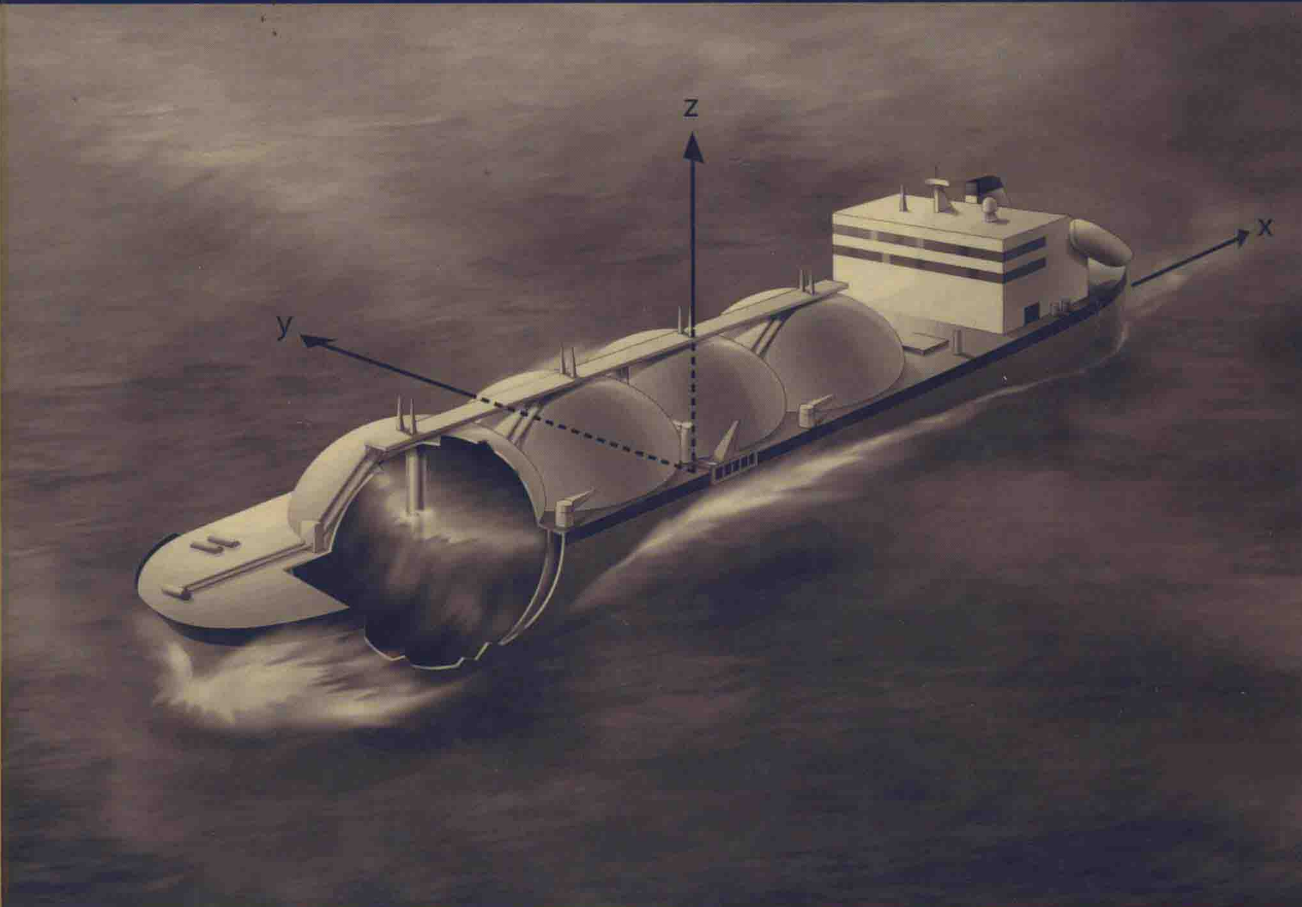
ODD M. FALTINSEN

ALEXANDER N. TIMOKHA



Sloshing

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This book presents sloshing with marine- and land-based applications, with a focus on ship tanks. It also includes the nonlinear multimodal method developed by the authors and an introduction to computational fluid dynamics. Emphasis is also placed on rational and simplified methods, including several experimental results. Topics of special interest include antirolling tanks, linear sloshing, viscous wave loads, damping, and slamming. The book contains numerous illustrations, examples, and exercises.

Odd M. Faltinsen received his Ph.D. in naval architecture and marine engineering from the University of Michigan in 1971 and has been a Professor of Marine Hydrodynamics at the Norwegian University of Science and Technology since 1976. Dr. Faltinsen has experience with a broad spectrum of hydrodynamically related problems for ships and sea structures, including hydroelastic problems. He has published approximately 300 scientific publications and is the author of the textbooks *Sea Loads on Ships and Offshore Structures* and *Hydrodynamics of High-Speed Marine Vehicles*, published by Cambridge University Press in 1990 and 2005, respectively. Faltinsen is a Foreign Associate of the National Academy of Engineering, USA, and a Foreign Member of the Chinese Academy of Engineering.

Alexander N. Timokha obtained his Ph.D. in fluid dynamics from Kiev University in 1988 and, later, a full doctorate in physics and a mathematics degree (habilitation) in 1993 at the Institute of Mathematics of the National Academy of Sciences of Ukraine. He is now Leading Researcher and Professor of Applied Mathematics at the Institute of Mathematics. Since 2004, he has been a Visiting Professor at CeSOS, Norwegian University of Science and Technology, Trondheim, Norway. In the 1980s, he was involved as a consultant of hydrodynamic aspects of spacecraft applications for the famous design offices of Yuzhnoye and Salut. Dr. Timokha's current research interests lie in mathematical aspects of hydromechanics with emphasis on free-surface problems in general and on sloshing in particular. He has authored more than 120 publications and 2 books.

CAMBRIDGE
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ISBN 978-0-521-88111-1

9 780521 881111 >

Cover design: Alice Soloway

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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore,
São Paulo, Delhi, Dubai, Tokyo

Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA

www.cambridge.org
Information on this title: www.cambridge.org/9780521881111

First published 2009

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication data

Faltinsen, O. M. (Odd Magnus), 1944–
Sloshing / Odd M. Faltinsen, Alexander N. Timokha. – 1st ed.
p. cm.
Includes bibliographical references and index.
ISBN 978-0-521-88111-1 (hardback)
1. Sloshing (Hydrodynamics) I. Timokha, A. N. II. Title.
TA357.5.S57F35 2009
620.1′064–dc22 2009006711

ISBN 978-0-521-88111-1 Hardback

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Nomenclature

pair A and $\overline{A}$, or A_i	dominant wave amplitudes in the steady-state analysis of nonlinear three-dimensional sloshing, or wave amplitudes in ocean wave problems
A_{ij}^{Name}	added mass coefficients for three-dimensional statement; *Name* specifies subject [$i, j = 1, \ldots, 6$ and Name = frozen, filled, slosh, etc.]
a_{ij}^{Name}	the same as A_{ij}^{Name}, but for a two-dimensional statement
B	beam (breadth) of a ship or catamaran
pair B and $\overline{B}$	dominant wave amplitudes in the steady-state analysis of nonlinear three-dimensional sloshing
$B_t = L_2$	breadth of tank for three-dimensional sloshing
Bo	Bond number
B_{ij}	elements of the damping matrix [$i, j = 1, \ldots, 6$]
b_{ij}	the same as B_{ij}, but for two-dimensional statement [$i, j = 1, \ldots, 6$]
b_s	effective sloshing breadth
c_0	speed of sound
Ca	Cauchy number
C_E	modified Euler number
C_D	drag coefficient
C_M	mass coefficient
C_v	modified cavitation number
C_{ij}^{Name}	restoring coefficients [$i, j = 1, \ldots, 6$; Name = frozen, filled, slosh, etc.]
D or d	diameter, draft of a ship
$D_0 = 2R_0$	diameter of spherical tank
d^*/dt	$*$-time derivative of a vector function in the body-fixed (noninertial) coordinate system; the superscript asterisk indicates that one should not time-differentiate the unit vectors (see eq. (2.50))
$\boldsymbol{e}_i$ or $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$	unit vectors of the body (tank)-fixed coordinate system [$i = 1, 2, 3$]
$\boldsymbol{e}_i'$	unit vectors of the Earth-fixed coordinate system [$i = 1, 2, 3$]

E	Young's modulus
$E(t)$, $\langle E\rangle$	energy, time-averaged energy
E_g	work done by gravitational force; bulk modulus of gas
E_k	kinetic energy
E_p	potential energy
E_l	bulk modulus of liquid
E_v	bulk modulus of elasticity
E_{ext}	work done by external forces
E_{in}	internal strain energy of deforming the object
E_{mem}	membrane elasticity
Eu	Euler number
$\boldsymbol{F}^{\text{Name}}(t)$	hydrodynamic force, where Name declares specific conditions on the considered fluid (e.g., filled, frozen) if needed $[= (F_1, F_2, F_3)]$
F_i^{Name}	for $i = 1, 2, 3$, components of $\boldsymbol{F}^{\text{Name}}(t)$; for $i = 4, 5, 6$, components of the hydrodynamic moment $\boldsymbol{M}_O(t)$ in the $Oxyz$-coordinate system
Fn	Froude number
$f_M(x, y)$	wave patterns defined by the natural sloshing modes, $f_M = \varphi_M(x, y, 0)$ [M is integer or a set of integers; e.g., i, j]
$\boldsymbol{g} = \mathfrak{g}$	gravitational acceleration vector $[= g_1\boldsymbol{e}_1 + g_2\boldsymbol{e}_2 + g_3\boldsymbol{e}_3]$
g	gravitational acceleration $[= 9.81 \text{ m s}^{-2}]$
g_i	components of $\boldsymbol{g}$ in the $Oxyz$-coordinate system ($i = 1, 2, 3$)
$\boldsymbol{G}_O(t)$	angular fluid momentum relative to the origin O
h	liquid depth
$\overline{h}$	nondimensional liquid depth scaled by tank breadth or length
H	wave height
H_t	tank height
$H_{1/3}$	significant wave height
$\boldsymbol{I}^0$	inertia tensor for a frozen liquid $[= \{I_{ij}^0\}]$
I	second moment of area with respect to the neutral axis for the beam problem
$\boldsymbol{J}^1(t)$	inertia tensor for sloshing $[= \{J_{ij}^1(t)\}]$
$\boldsymbol{J}_0^1$	linearized inertia tensor (time-independent) for sloshing $[\{J_{0ij}^1\}]$
$J_\alpha(\cdot)$	the Bessel function of the first kind [α is a real nonnegative number]
k or k_M	wave number; if M (integer or several integer indices, e.g., i, j, or a symbol) is present, the wave number for natural sloshing modes
KC	Keulegan–Carpenter number

l	characteristic linear dimension in two-dimensional statement; tank breadth for two-dimensional sloshing problem
l_b	length of a baffle
l_s	effective sloshing length
L	characteristic linear dimension in three-dimensional statement; the length of a ship; a typical dimension in some illustrative examples and exercises
L	Lagrangian
$L_t = L_1$	length of a tank in three-dimensional analysis
L_m	length in model scale
L_p	length in prototype scale
M	mass of an object in a three-dimensional statement
M_l	mass of a contained liquid in three-dimensional statement
$\boldsymbol{M}(t)$	fluid momentum
$\boldsymbol{M}_O^{\mathrm{Name}}(t)$	hydrodynamic moment relative to the origin O in the $Oxyz$-coordinate system; Name declares specific conditions on the considered fluid (e.g., filled, frozen) if needed $[= (M_{O1}, M_{O2}, M_{O3}) = (F_4^{\mathrm{Name}}, F_5^{\mathrm{Name}}, F_6^{\mathrm{Name}})]$
m	mass of an object in a two-dimensional statement, mass per unit length
m_k	spectral moments $[k = 0, 1, 2, \dots]$
m_l	mass of a contained liquid in two-dimensional statement
Ma	Mach number
$\boldsymbol{n} = (n_1, n_2, n_3)$	outer normal vector of a fluid volume
$\boldsymbol{n}^+$	normal vector with positive direction into a fluid volume $[= -\boldsymbol{n}]$
O	origin of the body-fixed coordinate system $Oxyz$
$O(\varepsilon)$	expresses the same order as a small parameter $\varepsilon \ll 1$
O'	the origin of the Earth-fixed (inertial) coordinate system $O'x'y'z'$
$Oxyz$	the body[tank]-fixed coordinate system
$O'x'y'z'$	the Earth-fixed [inertial] coordinate system
$o(\varepsilon)$	expresses higher order than a small parameter $\varepsilon \ll 1$
P	pressure impulse
$p(x, y, z, t)$	pressure
p_0	ullage pressure [= const]
p_a	atmospheric pressure
p_v	liquid vapor pressure
p_D	dynamic pressure
$Q(t)$	the liquid domain (in most cases, the tank liquid)
Q_0	the tank liquid domain in hydrostatic state

r	component of the cylindrical polar coordinate system (r, θ, z)
$\boldsymbol{r} = (x, y, z)$	radius vector of a point in the body-fixed coordinate system
$\boldsymbol{r}'$	radius vector of a point in the Earth-fixed coordinate system $[= \boldsymbol{r}'_O + \boldsymbol{r}]$
$\boldsymbol{r}_{lC}(t)$	radius vector of the mobile mass center of a contained liquid in the *Oxyz*-coordinate system $[= (x_{lC}(t), y_{lC}(t), z_{lC}(t))]$
$\boldsymbol{r}_{lC_0}$	radius vector of a contained liquid in the hydrostatic state in the *Oxyz*-coordinate system $[= (x_{lC_0}, y_{lC_0}, z_{lC_0})]$
$R_0[= \frac{1}{2}D_0]$	radius of a circular cylindrical tank or a circular spherical tank
r_0	radius of internal structures (e.g., poles) inserted into the liquid
$r_{jj}, j = 4, 5, 6$	radii of gyration
Ra	arithmetical mean roughness on the body surface
Rn and RE	Reynolds number, different definitions
Rn_{tr}	transition Reynolds number
$S(t)$	wetted tank surface
S_0	tank surface below the mean free surface
St	Strouhal number
S_Q	boundary enclosing the liquid volume Q [e.g., $\Sigma(t) + S(t)$]
t	time (s)
$\boldsymbol{t}$	tangential vector
T	period
T_0, T_1, and T_2	modal period and mean wave periods
T_M	for sloshing, natural sloshing periods [M is integer or a set of integers, e.g., i, j]
T_s	surface tension
T_d	duration of an external loading
T_{sc}	scantling draft
T_{mem}	membrane tension
T_{st}	tension of a string
u	the *Ox*-component of $\boldsymbol{v}$
u_1, u_2, u_3	see $\boldsymbol{v}$
u_r	see $\boldsymbol{v}_r$
U	characteristic velocity
U_g	gravity potential $[= -\boldsymbol{g} \cdot \boldsymbol{r} = -gz']$
$U_{sn} = U_n$	normal velocity component of a fluid surface; see $\boldsymbol{n}$
u_n	normal component of the fluid velocity on a fluid surface; see $\boldsymbol{n}$
$\boldsymbol{v}$	absolute fluid velocity $[= u\boldsymbol{e}_1 + v\boldsymbol{e}_2 + w\boldsymbol{e}_3 = (u, v, w) = (u_1, u_2, u_3)]$
$\boldsymbol{v}_r$	relative (with respect to the *Oxyz*-system) fluid velocity $[= u_r\boldsymbol{e}_1 + v_r\boldsymbol{e}_2 + w_r\boldsymbol{e}_3]$

v	the Oy-component of $\boldsymbol{v}$
v_r	see $\boldsymbol{v}_r$
v_O	velocity of the origin O [$= v_{O1}\boldsymbol{e}_1 + v_{O2}\boldsymbol{e}_2 + v_{O3}\boldsymbol{e}_3 = (v_{O1}, v_{O2}, v_{O3}) = (\dot{\eta}_1, \dot{\eta}_2, \dot{\eta}_3)$]
V	entry (vertical) velocity in slamming problems
Vol	fluid volume (area for two-dimensional case)
w	the Oz-component of $\boldsymbol{v}$
$\mathrm{w}(x, t)$	beam deflection
Wn	Weber number
w_r	see $\boldsymbol{v}_r$
W	the action; see eq. (2.80) [$= \int_{t_1}^{t_2} \mathsf{L}dt$]
(x_1, x_2, x_3)	(x, y, z)
$Y_\alpha(\cdot)$	Bessel function of the second kind [α is a real nonnegative number]

Greek symbols

α or α_i	used for definitions of different angles including the phase angle; auxiliary parameters
β	generalized coordinate in Lagrange variational formulation, deadrise angle
β_M	generalized coordinates in Lagrange variational formulation for multidimensional mechanical system, amplitudes of the natural sloshing modes in the modal representation of the free surface [M is integer or a set of integers, e.g., i, j]
χ	void fraction
δ	denotes variation of a functional value or generalized coordinate, e.g., $\delta\beta$, in variational formulations; boundary-layer thickness; a small distance when analyzing proximity effect of structures in Section 4.7.2.2
δ_{ij}	Kronecker delta
ε	formal small parameter in asymptotic analysis; the dimensionless forcing amplitude in multimodal method
$\Phi(x, y, z, t)$	velocity potential of the absolute velocity field $\boldsymbol{v}$ defined in the body-fixed coordinate system $Oxyz$
$\varphi_M(x, y, z)$	natural sloshing modes [M is integer or a set of integers, e.g., i, j]

γ	vortex density
$\eta_i(t)$	translatory ($i = 1, 2, 3$) and angular ($i = 4, 5, 6$) components of motions of the tank [body]-fixed coordinate system $Oxyz$ relative to an inertial coordinate system; also used for global ship motions [$i = 1, \ldots, 6$]
$\iota_{m,i}$	roots of the equation $J'_m(\iota_{m,i}) = 0$
$\kappa_M = \sigma_M^2/g$	spectral parameter of the problem on natural sloshing modes [M is integer or a set of integers, e.g., i, j]
κ	ratio of the specific heat
λ	wavelength
μ	dynamic viscosity coefficient
ν	kinematic viscosity coefficient
θ	component of the cylindrical polar coordinate system (r, θ, z)
Θ	angle measuring the wave propagating direction of elementary wave components in the sea relative to a main wave propagation direction
ρ	fluid density
ρ_l	liquid density
ρ_i	inner and exterior liquid density
ρ_o	ρ_o ullage gas density
ρ_g	gas density
ρ_c	gas density in the cushion
σ	circular forcing frequency or a frequency of an external wave
σ_M	wave frequencies; for sloshing, natural sloshing frequencies [M is integer or a set of integers, e.g., i, j]
σ_e	frequency of encounter
$\Sigma(t)$	free surface of a liquid during sloshing
Σ_0	mean free surface = hydrostatic liquid surface = unperturbed free surface
τ_l	laminar shear stress
τ_τ	turbulent shear stress
$\boldsymbol{\tau} = \{\tau_{ij}\}$	viscous stress components along the $(x_i - x_j)$-components $(i, j = 1, 2, 3)$
$\boldsymbol{\omega}(t)$	instant angular velocity of the tank (the $Oxyz$-coordinate system) with respect to an inertial coordinate system [$= (\omega_1(t), \omega_2(t), \omega_3(t))$]

$\omega_i(t)$	projections of the angular velocity $\boldsymbol{\omega}(t)$-vector in the $Oxyz$-coordinate system; equal to $\dot{\eta}_{i+3}(t)$, $i = 1, 2, 3$, for linear dynamics of the tank
$\boldsymbol{\Omega}(x, y, z, t)$	Stokes–Joukowski potential $[= (\Omega_1(x, y, z, t), \Omega_2(x, y, z, t), \Omega_3(x, y, z, t))]$
$\boldsymbol{\Omega}_0(x, y, z)$	Stokes–Joukowski potential for linear sloshing theory $[= (\Omega_{01}(x, y, z), \Omega_{02}(x, y, z), \Omega_{03}(x, y, z))]$
$\Omega(t)$	gas cushion volume
$\boldsymbol{\varpi}$	vorticity vector
ξ or ξ_M	(M is set of integers) damping ratio(s)
ζ	coefficient of bulk viscosity
ζ_a	amplitude of linear sea waves
$z = \zeta(x, y, t)$	normal representation of the free surface
$Z(x, y, z, t) = 0$	implicitly defined free surface

Preface and Acknowledgment

Our initial motivation for writing this book was to provide background on the analytically based *nonlinear* multimodal method for sloshing developed by the authors. We soon realized that we had to give a broader scope on sloshing and also present material on computational fluid dynamics (CFD), viscous flow, the effect of internal structures, and slamming. Furthermore, experimental results are to a large degree presented to validate the theoretical results and give physical insight.

A broad variety of CFD methods exist, and other textbooks provide details on different numerical methods. Our focus has been on giving an introduction to the many CFD methods that exist. An important aspect has also been to link the material to practical aspects. Our main application is for ship tanks, where sloshing can be very violent and slamming and coupling between sloshing and ship motions are important aspects. However, we have also emphasized links to other engineering fields with applications such as tuned liquid dampers for tall buildings, rollover of tanker vehicles, oil–gas separators used on floating ocean platforms, onshore tanks, and seiching in harbors and lakes; space applications are not addressed. Whenever possible we have tried to provide examples and have emphasized exercises where we provide hints and solutions. This fact has led to the development of simple analytical methods for analysis of, for instance, transient sloshing in spherical and horizontal circular cylindrical tanks, two-phase liquid flow, the effect of tank deformations, wave-induced hydroelastic analysis of a monotower with sloshing of water inside the shaft, flow through screens and swash bulkheads, and hydrodynamic analysis for automatic control of U-tanks.

Sloshing is a fascinating topic, and the first author was deeply involved in theoretical aspects of sloshing in liquefied natural gas tanks from the beginning of the 1970s, when he worked at Det Norske Veritas. Following that period was an approximately 20-year break in his activities with sloshing until he started again at the end of the past century. The second author has worked on spacecraft applications with particular emphasis on sloshing in fuel tanks, and since the beginning of the 1990s he has been involved with mathematical aspects of sloshing at the Institute of Mathematics, National Academy of Sciences of Ukraine, Kiev. It was their common interest in nonlinear multimodal methods for sloshing that brought them together at the Center for Ships and Ocean Structures (CeSOS), Norwegian University of Science and Technology (NTNU), Trondheim.

Mathematics is a necessity in reading the book, but we have tried to also emphasize physical explanations. Knowledge of calculus, including vector analysis and differential equations, is necessary to read the book in detail. The reader should also be familiar with dynamics and basic hydrodynamics of potential and viscous flow of an incompressible fluid. This book is more advanced from a theoretical point of view than the previous books *Sea Loads on Ships and Offshore Structures* and *Hydrodynamics of High-Speed Marine Vehicles* by the first author. Part of the book has been taught to graduate students at the Department of Marine Technology, NTNU. The book should be of interest for both engineers and applied mathematicians working with advanced aspects of sloshing. A pure mathematical language is avoided to better facilitate communication with readers with engineering backgrounds.

Quality control is an important aspect of writing a book, and we received help from both experts in different fields and graduate students. Dr. Svein Skjørdal of the Grenland Group, Sandefjord, and Dr. Martin Greenhow of Brunel University have been critical reviewers of all three books written by the first author. Dr. Skjørdal was helpful in seeing the topics from a practical point of view. The contributions by Dr. Olav Rognebakke, DNV, to several topics in the book are greatly appreciated.

Yanlin Shao read fastidiously through the text and asked many important questions that enabled us to clarify the text. In addition he has controlled calculations and provided solutions to all exercises. The detailed control of Dr. Hui Sun and Xiangjun Kong is also appreciated.

Professor Dag Myrhaug of NTNU and Professor J. M. R. Graham of Imperial College, London, critically reviewed Chapter 6 on viscous wave loads and damping.

Professor Marilena Greco of CeSOS and INSEAN and Professor G. X. Wu of University College provided important contributions to Chapter 10 on CFD and Chapter 11 on slamming. The expert help from Dr. Ould El Moctar of Germanischer Lloyds in reviewing Chapter 10 and by Professor Alexander Korobkin of University of East Anglia in reviewing Chapter 11 is also greatly appreciated.

Many other people should be thanked for their critical reviews and contributions, including Bjørn Abrahamsen, CeSOS; Professor Jørgen Amdahl, NTNU; Dr. Petter Andreas Berthelsen, CeSOS; Dr. Henrik Bredmose, University of Bristol; Dr. Claus M. Brinchmann, Rolls-Royce Intering Products; Dr. Chunhua Ge, Lloyd's Register of Shipping; Mateusz Graczyk, CeSOS; Xiaoyu Guo, CeSOS; Dr. Kjell Herfjord, StatoilHydro; Professor Changhong Hu, Kyushu University; David Kristiansen, CeSOS; Professor Carl Martin Larsen, CeSOS; Dr. Claudio Lugni, INSEAN; Professor J. N. Newman; Jan Arne Opedal, NTNU and DNV; Dr. Csaba Pakozdi, CeSOS; Professor Bjørnar Pettersen, NTNU; Dr. Hang Sub Urm, DNV; Tone Vestbøstad, CeSOS; and Dr. Zhu Wei, CeSOS.

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