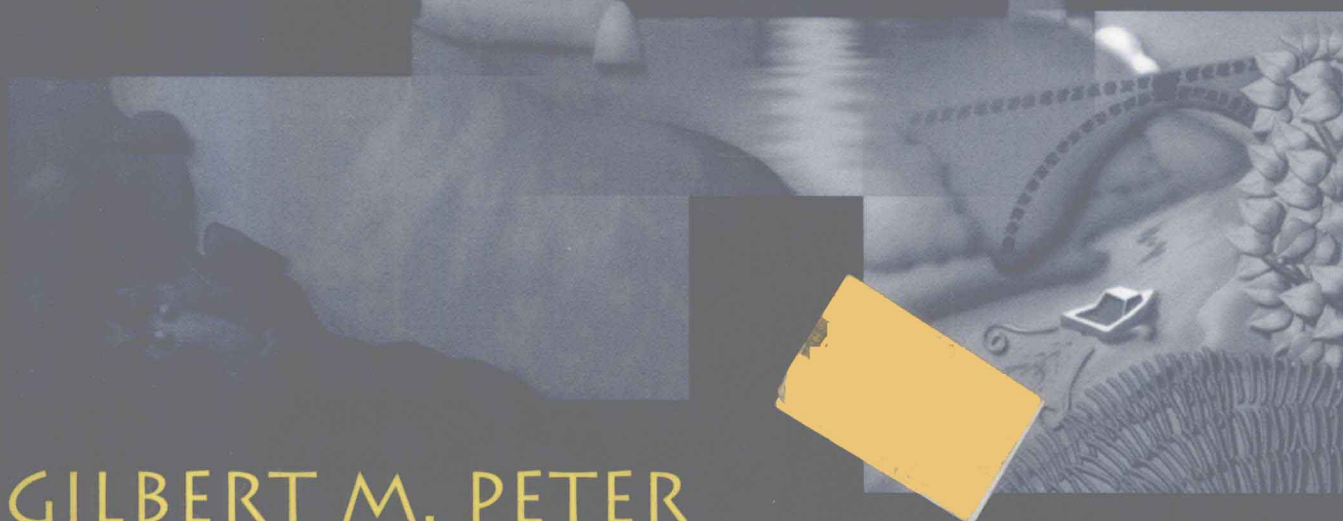
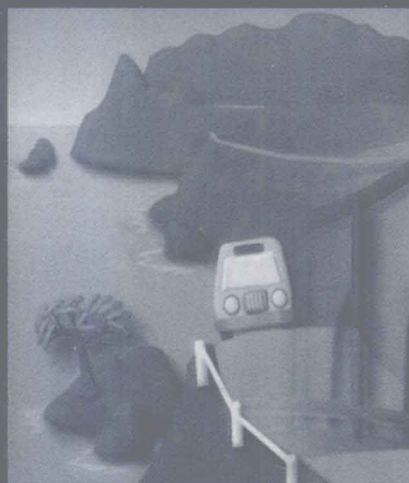


ALGEBRA FOR COLLEGE STUDENTS



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ALGEBRA FOR COLLEGE STUDENTS

DEDICATION

We dedicate this book to our wives Marlene and Pauline and to the re-entry students whose desire to change their lives and careers through education has been an inspiration to both of us.

PREFACE

This text is intended for a one-term course in intermediate algebra and is the companion text to *Elementary Algebra* by the same authors. A primary goal is to help to prepare students for success in college algebra, precalculus, or statistics. The last three chapters—Conic Sections; Exponential and Logarithmic Functions; and Sequences, Series, and the Binomial Theorem—build students' skills and understanding beyond the typical coverage of intermediate algebra and are appropriate for expanded intermediate algebra courses. For briefer courses, this text is designed to allow the exclusion of some material. Chapter 1 (The Real Number System) can be omitted if students are comfortable working with fractions and are familiar with the set of real numbers and its subsets. In addition, Sections 2.4 (Solving Linear Equations Involving Absolute Value), 2.6 (Solving Linear Inequalities Involving Absolute Value), 4.3 (Solving Linear Systems of Equations Using Matrices), and 4.6 (Linear Programming) can also be eliminated in a briefer course of instruction.

It has been our experience, and the experience of many instructors we know, that students often have a difficult time making the transition from the more “skills-oriented” intermediate algebra course to the more “concept-oriented” presentation typical of college algebra, precalculus, and statistics courses. Instructors in these courses often find that students are simply not adequately prepared to take college algebra (for example) in spite of evidence indicating that they had performed adequately (or better) in the prerequisite courses. The texts used in these preceding courses are often a key factor in this lack of preparation. This is particularly true when the intermediate text duplicates much of the content and the approach of an elementary algebra text. It isn't until the college algebra course that the student (and instructor) discovers to what extent the student is underprepared for success. This has sometimes been our experience with texts we have used, and some of the reviewers of this text volunteered their own experiences and concerns of this kind. Therefore, we have written an intermediate algebra text that makes reasonable demands of the students while preparing them for success at the college algebra level. We've accomplished this goal by organizing and presenting the traditional contents of intermediate algebra in a way that we've found to be effective in guiding students to success in this and any subsequent mathematics courses.

To master skills and apply them at a more conceptual level in later courses, students must use the key skills and concepts in a number of different circumstances.

In this way they will develop a more intuitive and reasoning-oriented ability to use mathematics. Understanding the concept of a function and the correspondence between the algebraic expression of a function and the graphic one is essential for success in college algebra. We believe that too often functions and graphing are covered late in intermediate algebra, where they are not given proper emphasis and when students have too few opportunities to practice the skills and understand the power of the function concept. Therefore we have introduced graphing and the concept of a function in Chapter 3. Once introduced, the function concept is then used in the development of other topics to help students gain a more powerful understanding of different types of equations and their solutions. This gives students many opportunities to use functions during the semester and to obtain insight into analyzing mathematical relationships. In addition, the early introduction of functions and graphing provides opportunities for students to use a graphing calculator or computer software. These powerful graphing technologies help students to visualize concepts and to develop a deeper understanding of various algebraic equations and functions. Although the text encourages and helps the students to explore with graphing technology, reference to this technology is not invasive and its use is completely optional.

A major theme of this book is problem solving. This emphasis appears in a variety of ways, but it is most evident in the number of applications and applied problems throughout the book, and in the “More Applications” sections at the ends of Chapter 2, 4, 6–9, and 11. While the problem-solving theme is not limited to these sections, they do provide major examples of the way in which the mathematical concepts and skills being presented can be used to model and solve real-life applied problems. In addition, the introduction of graphing and the function concept in Chapter 3 and especially of systems of equations in Chapter 4 provides students with powerful tools that they can use for solving applied problems.

It has been said that a good intermediate algebra text can be measured by its ability to use examples to show the “how and why” of an algebraic concept or skill, rather than by presenting lengthy discussions of the theory. We believe that both theory and practice are equally important and we have tried to strike a balance between the two. We make an effort to show students how to apply the skills and concepts with an abundance of examples, thereby encouraging their confidence and providing them with a substantial opportunity to see mathematics in action. Corresponding to the nearly 900 in-text examples is a wealth of exercises, over 6,200 of them. Any experienced instructor knows that exercise sets are vital to a good text. This is where the students have the opportunity to practice the mathematics that they have just learned and to discover where additional work is needed. The exercises are structured to give instructors many options in assigning problems, including group problems and writing exercises.

Key Features

APPLIED PROBLEMS

Applied problems appear early in Chapter 1 and are included in most exercise sets as well as in separate sections called “More Applications” at the end of most chapters. Experience has shown that students often learn how to manipulate algebraic expressions without knowing how to use them as a tool for solving applied problems. The introduction of such problems early in the text helps the student to see how

mathematics is used while helping them to develop the logical thinking necessary for problem-solving. Applied problems present a challenge that must be met to be successful in working with the more difficult material found in subsequent courses. To make the problems meaningful and not contrived, we have selected problems from physics, chemistry, business, finance, and areas of student aspirations or experience. Some of these problems may use units of measurement that are not familiar to the student, but such familiarity is not necessary to solve the problems successfully.

SYSTEMS OF EQUATIONS AND GRAPHING

Linear equations and inequalities are introduced in Chapter 2, expanded upon in Chapter 3, where graphing first occurs, and then reinforced in Chapter 4, when systems of linear equations and inequalities are developed. This organization provides the student with the option of using systems of equations or inequalities to solve applied problems throughout the remainder of the text. The introduction of graphing techniques early in the course gives the student a means to visualize the mathematics that they are learning and to use this technique of visualization to help them solve later problems.

EARLY FUNCTIONS

The concept of a function is central to the development of mathematics and as such is introduced in Chapter 3, earlier than many intermediate algebra texts. Functions are then used throughout the remainder of the text where appropriate. For example, they are used to discuss polynomial functions and their properties in Chapter 5, rational functions in Chapter 7, quadratic functions in Chapter 8, and exponential and logarithmic functions in Chapter 11.

APPLICATIONS OF GEOMETRY

Many students enroll directly in intermediate algebra after completing elementary algebra, without experiencing the benefit of a course in plane geometry. They are, however, aware of the geometry of the world about them, so we have included applications of algebra to geometry together with sufficient explanation so that they will understand. Common geometric figures together with the formulas for finding their perimeters, areas, and volumes are provided in the Appendix. For students who have completed a course in geometry, the problems will give them a chance to refresh and renew concepts already learned.

CALCULATORS

Although we have chosen not to focus on the calculator throughout the book, we are certainly aware of its utility and we have included problems where its use may be helpful. Such problems are identified by words or by a graphics calculator icon beside the particular exercise or group of exercises. We chose to use a graphics calculator logo because a graphics calculator can be used to carry out the same operations that a scientific calculator can in addition to having graphics capabilities. Periodically, instructions are given showing how to use a calculator to carry out an operation or a specific order of operations. Basic instructions for using the TI-81 and the TI-82 are contained in the Appendix.

THEORY VERSES PRACTICE

A careful balance is maintained between the “how” and the “why” of each mathematical concept. The discussion is generally intuitive but it is based on carefully stated mathematical definitions and theorems that are supported by a large number of examples. Proofs are included in the areas where they enhance the “why” of a concept.

SET NOTATION

Set notation is introduced in Chapter 1 and is used to show the relationship between the components of the number system. It is also used to provide a consistent means of identifying solution sets of equations or systems of equations throughout the text. Solution sets of inequalities are described using interval notation to prepare the student for more advanced courses.

FRACTIONS

Chapter 1 contains a review of fractions. The review can be either covered in class or used as a reference by students who feel they need additional work in this area.

AMATYC AND NCTM GUIDELINES

The text was written with the NCTM and AMATYC guidelines in mind. There is an abundance of exercises that require critical thinking, writing, and the use of modern technology, as well as cooperative group projects.

Key Chapter Pedagogy

CHAPTER OPENERS

Every chapter opens with an applied problem that appears later in the chapter. This gives the student an idea of the kind of problems they will learn to solve by using the concepts covered in the chapter.

SECTION OBJECTIVES

The objectives to be accomplished in each section are clearly stated at its beginning. They are written in a format that tells the student exactly what they are expected to learn by the time the section is completed.

HISTORICAL COMMENTS

All of us are indebted to those who preceded us in mathematics, so we have included many historical comments throughout the text. Some are brief glimpses of the lives of early mathematicians and describe the part they played in developing the topic at hand. Others trace the paths that have led to the mathematical symbolism that we use today. Still others present problems from ancient times or from texts that are hundreds of years old and that, except for the language, are still used today to teach important algebraic concepts. Complete solutions to the historical problems are presented at the end of the section in which the problem occurs. In every case, care has been taken to keep the historical comments brief enough so that they will be read by the students and interesting enough so that some students will want to learn more.

KEY CONCEPTS, DEFINITIONS, AND THEOREMS

Important theorems and other key concepts are boxed to emphasize their importance and to make them easier to find for future reference and review.

CAUTION WARNINGS

Students are warned about common misconceptions with special caution notations in the left margins of pages where such mistakes may occur. These warnings help the student avoid the frustration that naturally occurs when these errors are made and the student is unable to discover what is wrong.

CHAPTER SUMMARIES

A summary of the important concepts in each chapter is included at its end. The items discussed in the summaries are keyed to the chapter sections for quick reference.

CAN YOU MATCH THESE?

A short matching quiz concludes the text discussion in most sections. These self tests give students the opportunity to check whether they are ready to begin the exercise set for that section. Answers are supplied after the last question, providing them with immediate feedback about their level of understanding.

REVIEW PROBLEMS

Since algebraic concepts are best mastered and retained by constant practice, most exercise sets contain problems from previous sections and chapters. At the end of each chapter there is a complete set of review problems covering the topics in that chapter, and a cumulative review is included at the end of every third chapter. Complete solutions to all problems in the chapter reviews and the cumulative reviews are included in the Student's Solutions Manual.

EXERCISE SETS

Thousands of class-tested exercises appear at section ends and chapter ends. Odd- and even-numbered exercises are paired so that an instructor can assign just odd-numbered problems, with answers in the text and in the Student's Solutions Manual, or just even-numbered problems, with answers only in the Instructor's Solutions Manual.

STUDENT WRITING ASSIGNMENTS

We believe that students have not fully mastered a mathematical concept until they can discuss it and write about what they have learned. Where appropriate, the exercise sets contain a "Write in Your Own Words" section that directs the student to write about a particular concept. Responses can range from a single sentence to a paragraph or more, depending on the question.

FOR EXTRA THOUGHT

Many of the exercise sets contain problems that are designed to challenge the student by requiring them to think critically. These problems use the concepts of the section and require that the student extend the concept a step or two further to arrive at a solution.

COOPERATIVE EXERCISES

Each chapter contains a cooperative exercise after the chapter summary. Care has been taken, as much as possible, to coordinate the exercise with the concepts learned in the chapter. The contents of these exercises reflect situations relevant to everyday life.

Ancillaries

INSTRUCTOR'S SOLUTIONS MANUAL

An instructor's manual is available free of charge to all qualified adopters of the text. It contains the solutions to all problems not solved in the Student's Solutions Manual.

TEST BANK

A test bank is available free of charge to qualified adopters of the text. It contains 80 questions for each chapter, half of which are multiple choice and half problems that the student is required to solve. The Test Bank Manual is keyed to West Test, a computer test generator that can create tests based on the instructor's selections from the manual.

GROUP LEARNING RESOURCE MANUAL

A Group Learning Resource Manual for Elementary and Intermediate Algebra, by ViAnn Olson of Rochester Community College, contains 20 group learning exercises for both in-class and homework assignments. Each exercise is accompanied by suggestions for the instructor on how best to use that exercise. A section at the beginning of the manual contains guidelines on how to use group learning exercises effectively. The exercises are designed to be photocopied by instructors for distribution to the students.

STUDENT'S SOLUTIONS MANUAL

A student's manual, prepared by Ross Rueger of the College of the Sequoias, is available for purchase. It contains complete solutions to the odd-numbered exercises in the exercise sets, as well as all of the answers to the in-text chapter tests, chapter reviews, and cumulative reviews.

WEST MATH TUTOR

The West Math Tutor by Mathens is available for IBM and IBM-compatible PCs, as well as for Apple Macintosh computers. This software contains algorithmically based tutorials for most of the topics in the text. The tutorials are interactive in that they give feedback to the students when they make errors and provide hints that help the students when they try the questions again. The student is shown the correct solution after two incorrect attempts.

WEST MATH VIDEOS

The West Math Videos are designed to provide additional instruction for the students in the areas in which they may be experiencing difficulty. Worksheets accompany the videos so that the student can be actively involved in the lesson.

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PREFACE TO THE STUDENT

Learning mathematics requires dedication on your part. Try to adhere to the 2-for-1 rule: for every hour in class, follow it with two hours of quality study outside of class.

When you registered for the class that uses this text, you probably did not plan any conflicts. It is worth your effort to try to ensure that none develops. In order to be successful, we offer the following five suggestions:

1. Except in an emergency, attend every class session. No text is designed to cover all your questions. Your instructor can. If you must miss a class, get the notes as soon as possible.
2. Ask questions in class. There is no such thing as a stupid question! What does qualify as stupid is the failure to ask about something you don't understand when the opportunity presents itself.
3. Take advantage of your instructor's posted office hours. Failure to understand the contents of one section will often hinder your understanding of the sections that follow. College classes can be less personal than those you experienced in high school and it is up to you to get to know your instructor well.
4. Do the assigned homework as soon as possible after class. Time is your enemy. If you have a small amount of time and can do part of the assignment before your next class, do it. This will solidify the concepts you have learned, making it much easier to complete the rest of the assignment when your schedule permits.
5. Rework all problems that you miss on any test until you understand what you did wrong. This will keep you from making the same mistakes again.
6. The greatest threat to success in a mathematics class is procrastination. Many college students hesitate to admit they don't understand a concept, hoping that something magic will happen. What *will* happen is that insurmountable problems will develop, causing you to drop the class or, worse yet, to fail the class. Get help as soon as you need it.

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