” attitude, we detail several practice experiments in Chapter 12. No answers are provided because we do not want to bias your results, but you may contact us for data from our experiments.



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