TOPICS IN CONTEMPORARY MATHEMATICS

FIFTH EDITION

BELLO • BRITTON

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PREFACE

In this fifth edition we have continued in our goal of introducing the student to the many interesting mathematical concepts that are used in our contemporary world. We have tried to bring out the basic ideas and techniques as simply and clearly as possible and have related these ideas to other areas—such as sociology, psychology, and business—that will be attractive to the reader. Whenever feasible, elementary applications are given; these can be found in a new section feature called Getting Started, throughout the text's discussion and examples, and in the lesson problem sets.

The more abstract and theoretical aspects of the subject matter have been de-emphasized. Instead, emphasis has been placed on the understanding and use of the various concepts that are introduced. An important aid to this goal will be found in the exercises, which include over 4100 problems ranging from the necessary routine drill to challenges for the better students. The reader will find considerable support and explanation in the over 500 worked-out examples.

What Is New in the Fifth Edition?

We have followed the valuable suggestions of users of previous editions and the many reviewers contributing to this fifth edition to clarify the exposition, expand the coverage, and, in general, improve the book.

◆ We have completely redesigned the format of the book, now in four colors, and provided hundreds of new examples and exercises.

◆ Each chapter now begins with a Preview feature detailing the material to be covered in the chapter and the ways in which the topics are related to each other.

◆ Each section now begins with a Getting Started feature. These applications offer a motivating introduction for the techniques and ideas to be covered, and are drawn from a vast array of fields.

◆ Problem-solving examples have been added in pertinent sections to emphasize problem-solving methods throughout the text. These special examples use a unique two-column format to describe the general problem-solving method and then demonstrate a specific use.

◆ A new exercise feature called In Other Words gives students the opportunity to use writing to clarify and express ideas, concepts, and procedures. Students will think, talk, and write mathematics when they work these problems, which are included in every exercise set.

◆ A set of Research Questions is included at the end of each chapter to help students master research and library techniques as well as to explore how the topics under discussion were developed. These questions can be assigned to individual students or as group projects. A Research Bibliography detailing sources for researching these questions is provided at the end of the book.

◆ We have added a new section on Infinite Sets and a section on Game Theory.

◆ The presentation of rational, real, and complex numbers has been consolidated into a single chapter.

◆ Chapter 7, Geometry, has been extensively revised, giving a more detailed treatment of angles, and placing perimeter and circumference in the same section.

◆ We have placed mathematical systems and matrices together in Chapter 8.

◆ We have revised Chapter 12, Consumer Mathematics, to reflect real-world changes.

◆ The metric system has been moved to Chapter 13.

◆ Revision of Chapter 14, Computers, now makes it more current and more applicable to other computer work touched on throughout the text.

◆ We have made many significant efforts to address the NCTM curriculum recommendations regarding communication (In Other Words), reasoning (Chapter 2), connections (Discovery features and Mathematical Systems), Algebra (Chapters 5 and 6), Geometry (Chapter 7), Statistics (Chapter 11), Probability (Chapters 9 and 10), mathematical structures (Chapter 8), and problem solving (throughout the book).

Suggested Courses Using this Book

The book is quite flexible, with a large selection of topics available to suit various courses. The entire book can be covered easily in a full year's course, while many alternative choices can be made for a two-quarter or a one-semester course. Here are some of the courses for which the book is suggested:

◆ General education or liberal arts mathematics (the text follows most of the CUPM [Committee on the Undergraduate Program in Mathematics] recommendations for liberal arts mathematics)

◆ Topics in contemporary mathematics courses

◆ College mathematics or survey of mathematics courses

◆ Introduction to mathematics or applications of mathematics courses

There are a few more advanced topics that may be included or omitted at the instructor's discretion. These choices will not affect the continuity of any chapter presentation or syllabus as a whole. The topics include the following sections: 1.6 Infinite Sets; 2.6 Implication; 2.9 Switching Networks; 6.7 Linear Programming; 7.8 Networks; and 8.8 Game Theory.

Supporting Material and Supplements

This text has an extensive support package that includes:

◆ An *Instructor's Guide* containing commentary and teaching suggestions for each chapter, suggested course syllabi, answers to even-numbered problems in the exercises, a variety of five test forms per chapter, as well as answers to all test questions in the guide.

◆ A *Student's Solutions and Study Guide* containing complete solutions to all odd-numbered problems in the exercises and to all the problems in the chapter Practice Tests.

◆ Algorithmic testing from ips Publishing offering a total of 200 algorithms for creating a nearly unlimited variety of test forms for each chapter of the text. Available in IBM PC and Macintosh versions.

◆ ESATEST II computerized testing which allows instructors to edit and print fixed-item tests. The full range of user features includes pull-down menus, dialog boxes, random or manual item selection, and export/import capability. Available in IBM PC and Macintosh versions.

◆ Preparing for the CLAST—Mathematics, which is a competency-based study guide that reviews and offers preparatory material for the CLAST (College Level Academic Skills Test) objectives required by the State of Florida for mathematics.

◆ Algorithmic CLAST software, also from ips Publishing, for preparing practice test forms covering every CLAST mathematics objective.

◆ Videotapes reviewing key topics in the text.

◆ A disk containing the computer programs given in the appendix, PROGRAMS IN BASIC, and suitable for interactively working the appropriate problems in the text.

A Word About Problem Solving

Problem solving has become a fixture in mathematics textbooks. Lead by the teachings of George Polya, and following the recommendations of the NCTM and the MAA, most mathematics books at this level cover the topic. Many texts, however, front-load much of their presentation in the first chapter; all of the techniques, procedures, and pedagogy are paraded in Chapter One and then promptly forgotten. We have chosen to integrate problem solving where it is needed, and consequently, where it can be taught and learned most effectively.

For example, a few of the strategies suggested by Polya himself call for making a table, writing an equation, making a diagram, and accounting for all possibilities. Why not wait to present these techniques in the chapters dealing with truth tables, algebra, geometry, and counting, respectively, where the pertinent methods can be effectively displayed, rather than laboriously creating artificial solutions only to demonstrate strategies by solving artificial problems?

The artificial approach, after all, is not problem solving; it is problem making. It will make problems for the instructor, for the student, and (one might argue) for society at large. As Professor Beberman put it: "I think in some cases we have tried to answer questions that students never raise and to resolve doubts they never had. . . ." Therefore, we have worked very hard to dispell the unfortunate notion gained by many students that mathematics is an artificial subject riddled with uninteresting and contrived problems. As an ongoing theme of this text, problem solving is presented purposefully in meaningful and appropriate contexts where students can best understand and appreciate its methods. Above all, we hope that this integrated approach will help students learn how to apply problem-solving techniques in the real world once the course is over.

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Jack R. Britton

A USER'S GUIDE TO FEATURES

This new edition of *Topics in Contemporary Mathematics* contains a wide variety of features designed to help build the reader's understanding of mathematics by placing the work to be done **in context**. A student who utilizes the features in this book will gain a better understanding of the history behind each topic, how the topic relates to everyday life, how different topics in the course interrelate, and—most importantly—how to think about solving problems in the real world once the course is over.

The **features** in *Topics* have been carefully written to ensure that, while many are optional, they are **interrelated**. In addition, the text utilizes a new **four-color design** to highlight pedagogical features, emphasize each feature's function, and make them visually interesting.

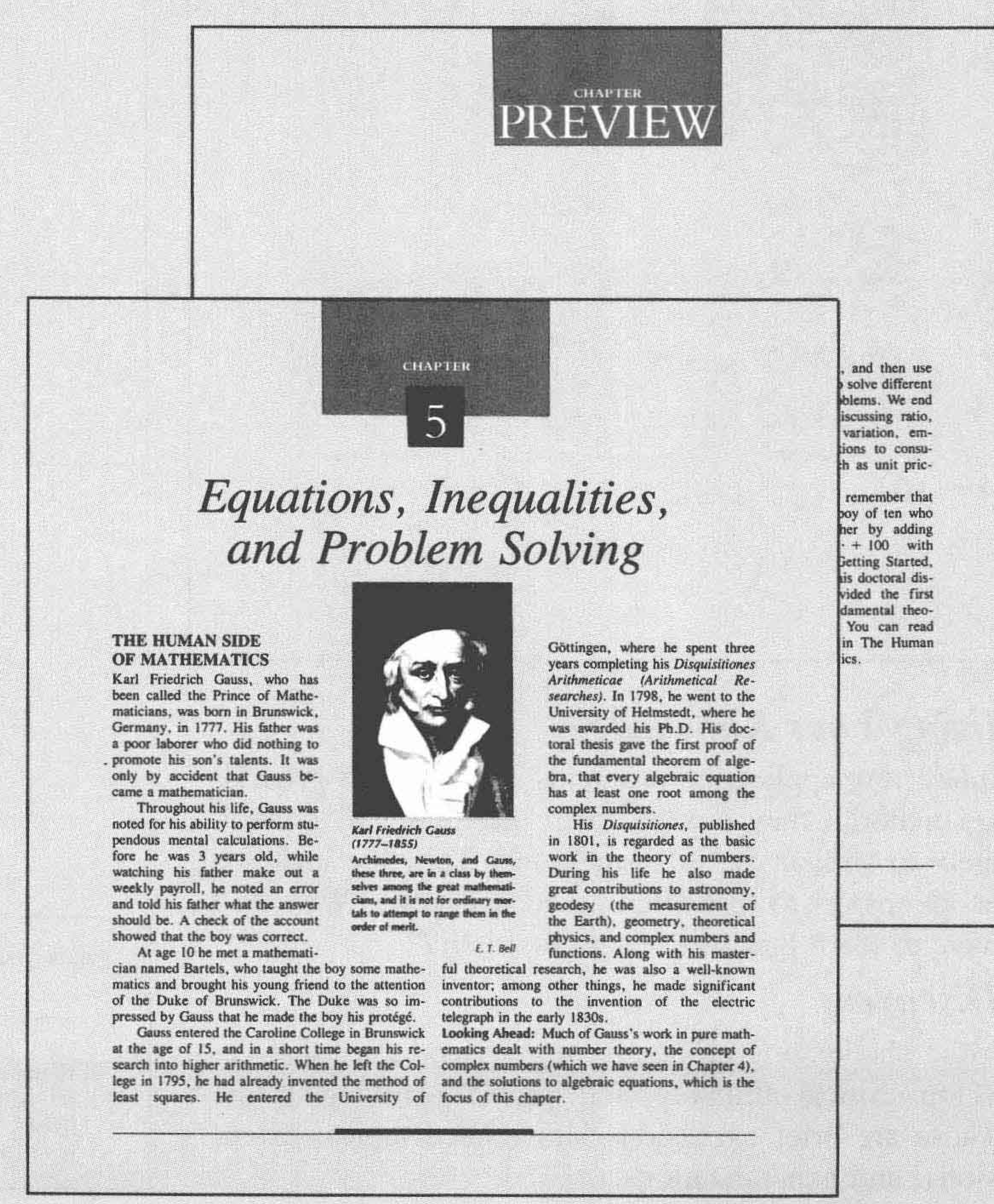
Putting Material in Context

Chapter Preview

Each chapter begins with a list of topics for quick reference. The introduction that follows provides an overview of the material being studied, and explains the ways in which the topics are related.

The Human Side of Mathematics

Who devised this material or contributed to its development? Placed at the beginning of each chapter to serve as motivation and offer historical perspective, this feature helps to communicate the message that mathematics is a growing body of knowledge, that it is a human endeavor, and that every topic studied began as part of a problem solving process. Remarks on **Looking Ahead** link the biography to upcoming material.



CHAPTER PREVIEW

CHAPTER 5

Equations, Inequalities, and Problem Solving

THE HUMAN SIDE OF MATHEMATICS

Karl Friedrich Gauss, who has been called the Prince of Mathematicians, was born in Brunswick, Germany, in 1777. His father was a poor laborer who did nothing to promote his son's talents. It was only by accident that Gauss became a mathematician.

Throughout his life, Gauss was noted for his ability to perform stupendous mental calculations. Before he was 3 years old, while watching his father make out a weekly payroll, he noted an error and told his father what the answer should be. A check of the account showed that the boy was correct.

At age 10 he met a mathematician named Bartels, who taught the boy some mathematics and brought his young friend to the attention of the Duke of Brunswick. The Duke was so impressed by Gauss that he made the boy his protégé.

Gauss entered the Caroline College in Brunswick at the age of 15, and in a short time began his research into higher arithmetic. When he left the College in 1795, he had already invented the method of least squares. He entered the University of Göttingen, where he spent three years completing his *Disquisitiones Arithmeticae (Arithmetical Researches)*. In 1798, he went to the University of Helmstedt, where he was awarded his Ph.D. His doctoral thesis gave the first proof of the fundamental theorem of algebra, that every algebraic equation has at least one root among the complex numbers.

His *Disquisitiones*, published in 1801, is regarded as the basic work in the theory of numbers. During his life he also made great contributions to astronomy, geodesy (the measurement of the Earth), geometry, theoretical physics, and complex numbers and functions. Along with his masterful theoretical research, he was also a well-known inventor; among other things, he made significant contributions to the invention of the electric telegraph in the early 1830s.

Looking Ahead: Much of Gauss's work in pure mathematics dealt with number theory, the concept of complex numbers (which we have seen in Chapter 4), and the solutions to algebraic equations, which is the focus of this chapter.

Karl Friedrich Gauss (1777–1855)

Archimedes, Newton, and Gauss, these three, are in a class by themselves among the great mathematicians, and it is not for ordinary mortals to attempt to range them in the order of merit.

E. T. Bell

Problem-Solving Skills

Problem Solving

Presented as an on-going theme throughout *Topics*, specific **Problem Solving** examples are clearly formatted using two columns. The left column uses the RSTUV method (Read, Select, Think, Use, and Verify) to guide the reader through the problem. In the right column, the solution is carefully developed. Similar standard examples follow and provide additional reinforcement.

Problem Solving:	Quadratic Equations
	Solve the equation $x^2 - 2x = 1$
1. Read the problem. **2. Select the unknown.**	We want to find the values of the variable x so that $x^2 - 2x = 1$.
3. Think of a plan. Is the equation a quadratic equation? If it is, write it in standard form. Can you factor it? If not, use the quadratic formula to solve it.	The equation $x^2 - 2x = 1$ is a quadratic equation. To write it in standard form, subtract 1 from both sides, obtaining $x^2 - 2x - 1 = 0$. Since we cannot factor this equation, we write $x^2 - 2x - 1 = 0$ and compare it with $ax^2 + bx + c = 0$. Thus, $a = 1$, $b = -2$, and $c = -1$.
3. Use the quadratic formula to carry out the plan. Find a, b, and c and substitute their values in $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ How many solutions should you get? Make sure the final answer is simplified.	Substituting these values in $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we obtain $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$ $= \frac{2 \pm \sqrt{4 + 4}}{2} = \frac{2 \pm \sqrt{8}}{2}$ $= \frac{2 \pm \sqrt{4 \cdot 2}}{2} = \frac{2 \pm 2\sqrt{2}}{2}$ This answer is *not* simplified. Divide each number in the numerator and denominator by 2 to obtain $1 \pm \sqrt{2}$. Thus, $x = 1 + \sqrt{2}$ or $x = 1 - \sqrt{2}$
4. Verify the result.	The verification is left to the student. Note that $(1 \pm \sqrt{2})^2 = 1 \pm 2\sqrt{2} + 2$.
TRY EXAMPLE 6 NOW.	Cover the solution, write your own, and then check your work.

EXAMPLE 6 Use the quadratic formula to solve $x^2 - 4x - 12 = 0$. (Compare Example 3.)

Solution To obtain the correct values of a, b, and c, we rewrite the equation in standard quadratic form as

$$1x^2 + (-4)x + (-12) = 0$$

which we compare with

$$ax^2 + bx + c = 0$$

Getting Started

Appearing at the beginning of every section, **Getting Started** is an application that demonstrates how the material relates to the real world. Hundreds of applications are used to introduce some of the techniques and ideas to be covered in each new section.

59. Not very much is known about the Greek algebraist Diophantus, except how old he was when he died. This fact has been preserved in the following 1,500-year-old riddle. Can you discover Diophantus' age at death?

    Diophantus' youth lasted $\frac{1}{6}$ of his life.
    After $\frac{1}{12}$ more, he grew a beard.
    After $\frac{1}{7}$ more of his life, he married, and 5 years later, he had a son.
    The son lived exactly $\frac{1}{2}$ as long as his father, and Diophantus died just 4 years after his son.
    [*Hint:* Let x be the number of years that Diophantus lived.]

5.2 SOLUTION OF FIRST-DEGREE SENTENCES

GETTING STARTED

A CAR RENTAL EQUATION

Have you rented a car lately? Some companies charge a flat fee each day and give you free mileage. Other companies have a flat fee *plus* mileage. Suppose your car breaks down and you wish to rent an economy car for 1 day. The cost C is \$25 per day plus 20¢ per mile. If m is the number of miles traveled in 1 day, the cost can be written as

$$C = 0.20m + 25$$

If at the end of the day you paid \$55, how far did you drive? To find the answer, you must solve the equation

$$55 = 0.20m + 25$$

An open sentence such as $55 = 0.20m + 25$ in which the unknown quantity has an exponent of 1, is called a **first-degree** sentence. We shall study two types of first-degree sentences in this section: equations and inequalities.

Car rental counters are a familiar sight at most airport terminals

Using Your Knowledge

Interesting application problems help students generalize material they have learned and apply it immediately to similar real-life situations. They are included as an answer to that often asked question, "Why do I have to learn this, and what good is it?"

Using Your Knowledge

Have you ever heard of **Chamberlain's formula,** *which purports to tell you how many years you should drive your present car before you buy a new one? If y is this number of years, then Chamberlain's formula reads*

$$y = \frac{GMC}{(G - M)DP}$$

where G is the new car's gas mileage, M is your present car's gas mileage, C is the cost in dollars of the new car, D is the number of miles you drive in a year, and P is the dollar price of gasoline per gallon.

For instance, suppose that the new car's gas mileage is 24 mi/gal, the old car's mileage is 12 mi/gal, the price of the new car is \$6400, you drive 12,000 mi/yr, and the cost of gasoline is \$1.40/gal. This means that G = 24, M = 12, C = 6400, D = 12,000, and P = 1.40, so

$$y = \frac{(24)(12)(6400)}{(12)(12{,}000)(1.4)} = \frac{64}{7} \approx 9.1$$

Thus, according to the formula, your old car should have been driven about 9.1 years before the purchase is justified.

Discovery

More challenging problems are provided to further develop critical thinking and problem-solving skills. These are brief excursions into related topics, extensions, and generalizations.

Discovery

82. A problem that comes from the Rhind papyrus, one of the oldest mathematical documents known, reads like this: "$\frac{2}{3}$ added and then $\frac{1}{3}$ taken away, 10 remains. . . ." The document then goes on to tell how to find the number with which you started. Can you discover what this number is? [*Hint:* If you let x be the unknown number, then the problem intends you to add $\frac{2}{3}x$ and then take away $\frac{1}{3}$ of the result to get a difference of 10.]

Examples, Exercise Sets, and Applications

The over 500 examples in *Topics* include a wide range of computational, drill, and applied problems selected to build confidence, competency, skill, and understanding.

Over 4100 carefully developed exercises provide extensive practice with drill and applied problems included in each section. Each exercise set is carefully graded to build student confidence in solving problems. Answers to odd-numbered exercises appear in the back of the text.

Extensive applications, a key strength of this text, are integrated throughout the examples and exercises, and have been carefully designed to show the relevance of the mathematics being studied.

Sometimes we do not have to find a numerical answer to a problem, but only an equation that the answer will satisfy, as shown in the next example.

EXAMPLE 5 A certain two-digit number is equal to seven times the sum of its digits. If the tens digit is x and the units digit is y, what equation must x and y satisfy?

Solution
1. Read the problem.
2. The unknowns are x (the tens digit) and y (the units digit).
3. Translate the problem. Since the tens digit is x and the units digit is y, the two-digit number must be $10x + y$. The sum of the digits is $x + y$, and seven times the sum of the digits is $7(x + y)$, so the required equation is

$$\underbrace{10x + y}_{\text{A two digit number}} \underset{\text{is equal to}}{=} \underbrace{7(x + y)}_{\text{seven times the sum of its digits}}$$

■

EXAMPLE 6 The Better Business Bank has two types of checking accounts, A and B. Type A has a monthly service charge of \$3 plus 25 cents for each check written. Type B has a monthly service charge of \$5 plus 10 cents for each check written. What is the greatest number of checks that can be written before type A becomes the more expensive of the two?

Solution Let x be the number of checks written. The cost of each account is

Type A $3 + 0.25x$ dollars

Type B $5 + 0.10x$ dollars

Hence we need to find the greatest value of x such that

$$3 + 0.25x \le 5 + 0.10x$$

This inequality can be solved as follows:

Given:	$3 + 0.25x \le 5 + 0.10x$
Subtract 3 from both sides:	$0.25x \le 2 + 0.10x$
Subtract $0.10x$ from both sides:	$0.15x \le 2$
Divide both sides by 0.15:	$x \le \frac{2}{0.15}$
	$x \le 13\frac{1}{3}$

Since x must be a whole number, it follows that 13 is the required answer. ■

Calculator and Computer Corners

Another problem solving skill students should develop is the ability to understand how and when to utilize technology. These features provide essential background on how to solve problems using a calculator or a computer.

67. What is the maximum number of letters Charlie could have received before throwing any away? [*Hint:* Let x^2 be the initial number of letters and let y^2 be the number he had left. Then $x^2 = y^2 + 13^2$. Remember that x and y are integers, and you will want to make y as large as possible relative to x.]

Calculator Corner

Your calculator can be extremely helpful in finding the roots of a quadratic equation by using the quadratic formula. Of course, the roots you obtain are being approximated by decimals. It is most convenient to start with the radical part in the solution of the quadratic equation and then store this value so you can evaluate both roots without having to backtrack or copy down any intermediate steps. Let us look at the following equation:

$$2x^2 + 7x - 4 = 0$$

Using the quadratic formula, the solution is obtained by following these key strokes:

[7] [x²] [−] [4] [×] [2] [×] [4] [+/−] [=]
[√x] [STO] [7] [+/−] [+] [RCL] [=] [÷] [2] [÷] [2] [=]

The display will show 0.5. To obtain the other root, key in

[7] [+/−] [−] [RCL] [=] [÷] [2] [÷] [2] [=]

which yields −4. In general, to solve the equation $ax^2 + bx + c = 0$ using your calculator, key in the following:

[b] [x²] [−] [4] [×] [a] [×] [c] [=]
[√x] [STO] [b] [+/−] [+] [RCL] [=] [÷] [2] [÷] [a] [=]

and

[b] [+/−] [−] [RCL] [=] [÷] [2] [÷] [a] [=]

Computer Corner

In this section, we learned how to solve quadratic equations by using the ***quadratic formula.*** *To do this, we must first write the equation in the form*

$$ax^2 + bx + c = 0$$

If you write the equation in this form, the Solving Quadratic Equations by Formula program found in the appendix will do the rest! You need only enter the coefficients a, b, and c. One word of caution: Make sure you enter a 1 as the coefficient of x^2 when you have an equation such as $x^2 + 2x + 1 = 0$.

In Other Words

Useful as a writing exercise, or for class discussion, these brief questions provide the opportunity to think about and clarify ideas, concepts, and procedures.

In Other Words

47. In your own words define the *replacement set* for an equation.
48. In your own words define the *solution set* for an equation
49. If a real number a is in the replacement set of an equation, will it always be in the solution set? Explain.
50. If a number s is in the solution set of an equation, will it always be in the replacement set? Explain.

EXAMPLE 7 Graph the solution set of $|x + 1| \ge 2$.

Solution $|x| \ge a$ is equivalent to $x \ge a$ *or* $x \le -a$. Therefore, $|x + 1| \ge 2$ is equivalent to $x + 1 \ge 2$ *or* $x + 1 \le -2$; that is, $x \ge 1$ *or* $x \le -3$. The graph of the solution set is shown in the figure.

Exercise 5.5

A. In problems 1–10, evaluate the given expression.

1. $|-10|$ 2. $|15|$ 3. $|-\frac{1}{8}|$
4. $|\frac{3}{4}|$ 5. $|5 - 8|$ 6. $|8 - 5|$
7. $|0| + |-2|$ 8. $|-2| - |-3|$ 9. $-|8|$
10. $-|3| + |-4|$
11. Determine which of the following are solutions of $|1 - 3x| > 3$.
(a) 2 (b) $-\frac{1}{2}$ (c) $\frac{5}{3}$ (d) 0
12. Determine which of the following are solutions of $|x - 2| < 2$.
(a) 0 (b) 1 (c) -1 (d) -2

B. In problems 13–18, find the set of integers for which the given sentence is true.

13. $|x| < 1$ 14. $|x| > -2$ 15. $|x| = 5$
16. $|x| \le 3$ 17. $|x| \ge 1$ 18. $|x| < 4$

In problems 19–36, graph the solution set of the given sentence (if possib and write the solution set in interval notation.

19. $|x| = 1$ 20. $|x| = 2.5$ 21. $|x| \le 4$
22. $|x| > 1$ 23. $|x + 1| < 3$ 24. $|x - 2| < 1$
25. $|x| \ge 1$ 26. $|x| > -1$ 27. $|x - 1| > 2$
28. $|x - 3| \ge 1$ 29. $|2x| < 4$ 30. $|3x| \le 9$
31. $|3x| \ge 6$ 32. $|2x| > 5$ 33. $|2x - 3| \le 3$
34. $|3x + 1| \le 8$ 35. $|2x - 3| > 3$ 36. $|3x + 1| > 8$

In problems 37–39, use $|b - a| \le c$, where b is the budgeted amount, a is the actual expense, and c is the variance.

37. A company budgets $500 for office supplies. How much money can they spend if their variance is $50?
38. A company budgets $800 for maintenance. How much money can they spend if an acceptable variance is 5% of their budgeted amount?

A. *Ratios*

Many quantities can be compared by using ratios. Here is the definition of a ratio.

Definition 5.1

A **ratio** is a quotient of two numbers. The ratio of a number a to another number b can be written as:

$$a \text{ to } b \quad \text{or} \quad a{:}b \quad \text{or} \quad \frac{a}{b}$$

The last form is used most often, but is frequently written as a/b.

EXAMPLE 1 Write the ratio of nutrients (139) to calories (100) in three different ways.

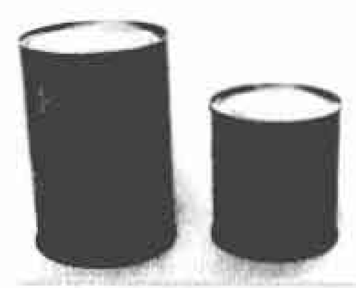
Solution 139 to 100 139:100 $\frac{139}{100}$

Of course, if the ratio in Example 1 had been 140 to 100, you could write it in reduced form as

$$7 \text{ to } 5 \quad \text{or} \quad 7{:}5 \quad \text{or} \quad \frac{7}{5}$$

You probably encounter many ratios in your everyday life. For example, the expression *miles per gallon* is actually a ratio, the ratio of the number of miles traveled to the number of gallons of gas used. Thus, if your car travels 294 miles on 12 gallons of gas, your miles per gallon ratio is $\frac{294}{12} = \frac{49}{2} = 24.5$.

Ratios can also be used to compare prices. For example, most people have the misconception that the more you buy of an item, the cheaper it is. Is this always true? Not necessarily. The picture shows two cans of Hunt's tomato sauce, bought in the same store. The 15-oz can costs 68¢, while the 8-oz can costs 32¢. Which can is the better buy? To compare these prices, we need to find the price of 1 oz of tomato sauce; that is, we need the **unit price** (the price per ounce). This unit price is given by the ratio



Unit pricing allows shoppers to quickly compare relative costs of the same or similar products when they are sold in different weights or volumes

$$\frac{\text{Price}}{\text{Number of ounces}}$$

For the 15-oz can, For the 8-oz can,

$$\frac{68}{15} = 4.5\overline{3} \qquad \frac{32}{8} = 4$$

Thus, the 15-oz can is more expensive. (Note that you could buy 16 oz, two 8-oz cans, for only 64¢, instead of 68¢ for the 15-oz can!)

Subsections Correlated to Exercise Sets

Before students can make decisions about how to solve a problem, they must first master the basic skills within each topic. To help the reader identify the different skills within a topic, each subsection is clearly titled and marked with an A, B, C, etc. These subsections are then correlated with the exercise sets to help students draw connections between subsection presentations and problems.

End-of-Chapter Study Aids

Summary

The Chapter Summary provides brief definitions and examples for key topics within a given chapter. Importantly, it also contains section references to encourage students to re-read sections, rather than memorizing a definition out of context.

Section	Item	Meaning
5.3	Finite intervals	$a \le x \le b$ Cl
		$a < x < b$ Op
		$a \le x < b$, $a < x \le b$ } Ha
5.5A	Absolute value	The distance line from 0 to
5.6	Quadratic equation	A second-deg can be writter $ax^2 + bx +$
5.6C	Quadratic formula	The solutions $ax^2 + bx +$ $x = \frac{-b \pm \sqrt{}}{}$
5.6D	Pythagorean theorem	In any right t square of the equal to the s of the other t
5.7	**Procedure for Solving Word Problem** 1. Read the problem carefully, and 2. Select a variable to represent this 3. Translate the problem into the la 4. Use the rules of algebra to solve 5. Verify the solution.	
5.8A	Ratio	A quotient of
5.8B	Proportion	An equality b
5.8C	Varies directly	y varies direc
5.8C	Varies inversely	y varies inver

34. The weight W of an object varies inversely as the square of i from the center of the Earth.
(a) Write an equation of variation.
(b) An astronaut weighs 121 pounds on the surface of the Ear dius of the Earth is 3960 miles, find the value of k for th (Do not multiply out your answer.)
(c) What will this astronaut weigh when she is 880 miles ab face of the Earth?

In Other Words

35. Explain the difference between a ratio and a proportion.
36. Explain the difference between direct variation and inverse va

Chapter 5 Summary

Section	Item	Meaning	Example
5.1	Variable	A symbol that may be replaced by any one of a set of numbers	x, y, z
5.1A	Open sentence	A sentence in which the variable can be replaced by a number	$x + 3 = 5; x - 1$
5.1A	Equation	Sentences in which the verb phrase is "="	$x + 7 = 9$
5.1A	Inequality	Sentences in which the verb is $>, <, \ne, \ge$, or $\le$	$x + 7 < 9; x > 8;$
5.1A	Solution set	The set of elements of the replacement set that make the open sentence a true statement	{3} is the solution s $x + 2 = 5$ when th replacement set is t whole numbers.
5.1A	Identities	Open sentences that are true for every number in the replacement set	$x + 0 = x; x + 2$ $a(b + c) = ab + c$
5.2	First-degree sentence	An open sentence in which the unknown quantity has an exponent of 1 only	$x + 7 = 8 - 2x; 2$
5.2A	Elementary operations	Operations that may be performed on a sentence to obtain an equivalent sentence	Addition or subtrac number on both sid equation

Research Questions

These questions provide an additional opportunity to explore how various mathematical topics were developed. This reinforces the message begun with **The Human Side of Mathematics** that the body of mathematical knowledge has evolved through human thought, experience, and communication. **Research Questions** combine with **In Other Words** to provide a strong, interesting writing component to the course, and are an excellent opportunity for group learning as well.

Practice Test

These tests are designed to help students check their comprehension. Practice Tests can help to further develop problem-solving and test-taking skills. Answers to all Practice Test items appear at the back of the book.

Research Questions

Sources of information for these questions can be found in the Bibliography at the end of the book.

1. Write a short essay about Gauss's childhood.
2. Find out and write a report about Gauss's proof regarding regular polygons in his *Disquisitiones Arithmeticae*.
3. Write a short paragraph about Gauss's inventions.
4. Aside from being a superb mathematician, Gauss did some work in the field of astronomy. Report on some of Gauss's discoveries in the field of astronomy.
5. In 1807, a famous French mathematician paid Gauss's involuntary 2,000-franc contribution to the French government. Find out who this famous mathematician was and the circumstances of the payment.
6. Another French mathematician asked the general commanding the French troops to send an officer to see how Gauss was faring during the war. This mathematician had submitted some results in number theory to Gauss under a pen name. Write a report on this incident and try to find the circumstances, the pen name, and the real name of the mathematician.

Chapter 5 Practice Test

1. If the replacement set is the set of integers, solve the following equations:
   (a) $x + 7 = 2$ (b) $x - 4 = 9$
2. If the replacement set is the set of integers, find the solution set for each of the following inequalities:
   (a) $x + 5 > 4$ (b) $2 + x \geq -x - 1$
3. Solve: $2x + 2 = 3x - 2$
4. Solve: $2x + 8 \geq -x - 1$
5. Graph the solution set of each of the following:
   (a) $x - 3 \leq 0$ (b) $-2x + 4 > x + 1$
6. Graph the solution set (if it is not empty) of each of the following:
   (a) $x + 2 \geq 3$ *and* $x \leq 4$ (b) $x - 3 \geq 1$ *and* $x \leq 0$
7. Graph the solution set of each of the following:
   (a) $x < 0$ *or* $x - 2 < 1$ (b) $x + 2 < 3$ *or* $x - 1 > 2$
8. Solve the equation $|x| = 3$.
9. Graph the solution set of $|x| < 2$.

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