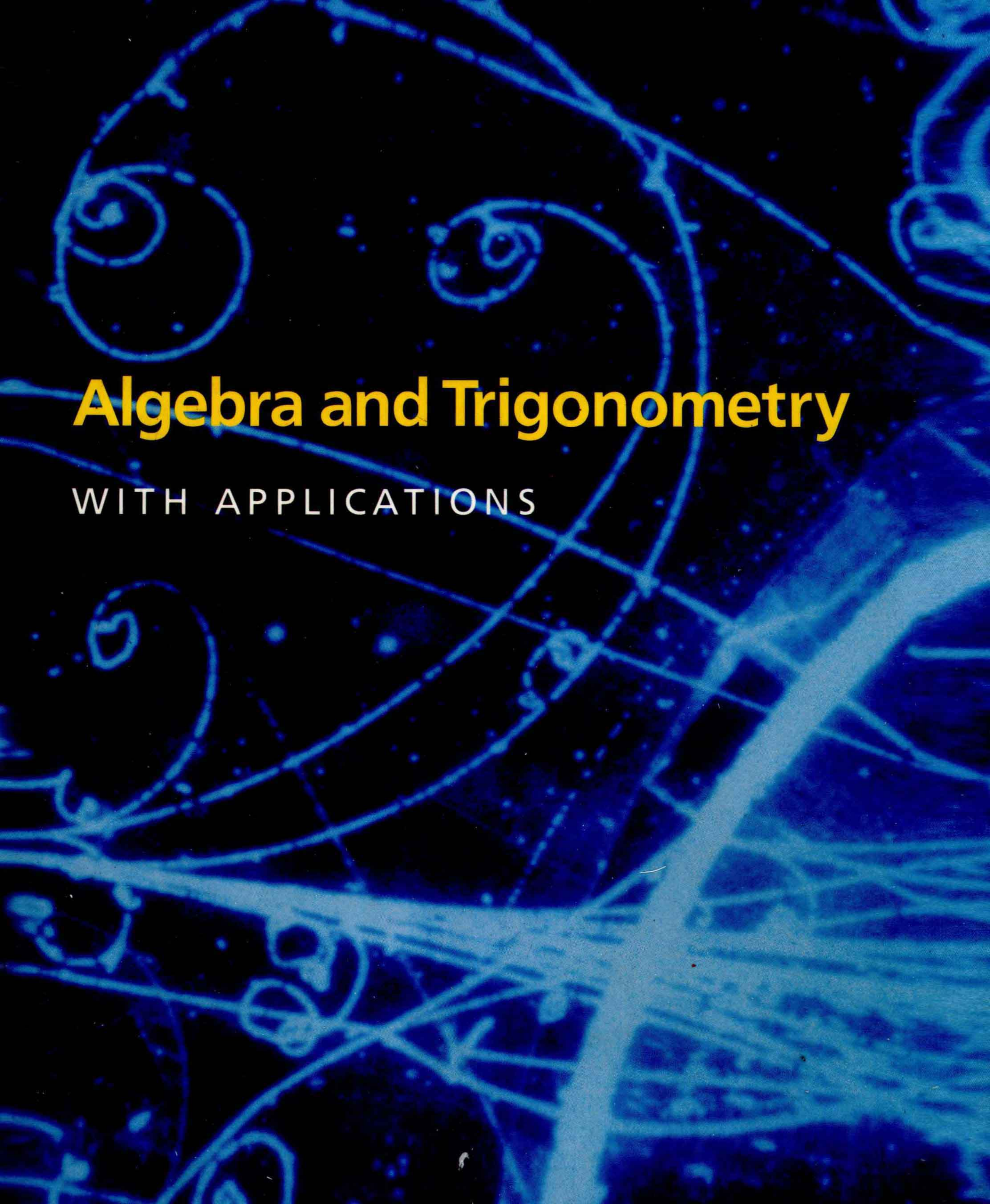
Algebra and Trigonometry

WITH APPLICATIONS



THIRD EDITION

Algebra and Trigonometry

WITH APPLICATIONS

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Preface

The third edition of *Algebra and Trigonometry* is being published at the beginning of a new era in the teaching of mathematics in our colleges and universities. Educators agree that this decade will see major changes in the way mathematics is taught—critical thinking will be nurtured, rote learning will be de-emphasized, and mathematical comprehension will be fostered by increased classroom use of electronic computers and calculators. In revising our textbook, we have tried to provide a measured response to these trends while continuing to offer students a straightforward, readable book.

Prerequisites

In writing and revising this textbook, we had in mind a typical reader with the equivalent of two years of college-preparatory mathematics in algebra and plane geometry, or one who has taken a college-level course in introductory algebra. Determined students with less preparation should be able to use the textbook successfully, particularly if they supplement it with the accompanying *Student Guide with Solutions.*

Presentation

Topics are presented in brief sections that develop logically from basic to more advanced skills and concepts. Motivation for new ideas is provided by showing their application in real-world situations. Numerous illustrative examples are worked out in detail. When appropriate, specific problem-solving procedures are given.

Problems

Problems at the end of each section begin with simple drill-type exercises to build student confidence. A gradual progression to more advanced problems invites students to work to the best of their abilities.

Odd-numbered problems Many of the odd-numbered problems, particularly those at the beginning of each problem set, are similar in scope to the worked-out examples in the text. Answers to most of these problems, with appropriate graphs, are given in the back of the book. In general, *odd-numbered problems can be assigned for homework with an expectation of success by most students.*

Even-numbered problems Some of the even-numbered problems, particularly those toward the end of each problem set, are considerably more challenging than the odd-numbered ones. These problems are meant to encourage critical thinking and to confirm mastery of mathematical concepts and techniques. *Even-numbered problems toward the ends of the problem sets should be assigned with care to avoid demoralizing students who are not prepared to handle them.*

Colored problem numbers Problems with red numbers constitute a good representation of the main ideas of each section and thus could provide a suitable basic set for homework assignments. Instructors using these problems for homework may wish to augment the basic set with a few carefully selected even-numbered problems.

Calculator problems Problems for which the use of a calculator is appropriate are marked with the symbol c. *In many of these problems we have purposely avoided using "round numbers" so students will be prepared to work with numbers that actually arise in applied mathematics.* In particular, this affords an opportunity for students to practice using correct rounding-off techniques.

Review problem sets The review problems at the end of each chapter can be used in a variety of ways: Instructors may wish to use them for supplementary or extra-credit assignments or as a source of problems for quizzes and exams. Students may wish to use them to pinpoint areas where further study is needed. Often, these problems are not arranged by section so that students can practice recognizing types of problems without using the placement of the problem in the problem set as a clue.

Major Changes in this Edition

In keeping with the current trend toward de-emphasizing rote learning and fostering understanding and critical thinking, we have made a slight reduction in the number of drill problems and added a few more problems that test for comprehension. In the same spirit, we have rewritten and reorganized textual material to serve these ends. We are placing even more stress on the important idea of a mathematical model, and thus have moved the section on ratio, proportion, and variation up to Section 3 of Chapter 1. Other significant changes are as follows:

Calculators We have moved the introductory material on the use of calculators up to Section 2 of Chapter 1 and made extensive use of calculators in examples and problems throughout the book. We assume that *all* students using the book have access to a basic scientific calculator.

Graphing calculators Although we have indicated how graphing calculators can be used to facilitate the solution of problems, *the use of a graphing calculator is optional* in this edition. Examples and problems for which it would be reasonable to use a graphing calculator are marked with the symbol gc.

Numerical methods The discussion of error involved in numerical calculations (pages 13 to 16) is now supplemented by a consideration of absolute, relative, and percent error (pages 129 to 130). More emphasis has been given to the bisection method for finding zeros (pages 263 to 265), and material on the use of function iteration and fixed points has been added (pages 130, 203 to 204, and 265 to 266).

Applications More applications of exponential functions to car and mortgage payments are now given in Section 5.1, applications involving right triangles have been moved up to Section 6.3, and the idea of mathematical expectation is applied to games of chance in Section 11.6. (Consult the *Index of Applications*, pages 721 to 725, to locate these and other applied examples and problems.)

Chapter tests New to this edition are brief tests at the end of each chapter. Students can use these for practice and review and to determine readiness for class tests. All answers for chapter tests are provided at the back of the book.

Preparation for calculus More examples and problems have been added pertaining to algebraic and trigonometric ideas that are used in calculus.

Available Student Aids

Student Guide with Solutions The student guide has been thoroughly revised. It now includes worked-out solutions to every other odd-numbered problem in the textbook as well as supplementary problems in a tutorial format for each chapter. In addition, there are two practice tests for each chapter—a multiple-choice test and a problem-solving test. All answers and solutions to the tutorial problems and to the chapter tests are provided in the guide itself.

Graphics Discoveries Alice M. Kaseberg, Steven L. Myers, and Robert B. Thompson (Lane Community College) have prepared a guide that will assist the student in exploring key topics in algebra and trigonometry using currently available graphing calculators.

Computer Software Robert J. Weaver of Mount Holyoke College has prepared instructional software on graphing polynomial functions, using the division algorithm, approximating zeros of polynomial functions using the bisection method, and solving general triangles. His disk, for use with IBM and compatible computers, is available to adopters on request.

Also, the *DERIVE*® computer software system is available at a special discount price to teachers and students in courses where this book has been adopted. For information regarding *DERIVE*, please call 1-800-255-2468.

Available Instructor Aids

Solutions Manual Step-by-step solutions to all problems in the textbook are available in this manual, prepared by Hyla Gold Foulis. A glance at the worked-out solutions will help the instructor to select homework problems at an appropriate level of difficulty for his or her class.

Test Bank For each of the eleven chapters in *Algebra and Trigonometry*, the *Test Bank* provides instructors with 25 to 30 multi-version base, or template, questions.

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These questions are keyed to the learning objectives from the student's guide and cover the main ideas in each chapter. Approximately two-thirds of the 25 to 30 base questions are problem-solving questions, and the remainder are multiple-choice. Each of the base questions appears in six different versions—three of them easy to medium and the other three medium to difficult—for a total of 150 to 180 questions per chapter.

Several versions of a computerized test-generation system make exam preparation quick and easy.

Test Manual Drawing on the questions in the *Test Bank*, the *Test Manual* provides six ready-made tests for each chapter. Four of these are short-answer tests—two easy to medium and two medium to difficult—and the remaining two are multiple-choice tests—one easy to medium and one medium to difficult. The tests are intended for use in a standard fifty-minute class.

The Video Tutor and The Video Tutor Learning Guides Thirty-five hours of videotaped lectures (in 15-minute segments) accompanied by learning guides for students have been prepared by A. E. T. Bentley (Capilano College). These offer detailed coverage of all topics in Chapters 1 to 7.

Overhead Transparencies A set of two-color acetate transparencies is available for use on an overhead projector. These include important figures in the textbook and statements of key definitions and theorems to use in lectures.

Acknowledgments

In revising our textbook, we have again drawn on our own experiences in teaching algebra and trigonometry and on feedback from our students. Especially valuable were the suggestions from other instructors who used the book and the thoughtful comments of our reviewers. We wish to thank all of these people and, in particular, to express our gratitude to the following:

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M. A. Munem
D. J. Foulis

A Note to the Instructor on the Use of Calculators

Problems and examples for which the use of a calculator is recommended are marked with the symbol [c]. Answers and solutions were obtained using an HP-32S calculator—other calculators that use different internal routines may give slightly different results. In some cases we have carried out computations to the full number of significant digits available on our calculator. This will permit students to check their own calculator work and to see for themselves that different calculators may give answers that differ in the last decimal place or so. Instructors should alert their students to this possibility.

Problems for which the (optional) use of a graphing calculator is reasonable are marked with the symbol [gc]. Instructors who require the use of a graphing calculator may wish to have their students use the *Graphics Discoveries* by Alice M. Kaseberg, Steven L. Myers, and Robert B. Thompson as a supplement to this textbook.

Instructors should stress that it is undesirable to use a calculator (or graphing calculator) for problems that are *not* marked with the symbol [c] (or [gc]). The use of a calculator (or graphing calculator) for such problems can actually hinder the student's understanding.

The rule for rounding off numbers presented in Section 1.2 is consistent with the operation of most calculators with round-off capability. Some instructors may wish to mention the popular alternative round-off rule: If the first dropped digit is 5 and there are no nonzero digits to its right, round off so that the last retained digit is even.

Instructors should encourage their students to learn to use calculators efficiently—for instance, to do chain calculations using the memory features of the calculator. In some of our examples, we have shown the intermediate results of chain calculations so the students can check their calculator work; however, it should be emphasized that it is not necessary to write down these intermediate results when using the calculator.

Because there are so many different calculators available, we have not given detailed key-stroke instructions for calculator operation in this textbook. Students should be encouraged to consult the instruction manuals furnished with their calculators. This is particularly important in connection with the use of Horner's method for evaluating polynomials (page 248).

Finally, we have made no attempt to provide a systematic discussion of the inaccuracies inherent in computations with a calculator. An excellent account of calculator inaccuracy can be found in "Calculator Calculus and Roundoff Errors" by George Miel in *The American Mathematical Monthly*, 1980, vol. 87, pp. 243–252.

Contents

Appendices

In algebra, the notation $x \times y$ for the product of x and y is not often used because of the possible confusion of the letter x with the multiplication sign $\times$. The preferred notation is $x \cdot y$ or simply xy. Similarly, the notation $x \div y$ is usually avoided in favor of the fraction $\frac{x}{y}$ or x/y.

Algebraic notation—the "shorthand" of mathematics—is designed to clarify ideas and simplify calculations by permitting us to write expressions compactly and efficiently. For instance, $x + x + x + x + x$ can be written simply as $5x$. The use of exponents provides an economy of notation for products; for instance, $x \cdot x$ can be written simply as x^2 and $x \cdot x \cdot x$ as x^3. In general, if n is a positive integer,

$$x^n \quad \text{means} \quad \overbrace{x \cdot x \cdot x \cdots x}^{n \text{ times}}$$

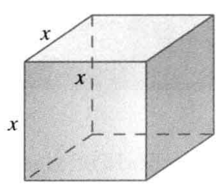
and

$$x^{-n} \quad \text{means} \quad \frac{1}{x^n}.$$

In using the **exponential notation x^n**, we refer to x as the **base** and n as the **exponent,** or the **power** to which the base is raised. When the exponent is negative, we must assume that the base is nonzero to avoid zero in the denominator.

By writing an **equals sign** (=) between two algebraic expressions, we obtain an **equation,** or **formula,** stating that the two expressions represent the same number. Using equations and formulas, we can express mathematical facts in compact, easily remembered forms. Formulas are used to express relationships among various quantities in such fields as geometry, physics, engineering, statistics, geology, business, medicine, economics, and the life sciences. Calculating the numerical value expressed by a formula when particular numbers are assigned to letters is known as **evaluation.**

Figure 1



Example 1 **(a)** Write a formula for the volume V of a cube that has edges of length x units (Figure 1).

(b)* Evaluate V when $x = 5.23$ centimeters.

Solution **(a)** $V = x \cdot x \cdot x = x^3$ cubic centimeters.

(b) When $x = 5.23$ centimeters, $V = (5.23)^3 = 143.055667$ cubic centimeters. ■

Example 2 A certain type of living cell divides every hour. Starting with one such cell in a culture, the number N of cells present at the end of t hours is given by the formula $N = 2^t$. Find the number of cells in the culture after 6 hours.

* Problems for which the use of a calculator is suggested are marked with the symbol [c]. Recommendations for the use of a calculator are given in Section 1.2.

CHAPTER

1

Concepts of Algebra

The gravitational forces that govern the motions of the planets can be calculated algebraically by using Newton's law of universal gravitation.

This chapter is designed as a review of the basic concepts and methods of algebra. Its purpose is to help you attain the algebraic skills that are required throughout the textbook. Topics covered include the language and symbols of algebra, polynomials, fractions, exponents, radicals, and complex numbers.

1.1 The Algebra of Real Numbers

Algebra begins with a systematic study of the operations and rules of arithmetic. The operations of addition, subtraction, multiplication, and division serve as a basis for all arithmetic calculations. In order to achieve generality, letters of the alphabet are used in algebra to represent numbers. A letter such as x, y, a, or b can stand for a particular number (known or unknown), or it can stand for any number at all. A letter that represents an arbitrary number is called a **variable.**

The sum, difference, product, and quotient of two numbers, x and y, can be written as

$$x + y, \qquad x - y, \qquad x \times y, \qquad \text{and} \qquad x \div y.$$

Solution Substituting $t = 6$ in the formula $N = 2^t$, we find that

$$N = 2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64 \text{ cells.}$$

Basic Algebraic Properties of Real Numbers

The numbers used to measure real-world quantities such as length, area, volume, speed, electrical charge, efficiency, probability of rain, intensity of earthquakes, profit, body temperature, gross national product, growth rate, and so forth, are called **real numbers.** They include such numbers as

$$5, \quad -17, \quad \frac{17}{13}, \quad -\frac{2}{3}, \quad 0, \quad 2.71828, \quad \sqrt{2}, \quad -\frac{\sqrt{3}}{2}, \quad 3 \times 10^8, \quad \text{and} \quad \pi.$$

The basic algebraic properties of the real numbers can be expressed in terms of the two fundamental operations of addition and multiplication.

Basic Algebraic Properties of Real Numbers

Let a, b, and c denote real numbers.

1. *The Commutative Properties*

(i) $a + b = b + a$ **(ii)** $a \cdot b = b \cdot a$

The commutative properties say that the *order* in which we either add or multiply real numbers doesn't matter.

2. *The Associative Properties*

(i) $a + (b + c) = (a + b) + c$ **(ii)** $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

The associative properties tell us that the way real numbers are *grouped* when they are either added or multiplied doesn't matter. Because of the associative properties, expressions such as $a + b + c$ and $a \cdot b \cdot c$ make sense without parentheses.

3. *The Distributive Properties*

(i) $a \cdot (b + c) = a \cdot b + a \cdot c$ **(ii)** $(b + c) \cdot a = b \cdot a + c \cdot a$

The distributive properties can be used to expand a product into a sum, such as

$$a(b + c + d) = ab + ac + ad,$$

or the other way around, to rewrite a sum as a product:

$$ax + bx + cx + dx + ex = (a + b + c + d + e)x.$$