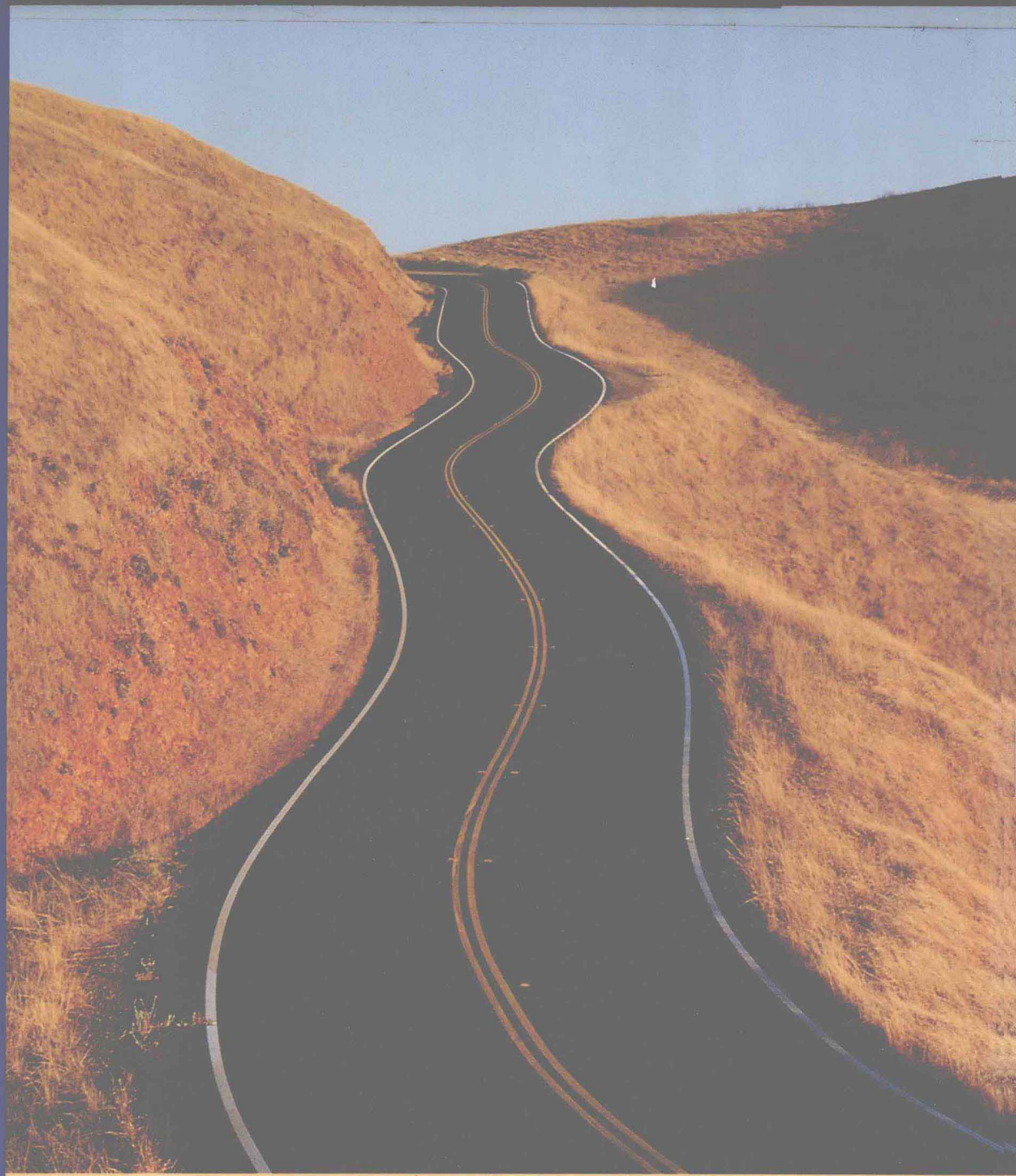
INTERMEDIATE ALGEBRA

fifth edition



JEROME E. KAUFMANN

Intermediate Algebra

FIFTH EDITION

Jerome E. Kaufmann

PWS Publishing Company

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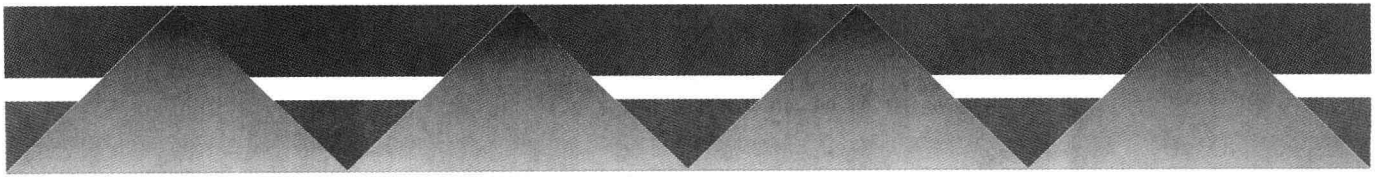

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Intermediate Algebra

PREFACE

Intermediate Algebra, Fifth Edition, is written for college students who need an algebra course that bridges the gap between elementary algebra and the more advanced courses in precalculus mathematics. It covers topics that are usually classified as intermediate algebra topics.

The basic concepts of intermediate algebra are presented in a simple, straightforward manner. Algebraic ideas are developed in a logical sequence, but in an easy-to-read manner without excessive formalism. Concepts are developed through examples, continuously reinforced through additional examples, and then applied in a variety of problem-solving situations.

In the preparation of this edition, special effort was made to incorporate improvements suggested by reviewers and by users of the earlier editions, while at the same time preserving the book's many successful features.

New in This Edition

- Problems called **Thoughts into Words** are now included in every problem set except the review exercises. These problems are designed to encourage students to express in written form their thoughts about various mathematical ideas. See, for example, Problem Sets 2.1, 3.5, 4.7, 5.5, and 6.6.
- The Miscellaneous Problems of previous editions, now called **Further Investigations**, have been enhanced by the addition of more problems that lend themselves to small group work. These problems remain as "extras" but add flexibility for the instructor. See, for example, Problem Sets 1.2, 2.7, 5.6, 6.5, and 7.5.
- A **Chapter Test** has been included at the end of each chapter. Along with the **Chapter Review Problem Sets**, these practice tests should provide students with ample opportunity to prepare for "real" examinations. **Cumulative Review Problem Sets** appear at the ends of Chapters 3, 5, 7, and 9.
- The **chapter introductions** have been rewritten in an effort to provide more motivation for students to study algebra. Each introduction begins with at least one application that leads into the material of the chapter.
- **Applications** have been added in several sections, including the following:

  Sections 3.1, 3.2, and 3.3: Examples and problems that connect geometry and the study of polynomials

  Section 5.2: Applications involving radicals

Section 5.5: Applications involving radical equations
Section 6.2: Applications of the Pythagorean theorem
Section 7.4: Applications of slope

- The use of a **graphing utility** is introduced in Section 7.1. Graphics calculator examples (designated by an icon) are then incorporated, as appropriate, throughout Chapters 7 through 10. These examples are written so that students without a graphing utility can read and benefit from them.
- A new section of problems called **Graphics Calculator Activities** has been added to many of the problem sets in Chapters 7 through 10. These activities, which are appropriate for either individual or small group work, have been designed to reinforce concepts already presented and lay the groundwork for concepts about to be discussed. They also help students to predict shapes and locations of graphs based on earlier graphing experiences. Through working these problems, students should become more familiar with the capabilities and limitations of a graphics calculator.
- Parts of **Chapter 8** have been reorganized and a new Section 8.3 has been added. Section 8.2 now discusses linear and quadratic functions with some applications. New Section 8.3 presents transformations of some basic curves. These transformations are then used as appropriate in subsequent sections. Section 8.4 covers the composition of functions in preparation for inverse functions presented in Section 8.5.
- A focal point of every revision is the **problem sets**. Users of the previous editions were very helpful in suggesting problems to be added, deleted, or changed in some way.

Other Special Features

- A common thread runs throughout the book: namely, *learn a skill,* next *use the skill to help solve equations and inequalities,* and then *use equations and inequalities to solve word problems.* This thread influenced some other decisions.
  1. Approximately 450 word problems are scattered throughout the text. These problems deal with a large variety of applications and constantly show the connections between mathematics and the real world.
  2. Many problem-solving suggestions are offered throughout, with special discussions in several sections. The problem-solving suggestions are demonstrated in more than 80 worked-out examples.
  3. Newly acquired skills are used as soon as possible to solve equations and inequalities, which are, in turn, used to solve word problems. Therefore, the concept of solving equations and inequalities is introduced early and developed throughout the text. The concepts of factoring, solving equations, and solving word problems are tied together in Chapter 3.

- As recommended by the American Mathematical Association of Two-Year Colleges, many basic geometric concepts are integrated in a problem-solving setting. Contained in this text are approximately 20 worked-out examples and 100 problems that connect algebra, geometry, and the real world. Specific discussions of geometric concepts are contained in the following sections:

  Section 2.2: Complementary and supplementary angles; the sum of the angles of a triangle equals 180°

  Section 2.4: Area and volume formulas

  Section 3.4: More on area and volume formulas, perimeter, and circumference formulas

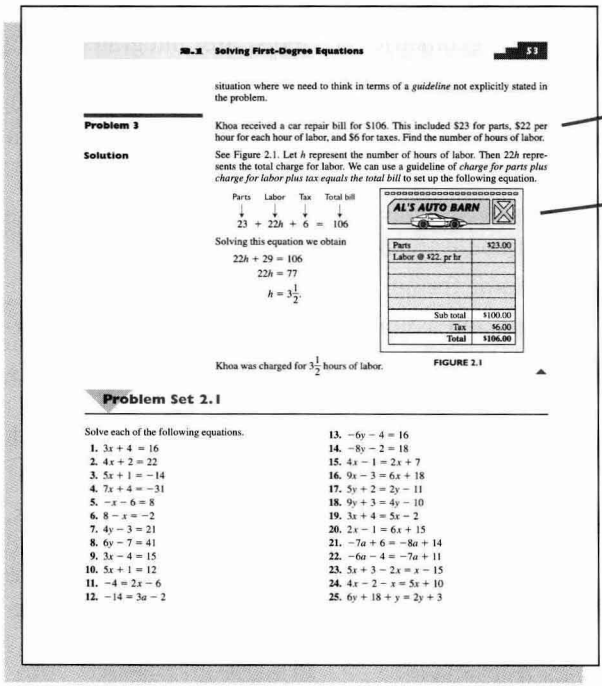
  Section 3.7: Pythagorean theorem

  Section 6.2: More on the Pythagorean theorem, including work with isosceles right triangles and 30°–60° right triangles

- Specific graphing ideas (intercepts, symmetry, restrictions, asymptotes, and transformations) are introduced and used throughout Chapters 7 and 8. In Section 8.3 the work with parabolas from Chapter 7 is used to develop definitions for translations, reflections, stretchings, and shrinkings. These transformations are then applied to the graphs of $f(x) = x^3$, $f(x) = \frac{1}{x}$, $f(x) = \sqrt{x}$, and $f(x) = |x|$.
- All answers for Chapter Review Problem Sets, Chapter Tests, and Cumulative Review Problem Sets appear in the back of the text.

Additional Comments About Some of the Chapters

- Chapter 1 is written so that it can be covered quickly, and on an individual basis if so desired, by those needing only a brief review of some basic algebraic concepts.
- Chapter 2 presents an early introduction to the heart of an intermediate algebra course. Problem solving and the solving of equations and inequalities are introduced early so they can be used as unifying themes throughout the text.
- Chapter 6 is organized to give students the opportunity to learn, on a day-to-day basis, different techniques for solving quadratic equations. The process of completing the square is treated as a viable equation-solving process for certain types of quadratic equations. The emphasis on completing the square in this setting pays dividends in Chapter 7 when we graph parabolas and circles. Section 6.5 offers some guidance as to when to use a particular technique for solving quadratic equations. In addition the often-overlooked relationships involving the sum and product of roots are discussed and used as an effective checking procedure.



2.2 Solving First-Degree Equations 53

situation where we need to think in terms of a *guideline* not explicitly stated in the problem.

Problem 3 Khoa received a car repair bill for $106. This included $23 for parts, $22 per hour for each hour of labor, and $6 for taxes. Find the number of hours of labor.

Solution See Figure 2.1. Let h represent the number of hours of labor. Then $22h$ represents the total charge for labor. We can use a guideline of *charge for parts plus charge for labor plus tax equals the total bill* to set up the following equation.

Parts Labor Tax Total bill
$23 + 22h + 6 = 106$

Solving this equation we obtain

$22h + 29 = 106$
$22h = 77$
$h = 3\frac{1}{2}$.

AL'S AUTO BARN

Parts	$23.00
Labor @ $22. pr hr	
Sub total	$100.00
Tax	$6.00
Total	$106.00

Khoa was charged for $3\frac{1}{2}$ hours of labor. FIGURE 2.1

Problem Set 2.1

Solve each of the following equations.

1. $3x + 4 = 16$
2. $4x + 2 = 22$
3. $5x + 1 = -14$
4. $7x + 4 = -31$
5. $-x - 6 = 8$
6. $8 - x = -2$
7. $4y - 3 = 21$
8. $6y - 7 = 41$
9. $3x - 4 = 15$
10. $5x + 1 = 12$
11. $-4 = 2x - 6$
12. $-14 = 3a - 2$
13. $-6y - 4 = 16$
14. $-8y - 2 = 18$
15. $4x - 1 = 2x + 7$
16. $9x - 3 = 6x + 18$
17. $5y + 2 = 2y - 11$
18. $9y + 3 = 4y - 10$
19. $3x + 4 = 5x - 2$
20. $2x - 1 = 6x + 15$
21. $-7a + 6 = -8a + 14$
22. $-6a - 4 = -7a + 11$
23. $5x + 3 - 2x = x - 15$
24. $4x - 2 - x = 5x + 10$
25. $6y + 18 + y = 2y + 3$

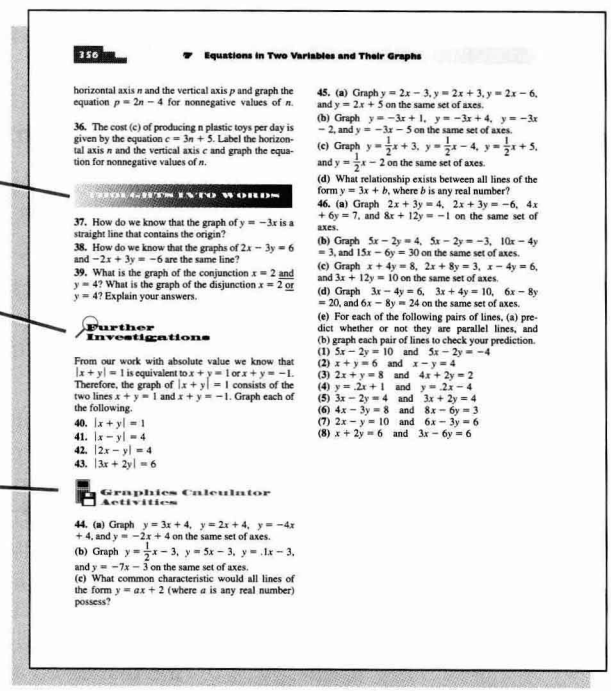
Many sample word problems are fully solved in sections specifically emphasizing problem solving.

Clearly rendered representational art lends interest and helps students visualize the problem.

"Thoughts into Words" problems encourage students to express their mathematical understanding verbally.

"Further Investigations" problems, which require skills learned in the section, are especially appropriate for group work.

"Graphics Calculator Activities," which reinforce concepts and lay the groundwork for new material, ask students to predict the shape and locations of graphs and to draw conclusions from what they see.



256 Equations in Two Variables and Their Graphs

horizontal axis n and the vertical axis p and graph the equation $p = 2n - 4$ for nonnegative values of n.

36. The cost (c) of producing n plastic toys per day is given by the equation $c = 3n + 5$. Label the horizontal axis n and the vertical axis c and graph the equation for nonnegative values of n.

Thoughts into Words

37. How do we know that the graph of $y = -3x$ is a straight line that contains the origin?

38. How do we know that the graphs of $2x - 3y = 6$ and $-2x + 3y = -6$ are the same line?

39. What is the graph of the conjunction $x = 2$ and $y = 4$? What is the graph of the disjunction $x = 2$ or $y = 4$? Explain your answers.

Further Investigations

From our work with absolute value we know that $|x + y| = 1$ is equivalent to $x + y = 1$ or $x + y = -1$. Therefore, the graph of $|x + y| = 1$ consists of the two lines $x + y = 1$ and $x + y = -1$. Graph each of the following.

40. $|x + y| = 1$
41. $|x - y| = 4$
42. $|2x - y| = 4$
43. $|3x + 2y| = 6$

Graphics Calculator Activities

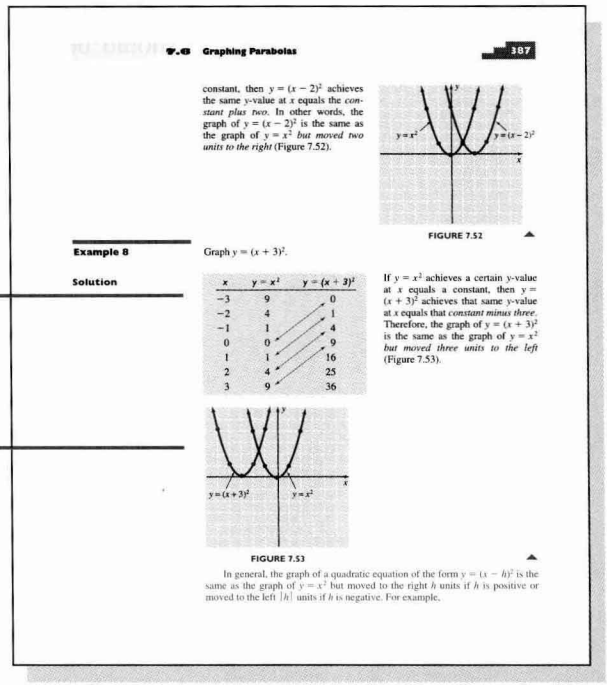
44. (a) Graph $y = 3x + 4$, $y = 2x + 4$, $y = -4x + 4$, and $y = -2x + 4$ on the same set of axes.
(b) Graph $y = \frac{1}{2}x - 3$, $y = 5x - 3$, $y = .1x - 3$, and $y = -7x - 3$ on the same set of axes.
(c) What common characteristic would all lines of the form $y = ax + 2$ (where a is any real number) possess?

45. (a) Graph $y = 2x - 3$, $y = 2x + 3$, $y = 2x - 6$, and $y = 2x + 5$ on the same set of axes.
(b) Graph $y = -3x + 1$, $y = -3x + 4$, $y = -3x - 2$, and $y = -3x - 5$ on the same set of axes.
(c) Graph $y = \frac{1}{2}x + 3$, $y = \frac{1}{2}x - 4$, $y = \frac{1}{2}x + 5$, and $y = \frac{1}{2}x - 2$ on the same set of axes.
(d) What relationship exists between all lines of the form $y = 3x + b$, where b is any real number?

46. (a) Graph $2x + 3y = 4$, $2x + 3y = -6$, $4x + 6y = 7$, and $8x + 12y = -1$ on the same set of axes.
(b) Graph $5x - 2y = 4$, $5x - 2y = -3$, $10x - 4y = 3$, and $15x - 6y = 30$ on the same set of axes.
(c) Graph $x + 4y = 8$, $2x + 8y = 3$, $x - 4y = 6$, and $3x + 12y = 10$ on the same set of axes.
(d) Graph $3x - 4y = 6$, $3x + 4y = 10$, $6x - 8y = 20$, and $6x - 8y = 24$ on the same set of axes.
(e) For each of the following pairs of lines, (a) predict whether or not they are parallel lines, and (b) graph each pair of lines to check your prediction.
(1) $5x - 2y = 10$ and $5x - 2y = -4$
(2) $x + y = 6$ and $x - y = 4$
(3) $2x + y = 8$ and $4x + 2y = 2$
(4) $y = .2x + 1$ and $y = .2x - 4$
(5) $3x - 2y = 4$ and $3x + 2y = 4$
(6) $4x - 3y = 8$ and $8x - 6y = 3$
(7) $2x - y = 10$ and $6x - 3y = 6$
(8) $x + 2y = 6$ and $3x - 6y = 6$

In sections on graphing, tables of values show numerical approaches to problems.

Large, clear graphs depict curves accurately and restate the equation being graphed.



7.6 Graphing Parabolas 387

constant, then $y = (x - 2)^2$ achieves the same y-value at x equals the *constant plus two*. In other words, the graph of $y = (x - 2)^2$ is the same as the graph of $y = x^2$ *but moved two units to the right* (Figure 7.52).

FIGURE 7.52 ▲

Example 8

Graph $y = (x + 3)^2$.

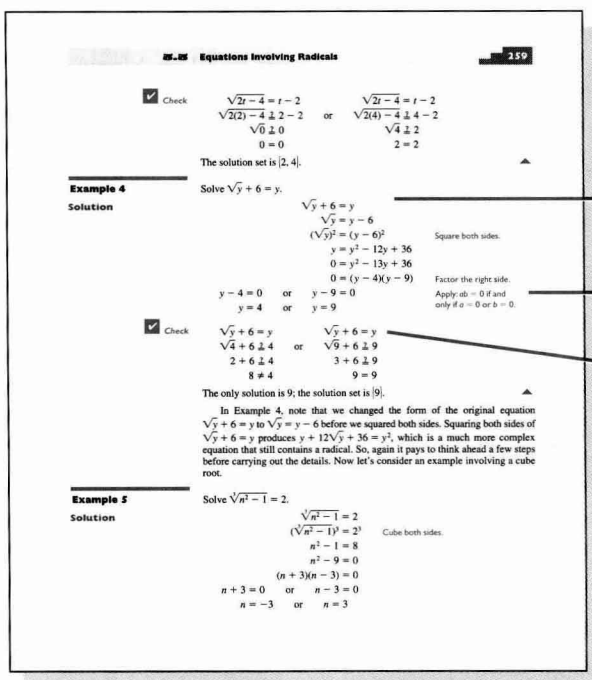
Solution

x	$y = x^2$	$y = (x + 3)^2$
−3	9	0
−2	4	1
−1	1	4
0	0	9
1	1	16
2	4	25
3	9	36

If $y = x^2$ achieves a certain y-value at x equals a constant, then $y = (x + 3)^2$ achieves that same y-value at x equals that *constant minus three*. Therefore, the graph of $y = (x + 3)^2$ is the same as the graph of $y = x^2$ *but moved three units to the left* (Figure 7.53).

FIGURE 7.53 ▲

In general, the graph of a quadratic equation of the form $y = (x - h)^2$ is the same as the graph of $y = x^2$ but moved to the right h units if h is positive or moved to the left $|h|$ units if h is negative. For example,



8.5 Equations Involving Radicals 259

Check $\sqrt{2t - 4} = t - 2$ $\quad\quad$ $\sqrt{2t - 4} = t - 2$

$\sqrt{2(2) - 4} \stackrel{?}{=} 2 - 2$ or $\sqrt{2(4) - 4} \stackrel{?}{=} 4 - 2$

$\sqrt{0} \stackrel{?}{=} 0$ $\quad\quad$ $\sqrt{4} \stackrel{?}{=} 2$

$0 = 0$ $\quad\quad$ $2 = 2$

The solution set is $\{2, 4\}$. ▲

Example 4

Solution

Solve $\sqrt{y} + 6 = y$.

$\sqrt{y} + 6 = y$

$\sqrt{y} = y - 6$

$(\sqrt{y})^2 = (y - 6)^2$ Square both sides.

$y = y^2 - 12y + 36$

$0 = y^2 - 13y + 36$

$0 = (y - 4)(y - 9)$ Factor the right side.

$y - 4 = 0$ or $y - 9 = 0$ Apply: $ab = 0$ if and only if $a = 0$ or $b = 0$.

$y = 4$ or $y = 9$

Check $\sqrt{y} + 6 = y$ $\quad\quad$ $\sqrt{y} + 6 = y$

$\sqrt{4} + 6 \stackrel{?}{=} 4$ or $\sqrt{9} + 6 \stackrel{?}{=} 9$

$2 + 6 \stackrel{?}{=} 4$ $\quad\quad$ $3 + 6 \stackrel{?}{=} 9$

$8 \neq 4$ $\quad\quad$ $9 = 9$

The only solution is 9; the solution set is $\{9\}$. ▲

In Example 4, note that we changed the form of the original equation $\sqrt{y} + 6 = y$ to $\sqrt{y} = y - 6$ before we squared both sides. Squaring both sides of $\sqrt{y} + 6 = y$ produces $y + 12\sqrt{y} + 36 = y^2$, which is a much more complex equation that still contains a radical. So, again it pays to think ahead a few steps before carrying out the details. Now let's consider an example involving a cube root.

Example 5

Solution

Solve $\sqrt[3]{n^2 - 1} = 2$.

$\sqrt[3]{n^2 - 1} = 2$

$(\sqrt[3]{n^2 - 1})^3 = 2^3$ Cube both sides.

$n^2 - 1 = 8$

$n^2 - 9 = 0$

$(n + 3)(n - 3) = 0$

$n + 3 = 0$ or $n - 3 = 0$

$n = -3$ or $n = 3$

Many worked examples show careful, step-by-step problem solving.

Annotations make clear each step of the problem.

The "Check" feature in worked examples and problems reminds students to complete this important problem-solving step.

- Chapter 7 is written on the premise that intermediate algebra students should become *very familiar* with straight lines, parabolas, and circles, and should be exposed in only a limited fashion to ellipses and hyperbolas. I intentionally kept the definition of a function out of Chapter 7. My experience indicates that intermediate algebra students need more work with coordinate geometry concepts—specifically graphing techniques—before being introduced to the concept of a function.
- Chapter 8 is devoted entirely to functions and the issue is not clouded by the jumping back and forth between functions and relations that are not functions. It includes some work on the composition of functions and the use of quadratic functions in problem-solving situations.
- Chapter 9 contains the various techniques for solving systems of linear equations. It is organized so that instructors can use as much of the chapter as needed for their particular course. Section 9.2 presents the elimination-by-addition method, which emphasizes equivalent systems and sets the stage for future work with matrices.
- Chapter 10 presents a modern-day version of the concepts of exponents and logarithms. The emphasis is on making the concepts and their applications understood. The calculator is used as a tool to help with the complicated computational aspects.

Ancillaries for Instructors

The following useful ancillaries are available to adopters of this text:

- **Annotated Instructor's Edition** includes answers to all problems in the text—most printed adjacent to the problem.
- **Instructor's Solutions Manual** contains solutions for even-numbered problems and answers for all odd-numbered problems.
- **Test Bank with Chapter Tests** contains all questions and answers from the computerized test bank and three sample tests (two multiple choice, one open ended) for each chapter. These tests may be duplicated for student testing by instructors using the text.
- **Computerized testing software** is available for the IBM and compatibles and for the Macintosh. The computerized testing programs contain multiple-choice and open-ended questions that allow users to edit, rearrange, and add to the question bank.
- **Videotape Series** is text-specific, following the organization and style of the textbook. Video lectures include basic instruction and worked examples.

Ancillaries for Students

- **Student's Solutions Manual** contains complete worked-out solutions for all odd-numbered problems.

- **Worksheets and Study Guide** is a text-specific study resource in worktext format. It includes examples and exercises for topics keyed to sections in the text so that students have the opportunity for additional practice and study assistance. The manual is designed to be integrated as an interactive component to lectures or for instructional use outside the classroom.
- **MathQuest Tutorial Software** is an interactive, text-specific intuitive tutorial that runs on both Windows and Macintosh platforms. The program provides fill-in, multiple-choice, and true/false questions. If a student answers a question incorrectly, the program will first respond with hints; if the student answers incorrectly a second time, the program will supply a step-by-step solution. Record-keeping capabilities enable students to monitor their progress.

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Jerome E. Kaufmann
Marble Falls, Texas

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