

**MECHANICAL
BEHAVIOUR
OF
COMPOSITES
AND
LAMINATES**

Edited by

W. A. GREEN and M. MIĆUNOVIĆ

ELSEVIER APPLIED SCIENCE

MECHANICAL BEHAVIOUR OF COMPOSITES AND LAMINATES

Edited by

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ELSEVIER APPLIED SCIENCE
LONDON and NEW YORK

ELSEVIER APPLIED SCIENCE PUBLISHERS LTD
Crown House, Linton Road, Barking, Essex IG11 8JU, England

Sole Distributor in the USA and Canada
ELSEVIER SCIENCE PUBLISHING CO., INC.
52 Vanderbilt Avenue, New York, NY 10017, USA

WITH 23 TABLES AND 126 ILLUSTRATIONS

© ELSEVIER APPLIED SCIENCE PUBLISHERS LTD 1987

British Library Cataloguing in Publication Data

European Colloquium 214 Mechanical Behaviour
of Composites and Laminates (1986:

Yugoslavia)

Mechanical behaviour of composites and
laminates

1. Composite materials

I. Title II. Green, W. A. III. Mićunović, M.

620'.1'18 TA418.9.C6

ISBN 1-85166-144-1

Library of Congress CIP data applied for

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Printed in Great Britain by Galliard (Printers) Ltd, Great Yarmouth

MECHANICAL BEHAVIOUR OF COMPOSITES AND LAMINATES

Proceedings of the European Mechanics Colloquium 214 'Mechanical Behaviour of Composites and Laminates' held in Kupari, Yugoslavia, 16-19 September 1986.

PREFACE

The first announcement for Euromech Colloquium 214 on the Mechanical Behaviour of Composites and Laminates indicated the intention of the Chairmen to provide a forum for experimentalists and theoreticians to report on the current state of development in this rapidly expanding field. The invitation called for contributions concerned with four main topics. These were: the formulation of constitutive equations; the experimental determination of mechanical response; wave propagation and vibrations; and methods of solution of boundary value problems.

We believe that the papers contained in this report of the proceedings amply demonstrate the wide-ranging response to this call. The participants at the Colloquium included materials scientists, engineers, physicists, applied mathematicians and pure mathematicians and the papers in turn reflect this variety of disciplines which contribute to the study of composites and laminates.

In compiling the papers for this volume we have arranged them into five groups. The first group consists of nine contributions dealing with the topics of edge effects, impact damage and fracture criteria including both experimental and theoretical aspects of these topics. This is followed by a set of six papers devoted to the theoretical study of wave propagation and vibration. The third group consists of six articles concerned with homogenization theory applied to derive mathematical models of inhomogeneous media and structures. The four papers comprising the next group are devoted to the derivation of constitutive equations and the solution of boundary value problems for non-linear and inelastic composites. Finally there is a set of five contributions concerned with numerical methods and optimization. This grouping is by no means exclusive and many of the articles which we have assigned under one of these headings could equally well have been allocated under another. We would therefore encourage the reader whose main field of interest may be covered by one of these group headings to explore the possibilities in the remaining groups.

In the discussion session which formed the closing stage of the Colloquium a number of the participants laid emphasis on the interdisciplinary nature of the study of composites. The topic covers materials science, mathematical modelling and mechanics and structural analysis. There was a feeling that these aspects had

in the past been considered in isolation but that some attempts at coordination were now under way. On the mechanics aspect, stress was laid on the interrelation between theory, experiment and analysis. In this respect it was felt that the meeting had been of value in bringing together practitioners from each of these three areas, in a relaxed, informal atmosphere which gave the opportunity for fruitful interaction. We believe that we speak for all the participants when we say that the presentations were both stimulating and informative. Each session of talks evoked a lively response in terms of questions and discussion and one of the exciting features of the meeting was the chance to see the wider aspects of the field in relation to one's own work. We are convinced that the papers contained in this compilation make a significant contribution to the research activity in this field and we welcome this opportunity to make them accessible to a wider audience. In doing so, we earnestly hope that the reader will experience the enthusiasm and enjoyment for their subjects displayed by the authors which was one of the features of the Colloquium.

It is a pleasure to thank Miss Gordana Avramović, Mr Nenad Grujović and Dr Dragan Milosavljević whose hard work and enthusiasm ensured the smooth running of the sessions and the welfare of the delegates. They were assisted in the preparatory work by Mr Miroslav Živković and to him also we express our thanks. We are grateful to the Serbian Scientific Council, the Regional Scientific Society of Sumadija and the University of Svetozar Marković, Kragujevac, for their financial assistance towards the costs of the meeting and to the management of the Hotel Kupari complex for affording us the use of their conference facilities. Finally we acknowledge with thanks the secretarial assistance of Mrs Dušanka Žugić and Mrs Anne Perkins.

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EDGE EFFECTS IN FAILURE OF COMPRESSION PANELS

by

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Summary

A theoretical and experimental investigation is conducted into the behaviour after initial buckling of a series of carbon-composite compression panels. The panels are loaded to failure which is found to be precipitated by interlaminar shear stresses near the edge due to both membrane forces and twisting moments. Failure is predictable using either a fine finite-element model or a simple boundary layer approximation. The effect of these edge stresses is to reduce the strength of the panels by about 40%.

Introduction

The Aircraft Industry has pioneered the use of high grade fibre-composite structures in the pursuit for optimistic performances. Carbon composites in particular should be attractive in view of their high specific stiffness, which is the governing material parameter for obtaining high buckling resistance in thin-walled or slender structures. However the development of carbon composite designs for structures in compression has been slow and their full potential has not been realised. 'Knock-down' factors of 2 have been used by industry on permissible strains simply to play safe on strength criterion, with the result that the new lithium-aluminium alloys look competitive even though their specific stiffness is a moderate 20% better than traditional alloys. The shortfall in compressive performance of carbon composites has been due largely to three factors:

- (a) Commonly used epoxy resin matrix materials do degenerate when warm and moist and this leads to destabilisation of the fibres. This environmental degradation can be accelerated if the structure suffers moderate impact damage.
- (b) Composite laminates do have in-built imperfections and this can lead to strength reductions in imperfection-sensitive structures like some shells or stiffened panels which are optimised for coincident buckling modes.
- (c) Thin-walled compression panels are usually stabilised by deploying discretely spaced stiffeners of some variety to reduce the effective breadth/thickness (b/t) ratio. Such discontinuities inevitably raise the spectre of three-dimensional stress fields - the Achilles heel of all composite laminate constructions.

The first weakness is being overcome with newer matrix materials, and the last two should be solved by better detailed design and production technology, particularly since the new high-strain carbon fibres offer potentially 50% more strength. There is therefore every incentive to understand, and be able to predict, the mechanisms involved in the failure of stiffened compression panels.

Buckling of Stiffened Panels

The behaviour of compression panels stiffened, or supported along their longitudinal edges, is well understood for homogeneous isotropic metals. Thus as the load is increased the panel will buckle and lose stiffness at a critical stress σ_{cr} proportional to $E(t/b)^2$ where E is an effective modulus. If the b/t ratio is large, this initial buckling stress may be much below any potential material failure, and considerable **post-buckling** strength may be exploited. The immediate drop in stiffness after initial buckling is of order $1/3$ (see Fig. 7) and as the panel is further loaded the central region remains dormant whilst the extra load is carried by the regions near the supports or stiffeners. Eventual failure may be a further instability mode of the support-stiffener, a pure material failure, or a combination of both. A characteristic buckling mode of a long panel is shown in Fig. 1, where the main feature to observe is the shedding of membrane load N_{xx} to the edge supports, particularly in the region of the buckle crests. Also shown is a typical distribution of twisting moment M_{xy} which is a maximum at the node lines. This feature will be shown to be important.

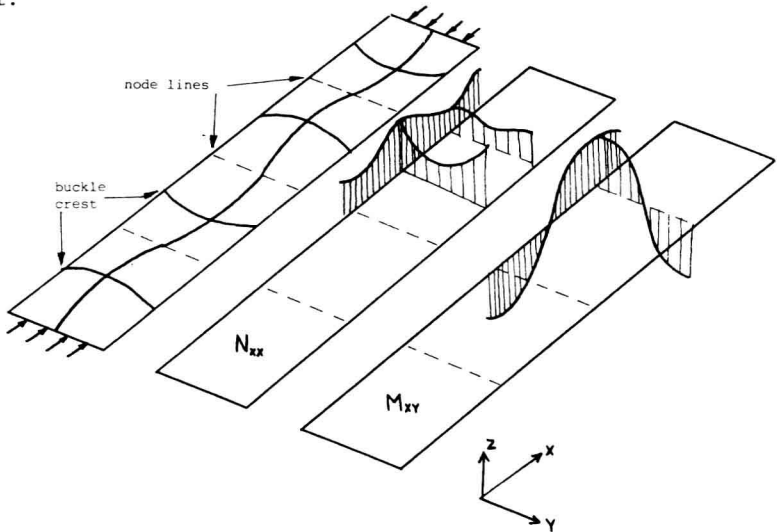


Fig. 1. Post-buckling Modes and Stress Distributions in a long panel

In carbon composite panels very little work has been reported but the behaviour after the initial buckling load must be radically different from familiar metal behaviour which usually involves a gradual reduction in the panel stiffness, leading to ultimate failure. The reduced post-buckled stiffness will remain fairly constant. Failure is likely to be sudden, without warning, and will probably destroy the evidence of the failure mechanism.

The prediction of post-buckling stiffness is possible using classical laminate theory and the plate equations for moderately large displacements. However, to predict the ultimate load capacity we need to know the location and the mechanism of the failure process, and in the case of post-buckled panels the source may be near a supported edge where the stress concentrations of Fig. 1 are highest. In addition to the membrane action

N_{xx} , the post-buckling deformation will feed upon the initial imperfections and produce twisting moments M_{xy} and edge shears Q_y all of which are likely to interact with three-dimensional edge effects - the object of this study.

Experimental work by Starnes (1) has identified the source of failure along the edge at the node lines of Fig. 1. This partially contradicts the above assertion since the membrane compression is not a maximum here, and the Kirchhoff shear ($Q_y - \partial M_{xy}/\partial x$) is actually zero! Industry has therefore been understandably cautious, and to the authors' knowledge no military aircraft wing-box has been designed with post-buckled strength. Other structures such as fuselage panels, control surfaces, and civil aircraft wings have b/t ratios in the range 20 to 50 where the ultimate strength may be up to 3 times the initial buckling value. Tests have been undertaken by the U.K. industry and failure originating at the node points has been confirmed (2). The source has been proposed as the twisting moments M_{xy} which produce high fibre strains in the 45° fibres near the surface. A finite element analysis was used to evaluate N_{xx} and M_{xy} and the consequent maximum strains were evaluated in the surface fibres. The ultimate strength was correctly predicted using a permissible fibre strain of 5300 microstrain. However this value is unrealistically low for multiply laminates and no attempt was made to predict failure for stacking sequences with 0° surface fibres. A programme has therefore been completed by the authors to identify the source of postbuckling failure and validate some analytical predictive capability (3,4).

Experimental Tests

Panels have been tested having b/t ratios in the range 20 to 60, and having aspect ratios between 3 and 8; consequently their behaviour is close to that of infinitely long panels. The loaded ends were clamped and the edges were simply supported. Three layups were used of 16-ply, medium stiffness XAS 914 prepreg, arranged in 'quasi-isotropic' sequences as follows:-

$$(0/90/+45)_2S; \quad (0/+45/90)_2S; \quad (45/0/-45/90)_2S$$

These three layups produce quite different through-thickness variations in any edge boundary layer.

A 'hard' displacement-controlled testing machine was used to avoid destroying the panels once failure started. Nevertheless it was very necessary to use an acoustic emission system to provide a signal of imminent failure and so enable us to examine and monitor the failure process. This technique was extremely successful. Fig. 2a shows that ring-down counting is an accurate prophet of composite failure, even though the dramatic increase in noise may occur only 5% below the failure load. The plates were removed for ultrasonic scanning and then reloaded again until further emission indicated new damage. The C-scans were most revealing and completely repeatable for several tests. They showed progressive delamination sites at the edges near the node lines and occurring at the 45° interfaces near the plate neutral axis as shown in Fig. 2b. Fig. 3 shows the pattern of delamination sites and a typical C-scan. Propagation of the delamination is extremely rapid in compression, since the plate is effectively reduced to two separate plates having only 1/8 of the original rigidity and buckling resistance.

The delamination interfaces occurred at positions above and below the neutral surface in an alternating fashion along the edge sides, and this pattern clearly corresponded to the changing sign of the twisting moment M_{xy} along the edges. The delamination source was confirmed to be a shear, rather than tensile peeling, from a fractographic survey of the face

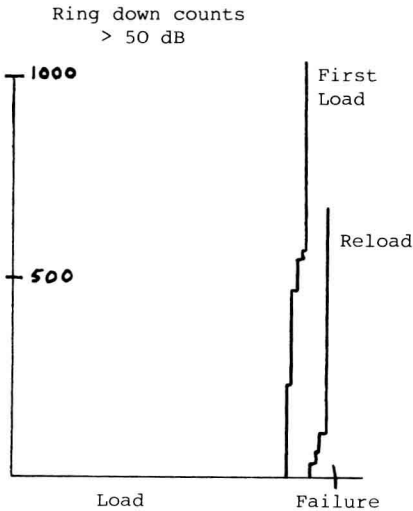


Fig. 2a. Acoustic Emission

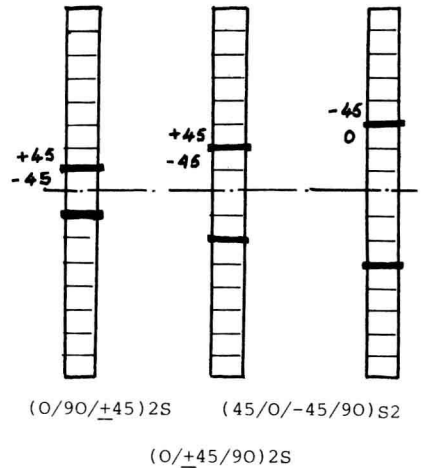


Fig. 2b. Delamination Sites

performed by the Royal Aircraft Establishment. [See ref. 5 for details of this technique.] The fact that shear delamination takes place between $+45^\circ$ layers and -45° or 0° layers is not surprising since these are the weaker interfaces, but the fractographic evidence showed the component σ_{xz} to be the driving shear. It is usual to look for the normal component σ_{yz} as a source of edge delamination.

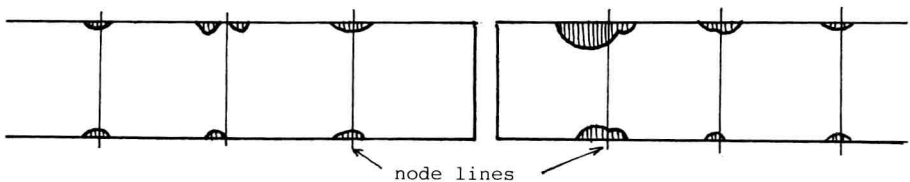


Fig. 3. Delamination Sites and Ultrasonic C-scan

The reasons for this behaviour, the dependence on M_{xy} , and an explanation for the sites near the neutral axis rather than the plate surface (where M_{xy} produces the largest fibre strains) needs an investigation into the nature of the edge effects and the three-dimensional stress field in a boundary layer at the plate edge.

Edge Effects in Composite Plates

The presence of a boundary layer zone, in which interlaminar stresses σ_z , τ_{xz} , τ_{yz} occur, is well known and has been covered by Pipes & Pagano amongst others (6,7). Full finite-element analyses have been used to substantiate the variation in these edge stresses and to confirm that the boundary layer extends an order of the plate thickness t from the edge. Previous arguments will not be repeated here but they have concentrated on the effects of longitudinal membrane loading, and in particular on the consequent peeling stress σ_z which is likely to cause internal delamination. A simplified analysis is possible using equilibrium arguments to evaluate the interlaminar stresses but some assumption has to be made on the profile of their variation away from the edge. We show later that this assumption is an important feature.

A similar edge effect due to the in-plane shears τ_{xy} , induced by M_{xy} , was suspected. Classical laminate and plate-bending theory will predict these as a linear variation with z . However, these shear stresses must decay to zero at the free edge, and it is well known (8) that they are turned through a right angle into τ_{xz} components in the "Kirchoff" Boundary Layer of thickness order t . The decay of τ_{xy} and the sympathetic rise of τ_{xz} was thought to be the mechanism for the onset of edge delamination.

The Kirchoff boundary layer in isotropic metals predicts both shear stresses to be of order M_{xy}/t^2 but this cannot be true in composites where the two components have differing order of stiffness. The in-plane shears τ_{xy} are resisted largely by the 45° angle-ply layers and have a respectable effective modulus G_{xy} compared with the transverse modulus G_{xz} which is largely dependent on the matrix stiffness. An approximate analysis in the Appendix shows this, and also demonstrates that the boundary layer now extends a distance of order $t(G_{xy}/G_{xz})^{1/2}$, which in the panels tested is in excess of $3t$.

It was decided therefore to resort to a three-dimensional finite element analysis to confirm these speculations, but it was clearly necessary to control the number of elements used. The failure locations had been identified as the regions near the node lines where both M_{xy} and N_{xx} have local maxima. The variation in these two forces occurs over a distance of order panel-width b which is large compared to t , and consequently it is justifiable to isolate a small region where the variations in M_{xy} and N_{xx} are ignored. Fortunately there exists a program developed by Bartholomew and Mercer (9) for analysing this two-dimensional flexure-torsion problem using a bolt-on constraint to any standard finite-element program - in this case MSC-NASTRAN was used. The problem was idealised as a doubly-symmetric finite strip of width $4t$ in which effectively the boundary layers on two edges are brought together and it is hoped that diffusion to plate behaviour is complete at the centre line. Fig. 4 shows the scheme. The elements used were the 8-node rectangular CHEXA, using 3 elements through each lamina to capture through-thickness variations. The full mesh shown deploys 1600 elements which taxed the resources of a CRAY 1S, and explains why a rather risky diffusion length of $2t$ was assumed. The values of N_{xx} and M_{xy} were taken from a separate finite element analysis of the whole plate corrected for the small shortfall between these

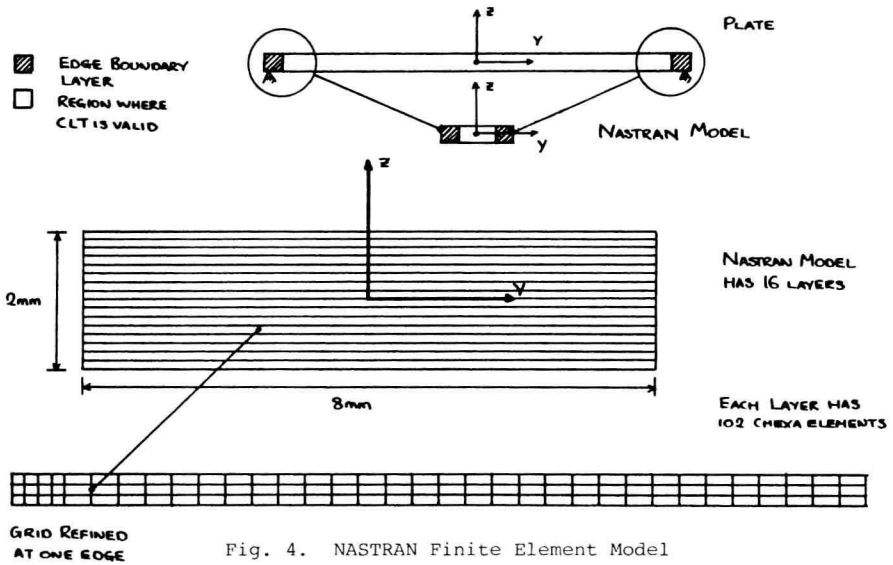


Fig. 4. NASTRAN Finite Element Model

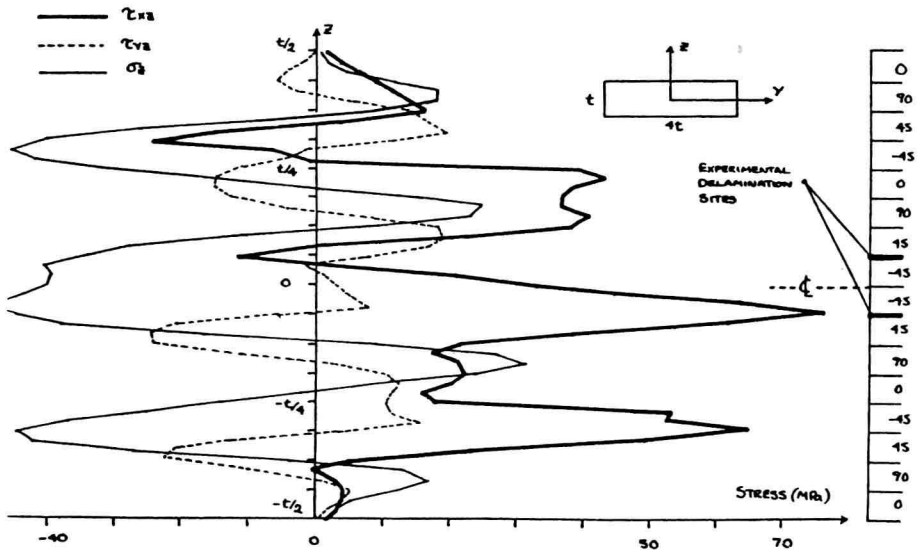


Fig. 5. Edge Stresses from Finite Element Model

$$(N_{xx} = -523 \text{ N/mm}; M_{xy} = 130 \text{ N})$$

predictions and the experimental results (see Fig. 7). It was found that M_{xy} and N_{xx} contributed equally to the interlaminar shears for large values of b/t , but that as b/t was reduced to 20, the postbuckling strength then exceeded the initial buckling load by less than 10% and the role of the twisting moment reduced to about a half that of the compressive force. A typical picture of the complex boundary layer stress field is shown in Fig. 5 for one lay-up. The shear stress τ_{xz} clearly peaks at one delamination site (the other would occur if the sign of M_{xy} was switched) at a value of about 80 MPa. This compares with an interlaminar shear strength of 85 MPa measured from a short beam test for this layup, and which failed at the same interface. A summary of all failure predictions for all panels tested is shown in Table 1 and expressed as a Reserve Factor [actual strength divided by predicted]. The agreement is very satisfactory considering the complex nature of the stress field and the simplified maximum-shear-stress failure criterion which ignores any coupling between τ_{xz} and σ_{zz} for example. Particularly striking is the comparison between the predictions using the maximum-fibre-strain criterion which are excessively optimistic for most stacking sequences.

Aspect Ratio	b/t	Stacking Sequence	Reserve Factor	
			1	2
3	57	(0/90/+45)2S	1.16	1.58
3	57	(0/+45/90)2S	1.14	1.54
3	57	(45/0/-45/90)S2	0.80	1.32
5	35	(0/90/+45)2S	1.02	1.43
5.5	31	(0/+45/90)2S	0.99	1.35
6.9	25	(0/+45/90)2S	0.90	1.27
7.8	22	(0/+45/90)2S	1.03	1.57

1. Using interlaminar shear criterion
2. Using maximum fibre strain criterion

Table 1. Prediction of Panel Strength

Simplified Analytical Predictions

The finite element results seem to have correctly identified the failure interfaces, however the associated computing costs are high, and industry is more likely to use a simpler estimation based on arguments similar to Pipes and Pagano (6,7) for estimating edge effects due to N_{xx} . In the Appendix a simple boundary layer analysis is presented for M_{xy} loading. It is shown that it is necessary to know the profile of $\tau_{xy}(y)$ and a method is given for evaluating this in terms of the two shear moduli. The resulting predictions for τ_{xz} using equation (4) of the Appendix are shown in Fig. 6 and agree well with finite element values. Also presented are the usual simplified predictions for N_{xx} loading using the Pipes and Pagano arguments. However it was necessary to assume a profile again and here we took a bilinear approximation having the same maximum gradient as the finite element results. Other assumptions can easily change the predictions by a factor of 2 either way, and an analysis similar to the torsion approach would be preferable. It should also be borne in mind that there is a limit to the precision sought in estimating gradients at the edge, where the idealised discontinuities in section properties of laminars would theoretically predict singular behaviour. It will probably be better to ignore spurious singularities and use an energy-release argument for predicting propagation of an assumed interlaminar flow (12).