Eberhard Zeidler

Nonlinear Functional Analysis and its Applications II/A

Linear Monotone Operators

非线性泛函分析及其应用

第2A卷

Springer

世界图书出版公司
www.wpcbj.com.cn

Nonlinear Functional Analysis and its Applications

II/A: Linear Monotone Operators

Translated by the Author and by Leo F. Boron†

With 45 Illustrations

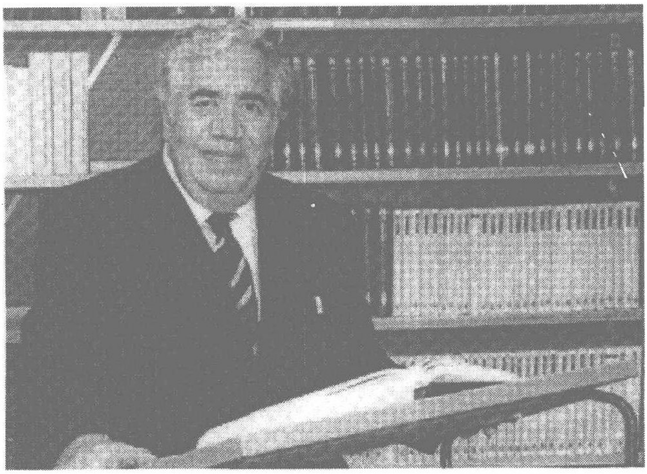
图书在版编目（CIP）数据

非线性泛函分析及其应用．第2A卷，线性单调算子
= Nonlinear Functional Analysis & It's Applications Vol. 2/A
Linear Monotone Operators：英文/（德）宰德勒著．
—北京：世界图书出版公司北京公司，2009. 8
ISBN 978-7-5100-0520-6

Ⅰ．非… Ⅱ．宰… Ⅲ．①非线性—泛函分析—英文
②非线性算子：单调算子—英文 Ⅳ．0177. 91

中国版本图书馆 CIP 数据核字（2009）第 126348 号

书　　名：	Nonlinear Functional Analysis & Its Applications Vol. 2/A
作　　者：	E. Zeidler
中 译 名：	非线性泛函分析及其应用 第2/A卷
责任编辑：	高蓉
出 版 者：	世界图书出版公司北京公司
印 刷 者：	三河国英印务有限公司
发　　行：	世界图书出版公司北京公司（北京朝内大街137号　100010）
联系电话：	010-64021602，010-64015659
电子信箱：	kjb@ wpcbj. com. cn
开　　本：	24开
印　　张：	20. 5
版　　次：	2009年08月
版权登记：	图字：01-2009-1011
书　　号：	978-7-5100-0520-6/O · 736　　定　　价：　69. 00元



Eberhard Zeidler
德国 Max Planck 研究所数学领头教授，Leipzig 大学数学系荣誉教授。

影 印 版 前 言

自 1932 年，波兰数学家 Banach 发表第一部泛函分析专著“Théorie des operations linéaires”以来，这一学科取得了巨大的发展，它在其他领域的应用也是相当成功。如今，数学的很多领域没有了泛函分析恐怕寸步难行，不仅仅在数学方面，在理论物理方面的作用也具有同样的意义，M. Reed 和 B. Simon 的“Methods of Modern Mathematical Physics ”在前言中指出：“自 1926 年以来，物理学的前沿已与日俱增集中于量子力学，以及奠定于量子理论的分支：原子物理、核物理固体物理、基本粒子物理等，而这些分支的中心数学框架就是泛函分析。”所以，讲述泛函分析的文献已浩如烟海。而每个时代，都有这个领域的代表性作品。例如上世纪 50 年代，F. Riesz 和 Sz.-Nagy 的《泛函分析讲义》(中译版，科学出版社，1985)，就是那个时代的一部具有代表性的著作；而 60 年代，N. Dunford 和 J. Schwartz 的三大卷“Linear Operators”则是泛函分析学发展到那个时代的主要成果和应用的一个较全面的总结。泛函分析一经产生，它的发展就受到量子力学的强有力的推动，上世纪 70 年代，M. Reed 和 B. Simon 的 4 卷“Methods of Modern Mathematical Physics”是泛函分析对于量子力学应用的一个很好的总结。

呈现在我们眼前这部 5 大卷鸿篇巨制——E. Zeidler 的“Nonlinear Functional Analysis and its Applications”是非线性泛函分析到了上世纪 80 年代的主要成果和最典型应用的一个全面的论述，是一部百科全书。该书写作思想是：

(1) 讲述什么样的概念是基本的具有支配地位的概念，它们之间的关系是什么？

(2) 上述思想与经典分析以及线性泛函分析已有结果的关系是什么？

（3）　最典型的应用是什么？

一般的泛函分析书往往注重抽象理论的阐述，写应用常常不够详尽。而 Zeidler 这部书大为不同，其最大的特点是，书中讲了大量的各方面的应用，而且讲得非常清楚深入。

首先，这部书讲清楚了泛函分析理论对数学其他领域的应用。例如，第 2A 卷讲述线性单调算子。他从椭圆型方程的边值问题出发，讲问题的古典解，由于具体物理背景的需要，问题须作进一步推广，而需要讨论问题的广义解。这种方法背后的分析原理是什么？其实就是完备化思想的一个应用！将古典问题所依赖的连续函数空间，完备化成为 Sobolev 空间，则可讨论问题的广义解。在这种讨论中间，我们可以看到 Hilbert 空间的作用。书中不仅有这种理论讨论，而且还讲了怎样计算问题的近似解（Ritz 方法）。

其次，这部书讲清楚了分析理论在诸多领域（如物理学、化学、生物学、工程技术和经济学等等）的广泛应用。例如，第 3 卷讲解变分方法和优化，它从函数极值问题开始，讲到变分问题及其对于 Euler 微分方程和 Hammerstein 积分方程的应用；讲到优化理论及其对于控制问题（如庞特里亚金极大值原理）、统计优化、博弈论、参数识别、逼近论的应用；讲了凸优化理论及应用；讲了极值的各种近似计算方法。比如第 4 卷，讲物理应用，写作原理是：由物理事实到数学模型；由数学模型到数学结果；再由数学结果到数学结果的物理解释；最后再回到物理事实。

再次，该书由浅入深地讲透了基本理论的发展历程及走向，它既讲清楚了所涉及学科的具体问题，也讲清楚了其背后的数学原理及其作用。数学理论讲得也非常深入，例如，不动点理论，就从 Banach 不动点定理讲到 Schauder 不动点定理，以及 Bourbaki-Kneser 不动点定理等等。

这套书的写作起点很低，具备本科数学水平就可以读；应用都是从最简单情形入手，应用领域的读者也可以读；全书材料自足，各部分又尽可能保持独立；书后附有极其丰富的参考文献及一些文献评述；该书文字优美，引用了许多大师的格言，读之你会深受启发。这套书的优点不胜枚举，每个与数理学科相关的人，搞理论的，搞应用的，搞研究的，搞教学的，都可读该书，哪怕只是翻一翻，都不会空手而返！

全书共有 4 卷（5 本）：

第 1 卷　不动点定理　　　第 2 卷　A. 线性单调算子，B. 非线性单调算子

第 3 卷　变分方法与优化　　第 4 卷　数学物理的应用

Zeidler 教授著述很多，他后来于 90 年代又写了两本“Applied Functional Analysis”（Springer-Verlag, Applied Mathematical Sciences, 108,109），篇幅虽然比眼前这套书小了很多，但特点没有变。近期，他又在写 6 大卷“Quantum Field Theory”，第 1 卷“Basics in Mathematics and Physics, a Bridge between Mathematicians and Physicists”，已经由 Springer-Verlag 出版社出版。

非常感谢刘景麟对本文建议。

南京理工大学　黄振友

To the memory of my parents

Preface to Part II/A

A theory is the more impressive,
the simpler are its premises,
the more distinct are the things it connects,
and the broader is its range of applicability.
Albert Einstein

This is the second of a five-volume exposition of the main principles of nonlinear functional analysis and its numerous applications to the natural sciences and mathematical economics. The presentation is self-contained and accessible to a broader audience of mathematicians, natural scientists, and engineers. The basic content can be understood even by those readers who have little or no knowledge of linear functional analysis. The material of the five volumes is organized as follows:

Part I: Fixed-point theorems.
Part II: Monotone operators.
Part III: Variational methods and optimization.
Parts IV/V: Applications to mathematical physics.

The main goals of the work are discussed in detail in the Preface of Part I. A Table of Contents of Parts I through V can be found on page 871 of Part I. The emphasis of the treatment is based on the following considerations:

(a) Which are the basic, guiding concepts, and what relationship exists between them?
(b) What is the relationship between these ideas and the known results of classical analysis and of linear functional analysis?
(c) What are some typical applications?

The present Part II is divided into two subvolumes:

Part II/A: Linear monotone operators.
Part II/B: Nonlinear monotone operators.

These two subvolumes form a *unit*. They consist of the following sections:

introduction to the subject;
linear monotone problems;
generalization to nonlinear stationary problems;
generalization to nonlinear nonstationary problems;
general theory of discretization methods.

The numerous applications concern differential equations and integral equations, as well as numerical methods to their solution.

The Appendix, the Bibliography, and the Index material to Parts II/A and II/B can be found at the end of Part II/B.

The modern theory of linear partial differential equations of elliptic, parabolic, or hyperbolic type is based on the so-called *Hilbert space methods*. In this connection, boundary value problems and initial value problems are transformed into operator equations in Hilbert space. The solutions of these operator equations correspond to *generalized solutions* of the original classical problems. Here, the generalized solutions live in so-called *Sobolev spaces*. Roughly speaking, Sobolev spaces consist of functions which have sufficiently reasonable generalized derivatives.

The theory of *monotone operators* generalizes the Hilbert space methods to *nonlinear* problems. We want to emphasize that the Hilbert space methods and the theory of monotone operators are connected with the *main streams* of mathematics. They are closely related to Hilbert's rigorous justification of the *Dirichlet principle*, and to the 19th and 20th problems of Hilbert which he formulated in his famous Paris lecture in 1900. The relevant historical background will be discussed in Chapter 18. From the physical point of view, the Hilbert space methods and the more general theory of monotone operators are based on the fundamental concept of *energy*. Roughly speaking, Sobolev spaces can be regarded as spaces of functions which correspond to physical states of finite energy. We will show that, in our century, the notion of monotone operators played, both implicitly and explicitly, a fundamental role in the development of the calculus of variations, in the theory of linear and nonlinear partial differential equations, and in numerical analysis.

In order to help the reader understand the basic ideas, the first chapters of this volume serve as an elementary introduction to the modern functional analytic theory of *linear* partial differential equations. In particular, Chapter 18 contains an elegant functional analytic justification of the Dirichlet principle, based on a generalization of the classical Pythagorean theorem to Hilbert spaces. An introduction to the theory of *Sobolev spaces* can be found in Chapter 21. Experience shows that students frequently have trouble with the technicalities of Sobolev spaces. In Chapter 21, for the benefit of the reader, we choose an approach to Sobolev spaces which is as elementary as possible. To this end, we first prove all the embedding theorems in an extremely simple manner in $\mathbb{R}^1$, before passing to $\mathbb{R}^n$. For the convenience of the reader, the basic properties of the *Lebesgue integral* are summarized in the Appendix to Part II/B. In this connection, we choose the simplest definition of the Lebesgue integral. In contrast to other definitions of the Lebesgue integral, our definition also applies immediately to functions with values in B-spaces. Such functions are needed in connection with evolution equations. Moreover, in Chapter 18 we discuss a number of important principles which are frequently used in modern analysis, for example, the smoothing principle via mollifiers, the localization principle via partition of unity, the extension principle, and the completion principle.

The *basic ideas* and basic principles of the theory of nonlinear monotone operators are discussed in detail in a special section at the beginning of Part II/B. Any reader who wishes to learn about *nonlinear* monotone operators as quickly as possible may immediately begin reading Part II/B.

A reference of the form $A_1(20)$ and $A_2(20)$ is to formula (20) in the Appendix of Part I and II/B, respectively; while (18.20) refers to formula (20) in Chapter 18. Omission of a chapter number means that the formula is in the current chapter. The References to the Literature at the end of each chapter are of the following form: Krasnoselskii (1956, M, B, H), etc. The name and the year relate to the Bibliography at the end of Part II/B. The letters stand for the following:

M: monograph;
L: lecture notes;
S: survey article;
P: proceedings;
B: extensive bibliography in the work cited;
H: comments on the historical development of the subject contained in the work cited.

A List of Symbols may be found at the end of Part II/B. We have tried to use generally accepted symbols. A few peculiarities, introduced to avoid confusion, are described in the remarks introducing the List of Symbols.

Basic material on linear functional analysis may be found in the Appendix to Part I.

The theory of monotone operators is related to the simple fact that the *derivative* f' of a *convex* real function f is a *monotone* function. However, it is quite remarkable that the idea of the monotone operator allows many diversified applications. For example, there are *applications* to the following topics:

(i) variational problems and variational inequalities;
(ii) nonlinear elliptic, parabolic, and hyperbolic partial differential equations;
(iii) nonlinear integral equations;

(iv) nonlinear semigroups;
(v) nonlinear eigenvalue problems;
(vi) nonlinear Fredholm alternatives;
(vii) mapping degree for noncompact operators;
(viii) numerical methods such as the Ritz method (e.g., the method of finite elements), the Galerkin method, the projection–iteration method, the difference method, and the Kačanov method for conservation laws and variational inequalities.

Concerning time-dependent problems we emphasize both the *Galerkin method* and the method of *semigroups*. We also discuss in detail the fact that the theory of monotone operators generalizes both the theory of *bounded* and *unbounded* linear operators. To this end we develop, in Chapters 18 and 22 through 24, the theory of linear partial differential equations based on bounded linear operators, and in Chapter 19, we study in detail the elegant method of the Friedrichs extension for unbounded linear operators and its applications to variational problems and to linear and semilinear elliptic, parabolic, and hyperbolic equations, as well as applications to the semilinear Schrödinger equation. As we shall show in Parts IV and V, unbounded linear operators play a decisive role in quantum mechanics and quantum field theory (elementary particle physics). In contrast to this, for example, bounded linear operators are related to elasticity and hydrodynamics.

At the center of the theory of monotone operators there stands the notion of the *maximal monotone* operator, which generalizes both the theory of bounded and unbounded linear monotone operators. The theory of maximal monotone operators will be studied in detail in Chapter 32.

A number of diagrams contained in the text should help the reader to discover interrelationships between different topics. In particular, we recommend Figure 27.1 in Section 27.5 of Part II/B where the reader may find interrelationships between many important operator properties in nonlinear functional analysis. A list of all these schematic overviews can be found at the end of Part II/B. A list of the basic theorems and of the basic definitions can also be found there.

In Part I we studied equations involving compact operators. The decisive advantage of the theory of monotone operators is that it is also applicable to *noncompact* operators. Along with abstract existence theorems we also stress the methods of *numerical* functional analysis. Chapters 20 through 22 (resp. Chapters 33 through 35) may serve as an introduction to linear (resp. nonlinear) numerical functional analysis. For example, in terms of numerical functional analysis, monotone operators allow us to justify the following fundamental principle: Consistency and stability imply convergence. In this connection, the general notion of A-proper maps is crucial.

The connection between the theory of monotone operators and general variational methods will be studied in detail in Part III. In Parts IV and V we will consider applications of the theory of monotone operators to interesting problems in *mathematical physics*. For example, the theory of monotone operators plays an important role in elasticity, hydrodynamics (the Navier-Stokes equations), gas dynamics (subsonic flow), and semiconductor physics.

I hope that the reader will enjoy discovering a number of interesting interrelationships in mathematics.

Leipzig
Spring 1989

Eberhard Zeidler

Contents (Part II/A)

Contents (Part II/B)