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Substance and Function & Einstein's Theory of Relativity

ERNST CASSIRER

SUBSTANCE AND FUNCTION *and* EINSTEIN'S THEORY OF RELATIVITY

Both Books Bound as One

BY

ERNST CASSIRER

Authorized Translation by

WILLIAM CURTIS SWABEY, PH.D.

and

MARIE COLLINS SWABEY, PH.D.

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PREFACE

The investigations contained in this volume were first prompted by studies in the philosophy of mathematics. In the course of an attempt to comprehend the fundamental conceptions of mathematics from the point of view of logic, it became necessary to analyse more closely the function of the concept itself and to trace it back to its presuppositions. Here, however, a peculiar difficulty arose: the traditional logic of the concept, in its well-known features, proved inadequate even to characterize the problems to which the theory of the principles of mathematics led. It became increasingly evident that exact science had here reached questions for which there existed no precise correlate in the traditional language of formal logic. The content of mathematical knowledge pointed back to a fundamental form of the concept not clearly defined and recognized within logic itself. In particular, investigations concerning the concepts of the series and of the limit, the special results of which, however, could not be included in the general exposition of this book, confirmed this view and led to a renewed analysis of the principles of the construction of concepts in general.

The problem thus defined gained more general meaning when it became clear that it was in no way limited to the field of mathematics, but extended over the whole field of exact science. The systematic structure of the exact sciences assumes different forms according as it is regarded in different logical perspectives. Thus an attempt had to be made to advance from this general point of view to the forms of conceptual construction of the special disciplines,—of arithmetic, geometry, physics and chemistry. It did not accord with the general purpose of the enquiry to collect special examples from the particular sciences for the support of the logical theory, but it was necessary to make an attempt to trace their systematic structures as wholes, in order that the fundamental unitary relation by which these structures are held together might be revealed more distinctly. I did not conceal from myself the difficulty of carrying out such a plan; I finally resolved to make the attempt only because the value and significance of the preliminary work already accomplished within the special sciences became increasingly evident to me.

Particularly in the exact sciences, the investigator has turned from the special problems to the philosophical foundations with ever clearer consciousness and energy. Whatever one may judge in detail of the results of these researches, there can be no doubt that the logical problem has thereby been greatly and directly advanced. I have, therefore, sought to base the following exposition upon the historical development of science itself and upon the systematic presentation of its content by the great scientists. Although we cannot consider all the problems that arise here, nevertheless, the special logical point of view which they represent must be carried through and verified in detail. What the concept is and means in its general function can only be shown by tracing this function through the most important fields of scientific investigation and representing it in general outline.

The problem receives new meaning when we advance from purely logical considerations to the conception of *knowledge of reality*. The original opposition of thought and being breaks up into a number of different problems, which are, nevertheless, connected and held in intellectual unity by their common point of departure. Whenever, in the history of philosophy, the question as to the relation of thought and being, of knowledge and reality, has been raised, it has been dominated from the first by certain *logical* presuppositions, by certain views about the nature of the concept and judgment. Every change in this fundamental view indirectly produces a complete change in the way in which the general question is stated. The *system* of knowledge tolerates no isolated "formal" determination without consequences in all the problems and solutions of knowledge. The conception, therefore, that is formed of the fundamental nature of the concept is directly significant in judging the questions of fact which are generally considered under "Criticism of Knowledge," ("*Erkenntniskritik*") or "Metaphysics." The transformation which these questions undergo when regarded from the general point of view that is gained by criticism of the exact sciences and the new direction which their solution takes, Part II of the book attempts to show. Both parts, though seemingly separate in content, are united, nevertheless, in a philosophical point of view; both attempt to represent a single problem which has expanded from a fixed center, drawing ever wider and more concrete fields into its circle.

ERNST CASSIRER.

TRANSLATORS' PREFACE

It was thought that there was need for some comprehensive work in the English language on the philosophy of the exact sciences which would do full justice to the newer developments in mathematical and physical speculation while showing at the same time the historical connections of these tendencies. It seemed that the two works of Professor Ernst Cassirer herewith presented fulfilled these requirements best of all. The reader will find here a constructive and systematic survey of the whole field of the principles of the exact sciences from the standpoint of a logical idealism, which is historically derived from Kant, but which lacks the fatal rigidity of the latter's system. As Professor Cassirer develops his logical or critical idealism it becomes a doctrine of creative intelligence. His doctrine is neither idealism, pragmatism nor realism as these terms are understood in our English-speaking philosophy; it is rather a positivistic and non-static rationalism, which seeks to preserve the *spirit* which unites Plato, Descartes, Leibniz and Kant and to show how this spirit reaches its fulfillment in the modern development of mathematical and physical theory.

The first part of the present book, *Substanzbegriff und Funktionsbegriff* was published in 1910, while the second part, which we have called the Supplement, *Zur Einstein'schen Relativitätstheorie*, appeared in 1921. The intervening period was, of course, one of immense importance for the philosophy of physics, since it marked the development of the new and revolutionary theory of relativity. In accordance with the fundamental maxim of his critical method Professor Cassirer based his analysis in 1910 on the historical state of science, which was still dominated by the Newtonian conceptions of space and time. On the ground of the same maxim, he has since taken account of the new theory of relativity and has, with good logical justification, seen in the latter the relative completion and realization of the historical tendency which he had described in his earlier works. Professor Cassirer's philosophy may be regarded as a fundamental epistemological "theory of relativity" which sets forth a general philosophical standpoint from which Einstein's theory is seen to be only the latest and most radical fulfillment of the motives

which are inherent in mathematical and physical science as such. While Professor Cassirer has had his fundamental principles confirmed rather than disproved by this recent development, his discussion in Chapter IV, Section VI, of *Substance and Function* must be taken in connection with his later statements.

With regard to the translation, the translators are aware that a good deal of the vigor and savor of the original has escaped in the process of substituting correct but colorless terms for the more vivid language of the original. Accuracy and clarity have been their chief aim. They alone are responsible for the italicized paragraph headings, which were inserted because it was thought that the book might be used as a text or reference work in connection with an advanced course in the Theory of Knowledge and that perhaps these guide-posts might help the student in finding his way through the difficult material.

Professor Cassirer himself kindly read the entire work in manuscript and, in a friendly letter, states that "*nach der Gesamteindruck . . . besteht für mich kein Zweifel dass der Sinn des Ganzen richtig getroffen und wiedergegeben ist.*" We wish herewith to express our hearty thanks to Professor Cassirer for permitting us to translate his works as well as for his trouble in reading the manuscript of the translation and for his courtesy in the whole transaction.

WILLIAM CURTIS SWABEY,
MARIE COLLINS SWABEY.

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