Shijun Liao

Homotopy Analysis Method in Nonlinear Differential Equations

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To my mother, wife and daughter

Preface

It is well-known that perturbation and asymptotic approximations of nonlinear problems often break down as nonlinearity becomes strong. Therefore, they are only valid for weakly nonlinear ordinary differential equations (ODEs) and partial differential equations (PDEs) in general.

The homotopy analysis method (HAM) is an analytic approximation method for highly nonlinear problems, proposed by the author in 1992. Unlike perturbation techniques, the HAM is independent of any small/large physical parameters at all: one can always transfer a nonlinear problem into an infinite number of linear sub-problems by means of the HAM. Secondly, different from all of other analytic techniques, the HAM provides us a convenient way to guarantee the convergence of solution series so that it is valid even if nonlinearity becomes rather strong. Besides, based on the homotopy in topology, it provides us extremely large freedom to choose equation type of linear sub-problems, base function of solution, initial guess and so on, so that complicated nonlinear ODEs and PDEs can often be solved in a simple way. Finally, the HAM logically contains some traditional methods such as Lyapunov's small artificial method, Adomian decomposition method, the δ-expansion method, and even the Euler transform, so that it has the great generality. Therefore, the HAM provides us a useful tool to solve highly nonlinear problems in science, finance and engineering.

This book consists of three parts. In Part I, the basic ideas of the HAM, especially its theoretical modifications and developments, are described, including the optimal HAM approaches, the theorems about the so-called homotopy-derivative operator and the different types of deformation equations, the methods to control and accelerate convergence, the relationship to Euler transform, and so on.

In Part II, inspirited by so many successful applications of the HAM in different fields and also by the ability of "computing with functions instead of numbers" of computer algebra system like Mathematica and Maple, a Mathematica package `BVPh` (version 1.0) is developed by the author in the frame of the HAM for nonlinear boundary-value problems. A dozen of examples are used to illustrate its validity for highly nonlinear ODEs with singularity, multiple solutions and multipoint boundary conditions in either a finite or an infinite interval, and even for some types of nonlinear PDEs. As an open resource, the `BVPh` 1.0 is given in this book with a simple users guide and is free available online.

In Part III, we illustrate that the HAM can be used to solve some complicated highly nonlinear PDEs so as to enrich and deepen our understandings about these interesting nonlinear problems. For example, By means of the HAM, an explicit analytic approximation of the optimal exercise boundary of American put option was gained, which is often valid for a couple of decades prior to expiry, whereas the asymptotic and perturbation formulas are valid only for a couple of days or weeks in general. A Mathematica code based on such kind of explicit formula is given in this book for businessmen to gain accurate results in a few seconds. In addition, by means of the HAM, the wave-resonance criterion of arbitrary number of traveling gravity waves was found, for the first time, which logically contains the famous Phillips' criterion for four waves with small amplitude.

All of these show the originality, validity and generality of the HAM for highly nonlinear problems in science, finance and engineering.

All Mathematica codes and their input data files are given in the appendixes of this book and available (Accessed 25 Nov 2011, will be updated in the future) either at

http://numericaltank.sjtu.edu.cn/HAM.htm

or at

http://numericaltank.sjtu.edu.cn/BVPh.htm

This book is suitable for researchers and postgraduates in applied mathematics, physics, finance and engineering, who are interested in highly nonlinear ODEs and PDEs.

I would like to express my gratitude to my collaborators for their valuable discussions and communications, and to my postgraduates for their hard working. Thanks to Natural Science Foundation of China for the financial support.

I would like to express my sincere thanks to my parents, wife and daughter for their love, encouragement and support in the past 20 years.

Shanghai, China
March 2011

Shijun Liao

Acronyms

2D	Two Dimensional
3D	Three Dimensional
APO	American Put Option
BEM	Boundary Element Method
BVP	Boundary Value Problem
BVPs	Boundary Value Problems
CPU	Central Processing Unit
DNS	Direct Numerical Simulation
FDM	Finite Difference Method
FEM	Finite Element Method
GBEM	Generalized Boundary Element Method
HAM	Homotopy Analysis Method
IVP	Initial Value Problem
IVPs	Initial Value Problems
ODE	Ordinary Differential Equation
ODEs	Ordinary Differential Equations
OHAM	Optimal Homotopy Analysis Method
PDE	Partial Differential Equation
PDEs	Partial Differential Equations

Contents

Part III Applications in Nonlinear Partial Differential Equations

Part I
Basic Ideas and Theorems

" *The essence of mathematics lies entirely in its freedom.* "

by Georg Cantor (1845 — 1918)

Chapter 1
Introduction

1.1 Motivation and purpose

It is well-known that nonlinear ordinary differential equations (ODEs) and partial differential equations (PDEs) for boundary-value problems are much more difficult to solve than linear ODEs and PDEs, especially by means of analytic methods. Traditionally, perturbation (Van del Pol, 1926; Von Dyke, 1975; Nayfeh, 2000) and asymptotic techniques are widely applied to obtain analytic approximations of nonlinear problems in science, finance and engineering. Unfortunately, perturbation and asymptotic techniques are too strongly dependent upon small/large physical parameters in general, and thus are often valid only for weakly nonlinear problems. For example, the asymptotic/perturbation approximations of the optimal exercise boundary of American put option are valid only for *a couple of days or weeks* prior to expiry, as shown in Fig. 1.1. Another famous example is the viscous flow past a sphere in fluid mechanics: the perturbation formulas of the drag coefficient are valid only for rather small Reynolds number $Re \ll 1$. Thus, it is necessary to develop some analytic approximation methods, which are independent of any small/large physical parameters at all and besides valid for strongly nonlinear problems.

In 1992, one of such kind of analytic approximation methods was proposed by the author (Liao, 1992), namely the homotopy analysis method (HAM) (Liao, 1997, 1999a,b; Liao and Campo, 2002; Liao, 2003a,b, 2004, 2005, 2006, 2009a, 2010a,b, 2011; Liao and Magyari, 2006; Liao and Tan, 2007; Li et al., 2010; Xu et al., 2010). Based on homotopy (Hilton, 1953) in topology (Sen, 1983), the HAM is independent of any small/large physical parameters. More importantly, unlike all other analytic techniques, it provides us a convenient way to guarantee the convergence of series solution of nonlinear problems by means of introducing an auxiliary parameter c_0, called the convergence-control parameter. In 2003, the basic ideas of the HAM and some applications mostly related to nonlinear ODEs were described systematically by the author in the book "Beyond Perturbation" (Liao, 2003b).

Thereafter, the HAM attracts attention of many researchers in about a dozen of countries, and has been successfully applied to solve a lot of nonlinear problems