ELEMENTARY ALGEBRA

EDGERTON and CARPENTER

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PREFACE

THIS new edition of *Elementary Algebra* has one supreme objective — to make the study of algebra a challenging, fascinating, intellectual adventure.

Algebra is far from being the impractical subject that outworn methods of teaching would make it appear. On the contrary, few other subjects are as basic to man's material or intellectual progress.

Where does the astronomer turn for symbols to express his discoveries of rhythm and proportion in the laws of the universe? To algebra. Where does the builder find the language to interpret the intricacies of his designs? In algebra. How does the student of social or economic affairs explain the laws of human relations? By means of the graph.

Each new adventure of the human mind, each new endeavor of the human hand at some point, finds its support in the principles of this ancient but ageless science.

Once the young student catches this vision of the subject, once he senses its practical, everyday values as well as its limitless ranges of exploration, he cannot resist the lure to learn more and more of its secrets.

This is the emphasis that *Elementary Algebra* seeks to secure for the teaching of the subject. This goal is the purpose of its new, historical introduction. The abundant drills, the matter-of-fact statements of rules, the simplicity of presentation, and clarity of coherence are means, not ends. They are merely the necessary aids to early competence in the use of symbols and understanding of principles. They will be taken in stride, without undue emphasis, if the supreme objective of algebraic adventure is kept in mind.

The treatment of *graphs* is unique. Graphs are of two kinds: illustrative arithmetical graphs, like the statistical graphs, and functional, algebraic graphs, like those of the formula and the equation. The former are disposed of in the general Introduction, where they form a link between the mathematics of the previous grades and the present algebra. The latter are inserted in the text in connection with the topic they exemplify, instead of being grouped in a separate chapter which breaks the sequence of the book.

Tests of all kinds abound throughout the book. Each chapter has at least two, and these are chosen (as selection, multiple-choice, completion, timed, true-false, matching, and so on) for their adaptability to the subject matter treated.

Geometry also finds an important place. Lines, angles, triangles, polygons, and circles are all treated in their algebraic applications at appropriate points.

The *illustrations* are meant to add life and interest to the work. In this pictorial age pupils want their work illustrated wherever possible. This is done by pictures, by graphs, and by blocked types of specific solutions.

Illustrative examples supplement the simple, orderly explanations by which each topic is introduced.

Cumulative reviews with each chapter apply in well graded problems all the principles previously treated. These reviews are supplemented by an equation review in each chapter.

The *examples and problems* are a special feature. There are more than in any other one book on the subject.

The text has profited by suggestions from teachers throughout the country. In particular, the authors acknowledge with gratitude the helpful advice of Mr. Joseph P. McCormack, Chairman of the Mathematics Department, Theodore Roosevelt High School, New York City.

E. I. E.
P. A. C.

THE BEGINNINGS OF ALGEBRA

WOULD it surprise you to know that the beginnings of algebra are in the very remote past? Or that a mere beginner in algebra today — perhaps just such a beginner as you — may know more about the subject than the wisest mathematician of those early days? Or that scholars of the Middle Ages were so proud of their solutions and proofs that they guarded them as carefully as inventors guard their secrets today? Or that the subject of algebra was such an interesting one, that points of dispute in the solutions of vexing problems were often debated in public before an audience of interested listeners?

These facts are all true, difficult as it may be to realize them.

The name *Algebra* comes from the title of an Arabic treatise of about 800 A.D. In this essay, algebra is defined as the "science of restoration and equation." Now we think of algebra simply as that branch of mathematics in which letters represent quantities, and signs (+, −, =, etc.) represent relationships between the quantities.

But algebra as a science is much older than the ancient article from which it got its name. Indeed, the subject is treated in the oldest written record of mathematics, the Ahmes Papyrus. This manuscript was written about 1650 B.C. and it was copied by its Egyptian scribe from a much older record, dating perhaps from 2500 B.C. However, algebraic solutions of these early dates were perhaps achieved by shrewd guess-work fully as much as by scientific principle.

Little was done to develop algebra as a science until the Italian mathematicians of the sixteenth century got interested in the subject. Strangely enough, such equations as the cubic and the biquadratic were solved long before it was discovered how easily the use of letters could be made to represent known, as well as unknown, quantities.

A knowledge of algebra was slow in growth, because of the wide variety of symbols used. The first appearance of a "plus" and "minus" was in a book by Widman, published in Leipzig in 1489. It is believed that he adopted these two symbols from an old Latin manuscript. The "Whetstone of Witte," an algebra printed in 1557 by the Englishman, Robert Recorde, also used the signs, plus and minus.

When algebra began to be applied to geometry by Descartes in 1637, the meaning of negative numbers began to be really appreciated.

In all the changes that have come in the world from two thousand years before Christ to two thousand years after, the study of algebra has never ceased. As part of the field of mathematics, it has made possible the world we live in today.

If some of your study of algebra seems a little hard, try to remember the thousands of years of progress behind it and the patient work of the famous men who discovered its laws. Be grateful to them for their never-ceasing efforts to advance the science. Be as patient and persistent in your study as they were. If you are, you will soon discover that algebra is one of the most fascinating and useful of all your studies.

ELEMENTARY ALGEBRA

INTRODUCTION. STATISTICAL GRAPHS

How do we get the news of what is going on in the world? One way is to hear about daily activities over the radio. But hearing news has one disadvantage. Never mind how

Courtesy Philadelphia Evening Bulletin

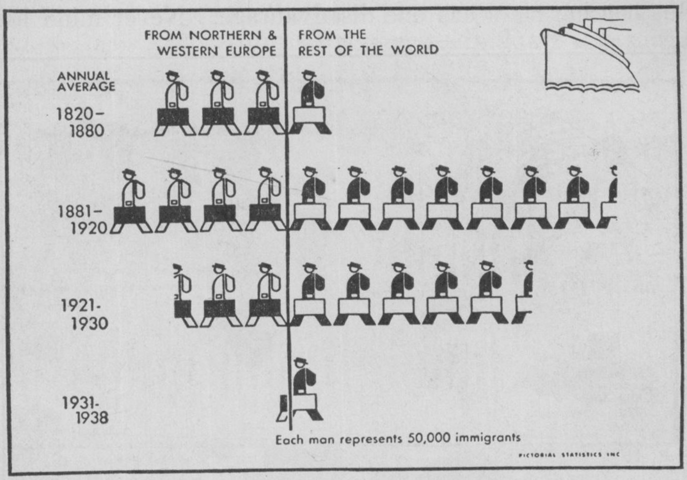
A WORLD TIME ZONE MAP

A pictograph of when and where news happened in one particular day.

dramatically or clearly such news is presented, the voice is gone in a flash and cannot be recalled.

Another important source of news works through our eyes — we see events in newsreels at the movies, or we read the news from a newspaper, or book, or magazine. The latter method has several advantages. We can take our time to consider it thoughtfully when the news is printed. We can compare it carefully with other accounts. We can turn back and reread any item as often as we please.

During the past few years an avalanche of news magazines of the pictorial type has descended upon our newsstands.



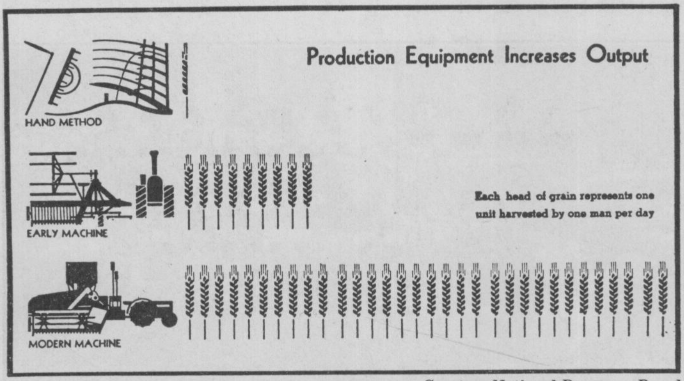
IMMIGRATION TO THE UNITED STATES

The popularity of this type of picture-book shows that the picture method of presenting facts is an excellent method of attracting interest. It insures clearness, and helps the reader to fix the facts more firmly in his mind.

Mathematicians observed the truth of this long ago. In showing relationships they found that a picture could tell a story much better and more vividly than any sentence or paragraph could possibly do. This is particularly true in algebra where letters stand for numbers and symbols stand for relationships between the numbers.

Graphs. — Any diagram which shows the relative size or magnitude of two or more quantities or the rate at which one quantity changes is called a *graph.*

Picture Graphs. — A mathematician makes use of *graphs* for his story-told-by-picture. A graph pictures the relation of two or more facts. It is often difficult to grasp all the relations in a numerical study of facts which is set up in tabulated form, but when these are expressed in picture form by means of a graph, the true relationships can be seen at once.



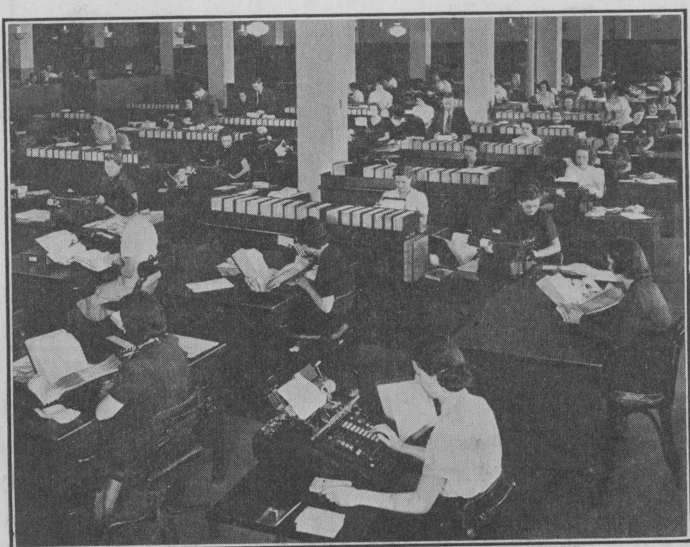
Courtesy National Resources Board

MACHINERY INCREASES THE OUTPUT OF AGRICULTURE

It is not only in mathematics books, however, that we find graphs used. They illustrate articles in magazines, daily papers, scientific magazines and official reports of various kinds. So it is quite necessary that we learn how to interpret them; that is, tell the meaning of the relations between the numbers graphed (pictured). As we go on in this book we shall learn how to graph formulas, equations, and other mathematical relations as well as the statistics in this Introduction.

The Statistical Graph. — The term *statistics* was first applied to collections of data relating to matters important to the state, such as numbers of population, yield of taxation, value of trade, and so on. Today, the field of statistics has widened, however, from affairs of state to data related to any branch of knowledge or investigation. We can, therefore, truthfully define statistics as a numerical study of social facts.

In business, in school, and in many other places facts about attendance, pupil's marks, wages, and other things must be recorded and tabulated. These facts become much clearer when pictured, because the *graph* shows vividly the relations between them.



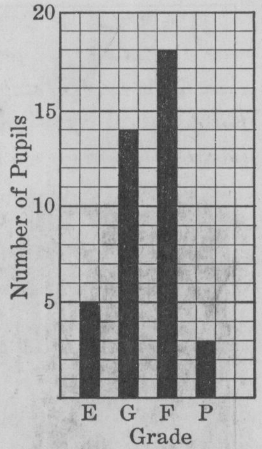
Courtesy National Cash Register Company

BILLING DEPARTMENT OF A LARGE DEPARTMENT STORE

The Bar Graph. — Another common form of graph consists of straight, heavy lines or bars of equal width, either vertical or horizontal, whose lengths represent the relative magnitude of the quantities compared. The graphs shown on the next page are typical bar graphs.

The bar graph we see here pictures the marks obtained in a certain test in mathematics. The figures at the left show the number of pupils. The letters at the bottom show the grades received. This is a *vertical* bar graph.

Interpretation.



This graph may be interpreted by studying the answers to the following questions.

How many pupils obtained

(a) E (90 to 100)?

(b) G (80 to 90)?

(c) F (70 to 80)?

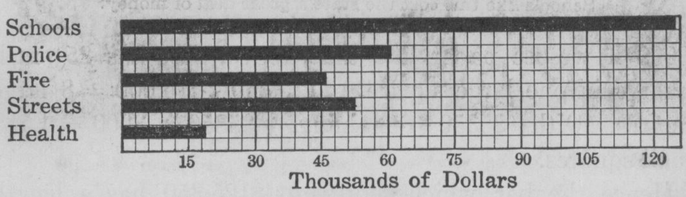
(d) P (Failed)?

(e) How many took the test?

(f) What per cent failed?

When bar graphs represent large numbers, they are not supposed to be absolutely accurate.

Below is a *horizontal* bar graph, to illustrate the following costs. In a certain city the cost of the public schools was \$125,350; police \$60,100; fire department \$45,800; streets and parks \$52,600; public health \$18,250. To picture the



relation of these costs by the use of bar graphs, we draw a heavy vertical line near the left side of our graph sheet. This is a specially ruled paper, very convenient in making graphs. We make all the bars start evenly at that line. The longest bar must represent \$125,350. The longest that we can make it with the paper that we are using is 9 large squares. Each large square then must represent at least $\frac{1}{9}$ of \$125,350, which is nearly \$14,000. Let us then make one large square represent \$15,000 and then each small square here will represent

Galloway

EAST HIGH SCHOOL, DENVER

Schools like this cost the state a great deal of money.

\$3000 if we use paper like that above, that has 25 small squares in one large square. \$125,350 ÷ 15,000 = 8 large squares, with \$5350 remaining. \$5350 ÷ 3000 = 1.78^+ small squares.

Hence the bar, which represents \$125,350, has a length equal to 8 large squares and a little more than $1\frac{3}{4}$ small squares. In a similar way we divide \$60,100 by 15,000 to find the bar representing police expenses; and so on for the other departments.

Interpreting the Graph on page xi.

(a) Which department costs the most?

(b) It costs about twice as much as the . . .

(c) Compare the costs of police, fire, and street departments (approximately).

Suggestions for Making Bar Graphs.

1. Use a sharp pencil, and a ruler.

2. Measure accurately.

3. Choose a unit fitted to the paper that you have. The unit is the number that the smallest square represents. In the preceding graph it was 3000.

4. In a *horizontal* bar graph, the scale begins at the left and goes to the right.

5. In a *vertical* bar graph, the scale begins at the bottom and goes upward.

6. As we have learned, in bar graphs the lengths of the bars are proportional to the magnitudes of the quantities represented. It is necessary, therefore, to choose a unit such that there will be room on the paper for the longest bar. Squared paper can be used to best advantage in doing this work, especially if it is ruled 10 squares to the inch. In all graph work try to arrange your chart on the paper in such a way as to have equal width of margin of the paper at the sides, and equal width of margin at the top and bottom of the graph.

Exercise

Draw bar graphs to represent each of the following. Interpret each graph.

1. The number of pupils present in a home-room for 5 successive days was 32, 30, 31, 32, 27.

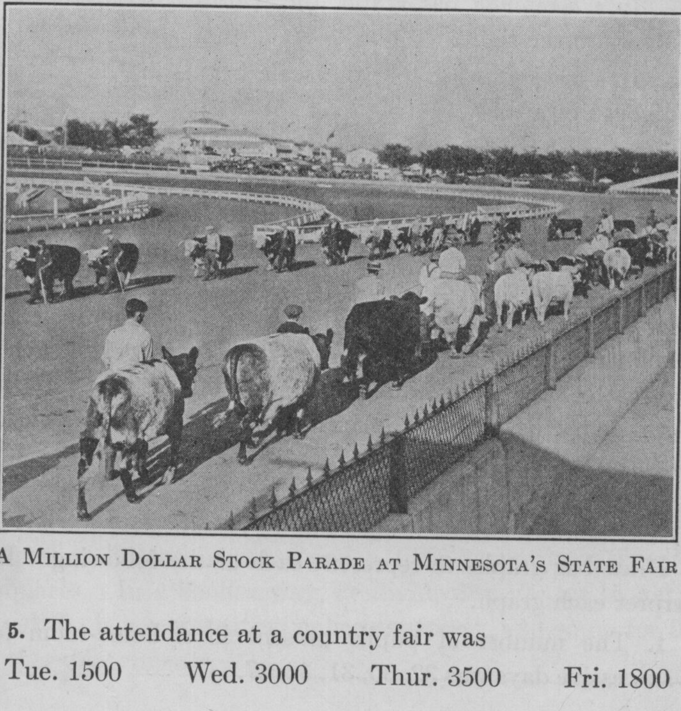
2. Ten pupils of an algebra class receiving the following marks in an examination: 100, 60, 90, 80, 95, 93, 80, 78, 92, 85.

3. The production of cotton in the U. S. to the nearest million bales.

1921 — 8	1923 — 10	1925 — 16	1927 — 13
1922 — 10	1924 — 14	1926 — 18	1928 — 14

4. The number of people in the U. S. with no education at all (called illiterate) as given by the U. S. Census reports in millions.

1880 — 6.2 1890 — 6.3 1900 — 6.2 1910 — 5.5 1920 — 4.9



A Million Dollar Stock Parade at Minnesota's State Fair

5. The attendance at a country fair was

Tue. 1500 Wed. 3000 Thur. 3500 Fri. 1800

6. The causes of deaths in N. Y. City in a recent year and the figures:

Cancer	7033	Pneumonia . . .	10521
Children's diseases .	1002	Tuberculosis . . .	4828
Heart	15784	Violence	5987
Kidney	4361	Other	2099

Use vertical bars. 1 inch might represent 2000.

7. A family's ice bill for 6 months beginning with June was

Month . . .	1	2	3	4	5	6
Cost . . .	\$1.25	\$2.50	\$2.25	\$1.75	\$1.50	\$1.00

8. The distances that different kinds of trucks can travel on \$1 expenditure has been reported as follows:

5-ton horse . .	1.6 miles	$3\frac{1}{2}$-ton gas . .	2.4 miles
5-ton gas . .	1.8 miles	$3\frac{1}{2}$-ton electric .	2.75 miles
5-ton electric. .	2.3 miles	2-ton horse . .	2.93 miles
$3\frac{1}{2}$-ton horse . .	2.2 miles	2-ton gas . .	2.6 miles

2-ton electric . 3.3 miles

9. Expenditures (cost of government) of N. Y. State, in millions.

1921 — 136	1923 — 136	1925 — 167	1927 — 189
1922 — 130	1924 — 151	1926 — 178	1928 — 218

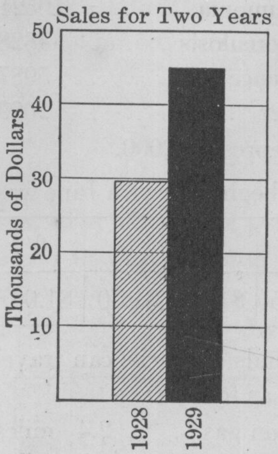
10. Debt in N. Y. State, in millions.

1921 — 268	1923 — 264	1925 — 318	1927 — 341
1922 — 267	1924 — 308	1926 — 315	1928 — 360

11. An object, falling freely, falls the following number of feet in the respective number of seconds.

Time (Sec.) .	0	1	2	3	4	5
Falls (Feet) .	0	16	64	144	256	400

The Double (Dual-) Bar Graph is used to compare two similar items for two respective years or two related items for the same year.



This graph shows the relation between the sales of the same salesman for two years, — the light bar representing the sales of one year and the shaded bar those of the other year.

Interpretation

How many thousand dollars' worth of goods did he sell each year?

His 1929 sales were how much more than his 1928 sales?

Do you think such a salesman should keep his job? Why?

Exercise

Make dual-bar graphs to represent:

1. The sales of one salesman whose sales for two different years were \$10,000 and \$14,000 respectively. The corresponding selling expenses for the two years were \$1400 and \$1625. Make two pairs of dual bars.

2. The rough materials used in manufacturing in a certain year in millions of dollars were exports 1.9, imports 1.8. (From and to the U. S.)

3. The capital and production in millions of dollars were: 1st year C, 78, P, 86; 2d year (40 years later) C, 5000, P, 1500.

4. The gasoline tax in the U. S. for 1927 and 1928 was 259 millions and 280 millions of dollars respectively. The