STRUCTURAL ANALYSIS OF SHELLS

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PREFACE

The purpose of this book is to provide instructions, procedures, and solutions for the static analysis of aerospace, civil, and mechanical engineering shell structures. This book also provides an introduction to and reference for the theory of shells.

To a great extent, much of the material from which this book was developed was obtained from the "Shell Analysis Manual," NASA CR 912. The "Shell Analysis Manual" was prepared for the National Aeronautics and Space Administration, Manned Spacecraft Center, Houston, Texas, by North American Rockwell Corporation, Space Division, Downey, California, under Contract NAS9-4387, for which Mr. Herbert C. Kavanaugh, Jr., was the NASA technical monitor.

Generally, the information presented in this book is a condensation of material published by U.S. Government agencies, universities, scientific and technical journals, text books, aerospace industries, including North American Rockwell Corporation, and foreign publications. Particular credit is given to the following publishers who granted permission to use their publications.

American Institute of Aeronautics and Astronautics, New York, New York.

American Rocket Society Journal, Vol. 31, No. 2, February 1961, pp. 237–246, "Stability of Orthotropic Cylindrical Shells Under Combined Loading" by T. E. Hess.

Journal of the Aerospace Sciences, Vol. 29, No. 5, May 1962, pp. 505–511, "Elastic Stability of Orthotropic Shells" by H. Becker and G. Gerard.

American Concrete Institute, Detroit, Michigan.

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Journal of the Structural Division, Vol. 82, July 1956, "Bending Stresses in Edge Stiffened Domes " by M. G. Salvadori and Shermann.

R. Oldenbourg Verlage, München, West Germany.

Drang and Swang by L. Föppl, 1941, 1944, and 1947.

Springer Verlag, Berlin, West Germany.

Stresses in Shells by W. Flügge, 1957.

Wilhelm Ernst and Sohn Verlag, Berlin, West Germany.

Beton Kalender by G. Worch, 1943 and 1958.

We also found the books *Elementary Statics of Shells* by A. Pflüger and *Statik Rotationssymmetrischer Flächentragwerke* by E. Hampe to be of great benefit.

Detailed derivations of formulas are limited because it is not believed to serve the purpose of this book. Numerous references to more detailed discussions are given.

The book has been developed primarily from existing material in the field of shells. The original works are referenced in the bibliography.

Chapter 2 outlines the force method for shells and simpler multishells which are combined from not more than two shell elements.

Chapter 3 presents the primary solutions needed for the force method for many shell geometries for many loadings.

Chapter 4 presents the secondary solutions for the same purpose.

Chapter 5 presents some special cases such as cylinders and spheres with different boundary conditions. Also the solution of interaction for a cylinder with abrupt change of wall thickness.

Chapter 6 finally presents the force method for complicated multishells with more than two shell elements.

Chapter 7 treats composite shells, reducing them to the same methods which were explained previously.

Chapter 8 presents the special cases of unsymmetrical shells (unsymmetrical due to geometry or loading).

Chapter 9 treats allowables and margins of safety for the biaxial state of stress as occurs in a shell structure. This chapter concludes the static analysis of multishells.

Chapter 10 is the first chapter in which stability is presented. The monocoque shells are discussed and formulas are presented.

Chapter 11 continues the stability analysis of shells, treating orthotropic shells in general.

Chapter 12 presents in more detail stability of stiffened shells.

Chapter 13 presents stability of sandwich shells.

This book was written by engineers for engineers and for the personal usage of the authors who participated in writing of this document and whose names are listed in alphabetical order. It is the authors' hope that this book will be not only useful to the practicing engineers but also for the students who would like to extend their knowledge of shell analysis. To help them, primarily, the introductory chapter is included which contains basic derivations which are needed for good understanding of shell analysis. An experienced engineer can simply omit this introductory chapter.

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CONTENTS

Chapter 1

INTRODUCTION TO THE THEORY OF SHELLS

1-1 General

The most common shell theories are those based on linear elasticity concepts. Linear shell theories adequately predict stresses and deformations for shells exhibiting small elastic deformations, that is, deformations for which it is assumed that the equilibrium-equation conditions for deformed elements are the same as if they were not deformed and Hooke's law applies.

The nonlinear theory of elasticity forms the basis for the finite-deflection and stability theories of shells. Large-deflection theories are often required when dealing with shallow shells, highly elastic membranes, and buckling problems. The nonlinear shell equations are considerably more difficult to solve and for this reason are more limited in use.

Development of more exact theoretical expressions does not necessarily assist in the solution of practical shell problems, since often the theoretical expressions can be solved only with great difficulty, and then only for special cases. The experimental approach is also limited because data are not available for every special case.

Practical difficulties in both theory and experiment have led to the development and application of applied engineering methods for the analysis of shells. While these methods are approximate and are valid only under specific conditions, they generally are very useful and give good accuracy for the analysis of practical engineering shell structures.

1-2 Linear Shell Theory

The theory of small deflections of thin elastic shells is based upon the equations of the mathematical theory of linear elasticity. The geometry of shells (i.e., one dimension much smaller than the other dimensions) does not warrant, in general, the consideration of the complete three-dimensional elasticity equations. In fact, the consideration of the complete elasticity equations leads to expressions and equations which are so complicated that it becomes impossible to obtain solutions for shell problems of practical interest.

Fortunately, however, sufficiently accurate analyses of thin shells can be obtained using simplified versions of the general elasticity equations. In the development of thin-shell theories, simplification is accomplished by reducing the shell problem to the study of the deformations of the middle (or reference) surface of the shell. In all cases, one begins with the governing equations in the three-dimensional theory of elasticity and attempts to reduce the system of equations, involving three independent space variables, to a new system involving only two space variables. These two variables are more conveniently taken as coordinates on the middle surface of the shell.

Shell theories of varying degrees of accuracy may be derived, depending upon the degree to which the elasticity equations are simplified. The approximations necessary for the development of an adequate theory of shells have been the subject of considerable controversy among investigators in the field. A brief discussion of the approximations is presented in Sec. 1-11. The theory presented in Secs. 11-8 and 11-9 is a first-order-approximation shell theory for axisymmetrically loaded shells of revolution.

1-3 Geometry of Shells

Before shell theory is discussed, the geometry of an arbitrary shell in three-dimensional space is defined. The geometry of a shell is entirely defined by specifying the form of the middle surface and the thickness of the shell at each point. To describe the form of the middle surface, it is necessary to present some of the important geometrical properties of a surface. A more detailed presentation of the theory of surfaces can be found in books on tensor analysis and differential geometry.

In the engineering application of thin shells, a shell whose reference surface is in the form of a surface of revolution has extensive usage. This discussion is restricted to surfaces of revolution. A surface of revolution is obtained by rotation of a plane curve about an axis lying in the plane of the curve. This curve is called the meridian, and its plane is the meridian plane. The intersections of the surface with planes perpendicular to the axis of rotation are parallel circles and are called parallels.

For such a shell the lines of principal curvature are its meridians and parallels. The following nomenclature is given in Fig. 1-1.

ϕ = angle between the axis of the shell and the shell normal at the point under consideration on the middle surface of the shell
θ = angle between r and any defined line ξ

The radii of curvature of a shell of revolution are

R_ϕ = radius of curvature of meridian
R_θ = length of the normal between any point on the middle surface and the axis of rotation
r = radius of curvature of the parallel
R_ϕ and R_θ = principal radii of curvature of the surface

The following geometrical relation is of fundamental importance:

$$r = R_\theta \sin \phi$$

1-4 External Loadings

The external loads consist of body forces that act on the element and surface forces that act on the upper and lower surfaces of the shell element.

All loadings under consideration at any point on the shell can be resolved into three components in the x, y, and z directions. The x direction is parallel to the tangent to the meridian. The y direction is parallel to the tangent to the parallel circle, and the z direction is normal to the surface of the shell. For example: The deadweight p (weight of shell per unit area) for a shell of revolution can be resolved into load per unit area in the x, y, and z directions, respectively, in the following manner (Fig. 1-2):

$$p_x = p \sin \phi \qquad p_y = 0 \qquad p_z = p \cos \phi$$

1-5 Internal Stresses

The external forces are resisted by internal forces, or stresses, which are in equilibrium with the external forces. It is convenient to investigate the stresses along a meridian and parallel, which are defined by the angles ϕ and θ.

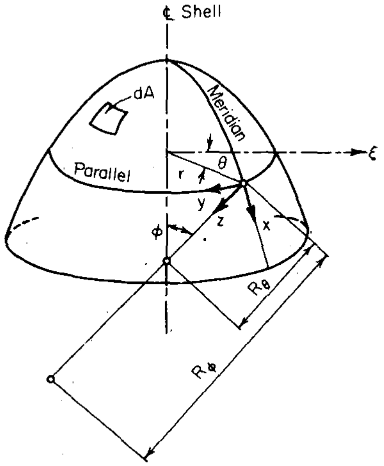

figure 1-1 *Shell of revolution.*

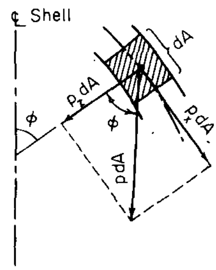

figure 1-2 *Loading components from deadweight.*

The internal forces consist of membrane forces, transverse shears, bending moments and twisting moments.

1. The membrane forces, which act in the plane of the surface of shell, are shown in Fig. 1-3.

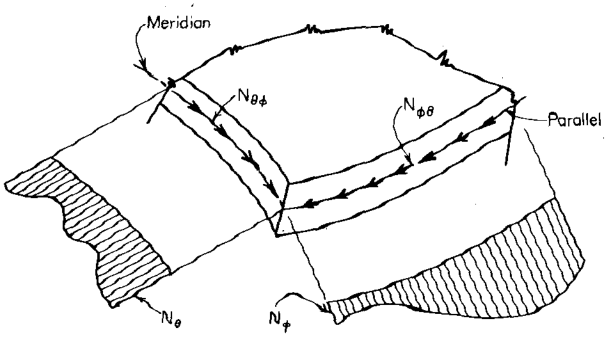

figure 1-3 *Membrane forces.*

N_ϕ, N_θ = normal inplane forces per unit length (load/unit length)
$N_{\phi\theta}$, $N_{\theta\phi}$ = inplane shear forces per unit length (load/unit length)
These forces can vary along the meridian and parallel (see Fig. 1-3).

2. The transverse shear forces per unit length Q_ϕ and Q_θ are shown in Fig. 1-4.

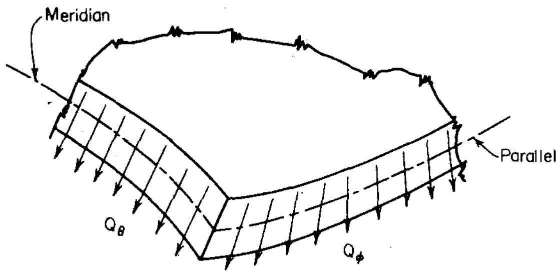

figure 1-4 *Transverse shear forces.*

3. Bending moments M_ϕ and M_θ per unit length and twisting moments $M_{\phi\theta}$ and $M_{\theta\phi}$ per unit length are shown in Fig. 1-5.

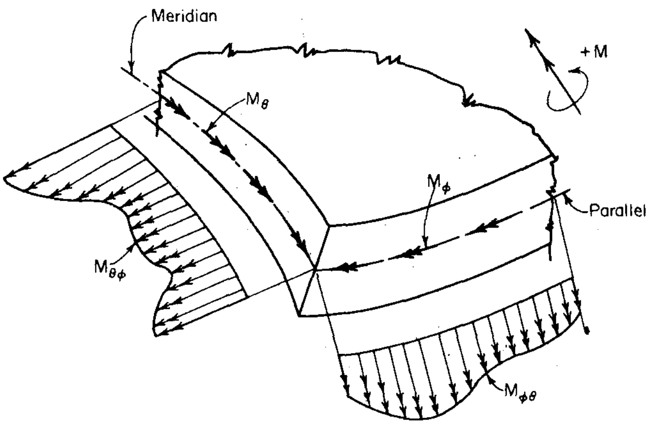

figure 1-5 *Bending and twisting moments.*

The positive directions of all stresses under 1, 2, and 3 are shown in the corresponding figures. All positive loadings act in the positive direction of the system of coordinates.

In the preceding section, all internal forces are replaced by statically equivalent forces and moments.

1-6 Condition of Equilibrium

The conditions for equilibrium of the shell element under external and internal loads will be determined. The equations arising by virtue of the demands of equilibrium and the compatibility of deformations will be derived by considering an individual differential shell element. These equations are relations between differential quantities or between differential changes in the internal forces and therefore are called differential equations. If a differential element is imagined separated from the loaded shell, it is stressed by 10 internal components which must be in equilibrium with the external loads.

$$N_\phi, N_\theta, N_{\phi\theta}, N_{\theta\phi}, Q_\phi, Q_\theta, M_\phi, M_\theta, M_{\phi\theta}, M_{\theta\phi}$$

To determine these components, there are known only six equilibrium equations:

$$\begin{aligned} \sum F_x &= 0 & \sum M_x &= 0 \\ \sum F_y &= 0 & \sum M_y &= 0 \\ \sum F_z &= 0 & \sum M_z &= 0 \end{aligned} \tag{1-1}$$

where $\sum F_i$ is the sum of the force in the i direction ($i = x, y, z$) and $\sum M_i$ is the sum of the moments about the i axis. This problem is four times internally statically indeterminate.

1-7 Membrane Theory for Shells of Revolution

Consider a truss structure, which is physically many times internally statically indeterminate. This complicated problem can be simplified by assuming all joints of the truss are pinned. This means that each member of the truss is stressed only axially. End moments and shears are zero, and the truss is analyzed as an internally statically determinate structure.

Similar assumptions may be introduced in the shell equations:

$$M_\phi = M_\theta = M_{\phi\theta} = M_{\theta\phi} = Q_\phi = Q_\theta = 0$$

Consequently, only four unknowns remain:

$$N_\phi, N_\theta; N_{\phi\theta}, N_{\theta\phi}$$

which are called the membrane forces. If a shell theory includes only the membrane forces in the analyses, it is called a membrane theory. Certain restrictions in the use of membrane theory will be discussed in Chap. 2.

Figure 1-6 shows a differential element of the shell whose area may be expressed

$$dA = r\, d\theta\, R_\phi\, d\phi$$

Figure 1-7 shows all forces in equilibrium which may act on a differential element in the membrane theory. The components of the external