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Weiyan Pan  
Kai Li

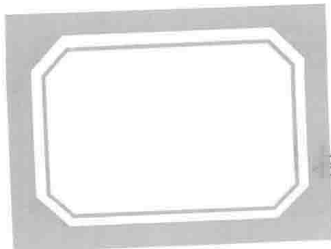
# Propagation of SLF/ELF Electro- magnetic Waves



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**ADVANCED TOPICS  
IN SCIENCE AND TECHNOLOGY IN CHINA**

## ADVANCED TOPICS IN SCIENCE AND TECHNOLOGY IN CHINA

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## Preface

Super low frequency (SLF: 30–300 Hz) and extremely low frequency (ELF: below 30 Hz) are the lowest frequency ranges in the applied radio spectrum. In SLF/ELF ranges, the wavelength is over 1,000 km, and the wave propagation in the Earth's space will be effected inevitably by the ground and the ionosphere. It is known that the excitation and emission in the space between the ground and the lower boundary of the ionosphere are of extreme difficulty. Meanwhile, because of the properties of low carried frequency, narrow bandwidth, and low information capacity, the applications of SLF/ELF waves do not range widely. However, in the past century the subject *Propagation of SLF/ELF Electromagnetic Waves* has been intensively investigated because of its useful applications in communication with submarines, geophysical prospecting and diagnostics, and seismic electromagnetic precursors monitoring. In the pioneering works of Wait, Budden, and Galejs, analytical solutions on SLF/ELF wave propagation are carried out. The subject has been investigated widely and the findings have been summarized in the classic book *Terrestrial Propagation of Long Electromagnetic Waves* by Galejs (1972). And some important findings are included in the book *Electromagnetic Waves in Stratified Media* by Wait (1970). In the past decades, some important progress has also been made. In the book *Resonances in the Earth–Ionosphere Cavity* by Nickolaenko and Hayakawa (2002), the fundamentals of ULF/ELF wave propagation and lightning problems are well summarized.

In this book it is investigated with an emphasis placed on the solution for SLF/ELF wave propagating in different regions, especially including Earth–ionosphere cavity or waveguide, anisotropic ionosphere, sea water, and layered Earth or sea floor. It is concerned with the approximated analytical solutions of the electromagnetic field radiated by a vertical or horizontal dipole. Usually, a simplified or idealized physical model is meant for solving a practical problem. From Maxwell's equations, and subject to the boundary conditions, the formulas of the electromagnetic field are always represented in the exact form of general integrals or in terms of special functions. Obviously, it is necessary to treat these by using mathematical techniques. The corresponding computations are also carried out, and some new conclusions are obtained.

In Chap. 1, the historical and technical overview is addressed for SLF/ELF wave propagation in the past century. Chapter 2 presents the fundamental theory of SLF/ELF wave propagation in Earth–ionosphere waveguide or cavity. Especially, the new algorithm is addressed to ELF range and the lower end of SLF range specifically. In Chap. 3, we treat the spherical harmonic series solutions for SLF/ELF field in the non-ideal Earth–ionosphere cavity and the speed-up numerical convergence algorithm. In Chaps. 4 and 5, we are concerned with SLF/ELF wave propagation in the regions consisting of the Earth, air, and ionosphere. Chapter 6 deals with SLF/ELF wave propagation along the boundary between sea water and ocean floor and the marine controlled-source electromagnetics (mCSEM) method. Chapter 7 addresses SLF field on the sea surface generated by a space borne transmitter. In Chap. 8, the atmosphere’s noise in SLF/ELF ranges is summarized.

The authors would like to express their gratitude to Professor Qingliang Li and Professor Hongqi Zhang of the China Research Institute of Radiowave Propagation (CRIRP). Many thanks are also due to Professor Kangsheng Chen, Professor Xianmin Zhang, and Professor Erping Li of Zhejiang University for their helpful support and encouragement. The authors are also grateful to their graduate students, especially including Huaiyun Peng, Yu Chen, Hong Lu, Yuanxin Wang, Xiqian Wang, and Xiaofei Zhao at China Research Institute of Radiowave Propagation, Yin Lin Wang, Yun Long Lu, Peng Fei Yu, and Bing Jun Xia at Zhejiang University. Finally, the authors would like to acknowledge the support and guidance of the editorial staffs of Zhejiang University Press and Springer.

The authors sincerely appreciate the financial support in part of the National Science Foundation of China under Grants 61271086 and 60971057, and the National Key Lab of Electromagnetic Environment, China Research Institute of Radiowave Propagation. The authors hope that the book will help stimulate new ideas and innovative approaches to radio wave propagation and applications in the years to come.

Qingdao, China  
Hangzhou, China  
July 2013

Weiyan Pan  
Kai Li

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# Historical and Technical Overview of SLF/ELF Electromagnetic Wave Propagation

In general, very low frequency (VLF) refers to electromagnetic waves with frequencies from 3 kHz to 30 kHz; ultra low frequency (ULF) refers to frequency range between 300 Hz to 3 kHz; the frequency range of 30 Hz to 300 Hz is defined as super low frequency (SLF); and the frequency below 30 Hz is referred to as extremely low frequency (ELF). In some early studies, alternative definitions may have been used with frequency range of 3 Hz to 3 kHz generally referred to ELF. In this book, the emphasis is on SLF (30–300 Hz) and ELF (below 30 Hz) ranges, using the MKS system of units and the time dependency  $e^{-i\omega t}$ .

## 1.1 Medium Characteristics of SLF/ELF Wave Propagation

Due to the long wavelength, SLF/ELF waves, when excited and propagated on and near the Earth's surface, will cover lithosphere, atmosphere and ionosphere, whose electromagnetic characteristics differ significantly in their propagation paths. The permeability of the atmosphere and ionosphere is approximately  $\mu_0$ , the permeability in free space, as well as that of the lithosphere, except in the regions that are rich in iron, nickel, cobalt, etc. Therefore, the permeability of the lithosphere can be also treated as  $\mu_0$ , if neither the transmitter nor the receiver is located in mineral-rich areas. The atmosphere is a non-conductive medium with negligible conductivity, whose permittivity is close to  $\epsilon_0$ , the permittivity of free space. While the lithosphere is conductive, the conductivity of sea water  $\sigma_{\text{sea}}$  is in the range of 2.5–5.5 S/m, and the conductivity  $\sigma_g$  of rock and soil in the range of  $10^{-2}$ – $10^{-4}$  S/m. The dielectric constant  $\epsilon_r$  of the lithosphere does not have large effects on the propagation of SLF/ELF waves.

The ionosphere is composed of partially ionized gas, which includes electrons, ions, and electrically neutral particles, above 70 km or so from the sea level. The electrons and ions make the ionosphere conductive, thus the majority of the energy of the incident VLF/SLF/ELF waves will be reflected by the ionosphere. When both the transmitter and the receiver are located in the space between the ground and the

lower boundary of the ionosphere, the excited wave will be reflected back and forth between the ground and the ionosphere. Due to the conductivity of the lithosphere and that of the ionosphere, the electromagnetic waves in the VLF range and below will have the reflection coefficients being close to 1 with low loss, the waves will thus be guided in the space between the ground and the lower boundary of the ionosphere, forming the Earth–ionosphere waveguide or cavity.

## 1.2 The VLF Waveguide Propagation Theory and Its Applications in Submarine Communication

In a conductive medium, the attenuation rate of the electromagnetic wave is proportional to the square root of its operating frequency, thus the low frequency wave is capable of penetrating a certain depth of sea water and can be picked up by a submerged submarine. The developments of the VLF submarine communication and navigation system began in both the United States and the Soviet Union in the 1950s. Driven by the defense requirements, massive effort was made and many resources were devoted to study the theory and application of VLF wave propagation, and numerous achievements were accomplished (Sommerfeld 1949; Bremmer 1949; Budden 1961a; Wait 1957, 1960, 1962b, 1968; Wait and Spies 1965; Galejs 1964a, 1967, 1968, 1972b). Especially, the two classic books *The Wave-Guide Mode Theory of Wave Propagation* (Budden 1961a) and *Electromagnetic Waves in Stratified Media* (Wait 1962a) laid the foundation of the VLF waveguide propagation theory.

In the works by Wait, the space between the ground and the lower boundary of the ionosphere was modeled as a waveguide with two concentric spherical reflective walls, and the ground-based VLF antenna as VED, as shown in Fig. 1.1. For this model, the VLF field components in the Earth–ionosphere waveguide can be expressed in Eq. (1.1) (Wait 1962a):

$$\begin{bmatrix} E_r \\ E_\theta \\ \eta H_\phi \end{bmatrix} = E_0(\pi x)^{\frac{1}{2}} \exp\left(-i\frac{\pi}{4}\right) \frac{2}{y_0} \times \sum_{n=0}^{\infty} \begin{bmatrix} A_n G_n(z) \\ A_n G'_n(z) \cdot (-i\gamma_n) \\ A_n G_n(z) \cdot (-\gamma_n) \end{bmatrix} G_n(z_0) \exp(-it_n x), \quad (1.1)$$

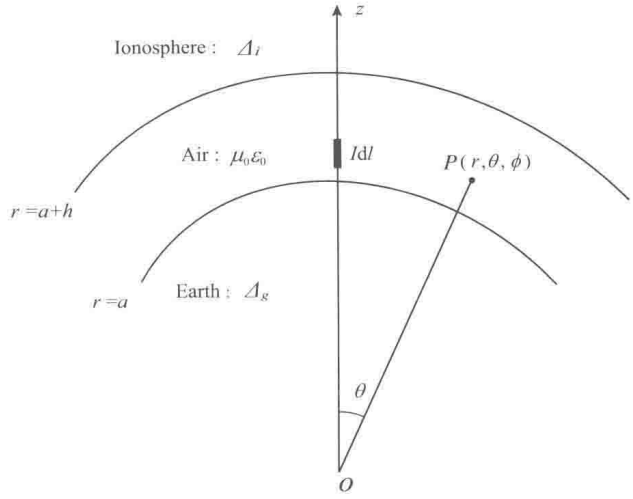
where

$$\gamma_n = \frac{1}{k[1 + \frac{t_n}{2}(\frac{2}{ka})^{\frac{2}{3}}]}, \quad (1.2)$$

$$E_0 = \frac{iI d l \eta \times 10^3}{\lambda} \cdot \frac{\exp(-ika\theta)}{a(\theta \sin \theta)^{\frac{1}{2}}} = i300 \times (P_{kW})^{\frac{1}{2}} \cdot \frac{\exp(-ika\theta)}{a(\theta \sin \theta)^{\frac{1}{2}}}. \quad (1.3)$$

Here  $E_0$  could be understood as the electric field in mV/m on the ideal ground radiated by a VDM. The angle  $\theta$  is the circle angular distance between transmitter

**Fig. 1.1** A VED in the Earth-ionosphere waveguide



and receiver in rad;  $P_{kW}$  is the transmitting power in kW, and  $a$  is the Earth's radius, which is 6,370 km. The parameter  $x$  in Eq. (1.1) is determined by the angular distance  $\theta$  from the transmitting source to the receiving point. It is

$$x = \left( \frac{ka}{2} \right)^{\frac{1}{3}} \cdot \theta. \quad (1.4)$$

Here  $G_n$  is the height-gain function, and  $z = r - a$  is the height of the observation point.  $t_n$  is the  $n$ th root of the modal equation, which determines the relative phase velocity and the attenuation rate of the  $n$ th mode.  $A_n$  is the excitation factor of  $n$ th mode in the Earth-ionosphere waveguide excited by a VED.

From Eq. (1.1), the VLF wave excited by a vertical electric dipole in the Earth-ionosphere waveguide can be treated as the summation of each mode, which has only the three components  $E_r$ ,  $E_\theta$ , and  $H_\phi$  traveling along the waveguide with the same phase velocity and attenuation rate. The relative phase velocity and the attenuation rate for the  $n$ th mode are expressed by

$$\frac{v_n}{c} - 1 = \text{Re} \left[ \frac{-t_n}{2^{\frac{1}{3}} (ka)^{\frac{2}{3}}} \right], \quad (1.5)$$

$$\alpha_n = 8.68 \times \text{Im} \left[ \frac{t_n \pi \left( \frac{2}{ka} \right)^{\frac{2}{3}}}{\lambda} \right] \text{ (dB/km)}, \quad (1.6)$$

where  $\lambda$  in km is the wavelength in free space.

It is noted that multiple modes exist in the Earth-ionosphere waveguide in the VLF range and the high-order modes decay significantly as the mode number increases. Thus the  $\text{TM}_1$  mode is dominant in the far-field region, which is the most important one in engineering.

In 1967, Watt reviewed the study of US researchers in VLF communication and navigation, and published the monograph *VLF Radio Engineering*. The monograph

was a summary of the contemporary accomplishments in VLF propagation researching, and discussed VLF communication and navigation system design, antenna design, and atmospheric noise in the VLF range as well.

### 1.3 SLF Communication System and SLF/ELF Propagation Theory

The operating frequency of SLF wave is even lower than that of VLF wave, thus SLF waves decay slower in sea water. The loss in sea water for the wave at 100 Hz is about 0.3 dB/m. With the development of modern submarine-detection technology, VLF receiving depth is no longer safe for a submarine. Project *Sanguine*, a plan to develop the submarine communication system in SLF range, thus started in the 1960s (Bannister 1975; Wait 1977). At that time, the properties of SLF/ELF wave propagation were investigated extensively. An excellent summary was given in the monograph *Terrestrial Propagation of Long Electromagnetic Waves* by Galejs (1972a).

Considering long wavelengths, no antenna can be put vertically on the ground in SLF range. Galejs therefore concentrated on analyzing electromagnetic waves excited by an HED in the Earth-ionosphere waveguide (Galejs 1972a). In SLF range, the wavelength in the atmosphere exceeds significantly the altitude between the ground surface and the lower boundary of the ionosphere, thus there only exists one propagating mode, which is a quasi-TEM mode, while all high-order modes are evanescent. In the case that both the transmitter and the receiver are located on or near the ground, ignoring high-order modes, SLF components radiated by an HED can be expressed as follows (Galejs 1972a):

$$E_r^{\text{he}}(a, \theta, \phi) = -\frac{I ds \eta \Delta_{gs}}{2ha} \cos \phi \Lambda_0^e \frac{\partial P_{v_0}(\cos(\pi - \theta))}{\sin v_0 \pi \partial \theta}, \quad (1.7)$$

$$E_\theta^{\text{he}}(a, \theta, \phi) = -\frac{i I ds \eta \Delta_{gs}}{2ha} \Delta_{gr} \cos \phi \frac{\Lambda_0^e}{ka} S_0^{-2} \frac{\partial^2 P_{v_0}(\cos(\pi - \theta))}{\sin v_0 \pi \partial^2 \theta}, \quad (1.8)$$

$$E_\phi^{\text{he}}(a, \theta, \phi) = -\frac{i I ds \eta}{2ha} \sin \phi \Delta_{gr} \Delta_{gs} \frac{\Lambda_0^e}{ka \sin \theta} S_0^{-2} \frac{\partial P_{v_0}(\cos(\pi - \theta))}{\sin v_0 \pi \partial \theta}, \quad (1.9)$$

$$H_\theta^{\text{he}}(a, \theta, \phi) = -\frac{i I ds}{2ha} \Delta_{gs} \frac{\sin \phi}{ka \sin \theta} \Lambda_0^e S_0^{-2} \frac{\partial P_{v_0}(\cos(\pi - \theta))}{\sin v_0 \pi \partial \theta}, \quad (1.10)$$

$$H_\phi^{\text{he}}(a, \theta, \phi) = -\frac{i I ds}{2ha} \Delta_{gs} \frac{\cos \phi}{ka} \Lambda_0^e S_0^{-2} \frac{\partial^2 P_{v_0}(\cos(\pi - \theta))}{\sin v_0 \pi \partial^2 \theta}. \quad (1.11)$$

In these formulas,  $\Delta_{gs}$  and  $\Delta_{gr}$  are the normalized impedances of the ground in the transmitting and receiving points, respectively.  $v_0$  is the eigenvalue of quasi-TEM mode, which is determined by the modal equation. For the first-order approximation, we have

$$v_0(v_0 + 1) \approx (ka)^2 \left[ 1 + \frac{i(\Delta_i + \Delta_g)}{kh} \right], \quad (1.12)$$

where  $\Delta_g$  and  $\Delta_i$  are the normalized surface impedance of the ground and that of the ionosphere, respectively.

In the operating range of 65 to 140 Hz for SLF communication system, the condition  $\nu_0 \gg 1$  is satisfied, thus the Legendre function is expressed in the asymptotic form (Galejs 1972a). It is

$$\begin{aligned} P_\nu(\cos(\pi - \theta)) &\approx \left( \frac{2}{\pi \nu_0 \sin \theta} \right)^{\frac{1}{2}} \cos \left[ \left( \nu_0 + \frac{1}{2} \right) (\pi - \theta) - \frac{\pi}{4} \right] \\ &= \left( \frac{1}{2\pi \nu_0 \sin \theta} \right)^{\frac{1}{2}} \left\{ e^{-i[(\nu_0 + \frac{1}{4})\pi - (\nu_0 + \frac{1}{2})\theta]} \right. \\ &\quad \left. + e^{i[(\nu_0 + \frac{1}{4})\pi - (\nu_0 + \frac{1}{2})\theta]} \right\}. \end{aligned} \quad (1.13)$$

When the receiving point is located neither near the source point nor its antipodal point, the electromagnetic wave propagating along the path of large arc can be neglected, and the components at the receiving point can be further approximated as follows (Galejs 1972a):

$$\begin{aligned} E_r^{\text{he}}(a, \theta, \phi) &= \frac{i I d s \eta}{2 h a} \Delta_{gs} \cos \phi \sqrt{\frac{2}{\pi \sin \theta}} \\ &\quad \times (k a S_0)^{\frac{1}{2}} \Lambda_0^e \exp \left( i k a S_0 \theta + i \frac{\pi}{4} \right), \end{aligned} \quad (1.14)$$

$$\begin{aligned} E_\theta^{\text{he}}(a, \theta, \phi) &= \frac{i I d s \eta}{2 h a} \Delta_{gs} \Delta_{gr} \cos \phi \sqrt{\frac{2}{\pi \sin \theta}} \\ &\quad \times (k a)^{\frac{1}{2}} S_0^{-\frac{1}{2}} \Lambda_0^e \exp \left( i k a S_0 \theta + i \frac{\pi}{4} \right), \end{aligned} \quad (1.15)$$

$$\begin{aligned} E_\phi^{\text{he}}(a, \theta, \phi) &= -\frac{I d s \eta}{2 h a} \sin \phi \Delta_{gs} \Delta_{gr} \sqrt{\frac{2}{\pi \sin \theta}} \\ &\quad \times (k a)^{-\frac{1}{2}} S_0^{-\frac{3}{2}} \Lambda_0^e \frac{\exp(i k a S_0 \theta + i \frac{\pi}{4})}{k a \sin \theta}, \end{aligned} \quad (1.16)$$

$$\begin{aligned} H_\theta^{\text{he}}(a, \theta, \phi) &= -\frac{I d s}{2 h a} \Delta_{gs} \frac{\sin \phi}{k a \sin \theta} \sqrt{\frac{2}{\pi \sin \theta}} \\ &\quad \times (k a)^{-\frac{1}{2}} S_0^{-\frac{3}{2}} \Lambda_0^e \exp \left( i k a S_0 \theta + i \frac{\pi}{4} \right), \end{aligned} \quad (1.17)$$

$$\begin{aligned} H_\phi^{\text{he}}(a, \theta, \phi) &= -\frac{i I d s}{2 h a} \Delta_{gs} \sqrt{\frac{2}{\pi \sin \theta}} \\ &\quad \times (k a)^{\frac{1}{2}} S_0^{-\frac{1}{2}} \Lambda_0^e \cos \phi \exp \left( i k a S_0 \theta + i \frac{\pi}{4} \right), \end{aligned} \quad (1.18)$$

where  $S_0$  is sine of incident angle to the ionosphere for the quasi-TEM mode, and  $\nu_0(\nu_0 + 1) \approx (k a S_0)^2$ .



From Eqs. (1.14)–(1.18), it is seen that the lower conductivity (thus higher impedance) of the ground can place a boost to the radiation efficiency of the ground-based SLF transmitter, thus making such region a good candidate for construction site of SLF horizontal antenna.

Wave propagation in SLF/ELF ranges has been studied widely in the 1990s and 1970s by many investigators (Schumann 1952a, 1952b; Galejs 1961, 1962, 1964b, 1965, 1972b; Bannister 1974, 1984a; Greifinger and Grifinger 1978; Behroozi-Toosi and Booker 1980; Yuan 2011), especially including Galejs and Wait. Galejs, who made outstanding contributions to the SLF communication engineering, passed away at an early age of 49 in 1972. In April 1974, a special issue on ELF/SLF propagation and communication was published by *IEEE Transactions on Communications*, especially including papers on SLF antenna design and site locating (Burrows 1974a, 1974b), propagation computation and measurement (Pappert and Moler 1974; Bannister 1974), signal and noise processing (Evans and Griffiths 1974; Ginsberg 1974), and environmental effect of a transmitting antenna (Valentino et al. 1974). In this issue, Wait wrote a mourning article “Dedication to Janis Galejs”. Lately, the ground-based SLF communication systems were completed and put in use in both the United States and the former Soviet Union (Russia).

In the whole ELF range, even the lower end of SLF range (below 50 Hz), the wavelength is very long, which can be compared with the Earth’s circumference. For instance, at  $f = 10$  Hz, the wavelength is about  $3/4$  times of the Earth’s circumference. It is noted that the space between the Earth’s surface and the ionosphere is usually regarded as a cavity. In this case, it is found that the condition of  $v_0 \gg 1$  is not satisfied for the mode eigenvalue  $v_0$  in the cavity, therefore Eq. (1.13) is no longer valid for calculating electromagnetic field in the Earth–ionosphere cavity excited by HED. In 2012, Peng et al. showed that in the lower SLF and ELF regions, the Legendre function  $P_\nu(\cos(\pi - \theta))$ , the distance factor of the field, can be better calculated by using numerical integrated algorithm (Peng et al. 2012, 2013). This new algorithm will be addressed in Chap. 2.

In 1999, Berrick presented a spherical harmonic series expression for electromagnetic field in atmosphere, with the lithosphere and the ionosphere being idealized as perfect conductors, and SLF/ELF radiation source as a VED (Barrick 1999). In the research group of Pan, the first author of this book, further works are concerned with the convergence acceleration algorithm for the electromagnetic field by HEDs and VEDs in SLF/ELF ranges, under the condition that the ground and the ionosphere are finitely conductive (Wang et al. 2007a, 2007b, 2009b). SLF/ELF electromagnetic field in Earth–ionosphere waveguide or cavity and the speed-up numerical convergence algorithm will be addressed in Chap. 3.

It should be pointed out that FDTD modeling of SLF/ELF wave propagation has greatly advanced (Simpson and Taflove 2004, 2007; Simpson 2006, 2009). However, this book mainly deals with the analytical solutions of SLF/ELF wave propagation, which is not concerned with the content of the FDTD modeling.