CALCULUS CONCEPTS

First Edition

An Informal Approach to the Mathematics of Change

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An Informal Approach to the Mathematics of Change

First Edition

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TO INSTRUCTORS

What This Book is About

This book presents a fresh, new approach to the concepts of calculus for students in fields such as business, economics, liberal arts, management, and the social and life sciences. It is appropriate for two-semester courses generally known as "brief calculus" or "applied calculus."

Philosophy

Our overall goal is to improve learning of basic calculus concepts by involving students with new material in a way that is significantly different from traditional practice. The development of conceptual understanding, not mastery of algebraic skill and technique, is our guiding force coupled with a commitment to make calculus meaningful to the student. Thus, the material in this book is data-driven and technology-based, with a unique modeling approach. It considers the ability to correctly interpret the mathematics of real-life situations of equal importance to the understanding of the concepts of calculus in the context of change.

Data-Driven

Many everyday, real-life situations involving change are discrete in nature and manifest themselves through data. Such situations often can be represented by continuous or piecewise continuous mathematical models so that the concepts, methods, and techniques of calculus can be brought to light. Thus we seek, when appropriate, to make real-life data a starting point for our investigations.

The use of real data and the search for appropriate models also exposes the students to the reality of uncertainty. We emphasize that sometimes there can be more than one appropriate model and that answers derived from models are only approximations. We believe that exposure to the possibility of more than one correct approach or answer is valuable.

Technology-Based

Calculus has traditionally relied upon a high level of algebraic manipulation. However, many non-technical students are not strong in algebraic skills, and an algebra-based approach tends to overwhelm them and stifle their progress. Today's easy access to technology in the form of graphing calculators and microcomputers breaks down barriers to learning imposed by the traditional reliance on algebraic methods. It creates new opportunities for learning through graphical and numerical representations. We welcome these opportunities in this book by assuming continual and immediate access to technology.

This book requires that students use graphical representations (scatter plots of data and graphs of functions) freely, make numerical calculations routinely, and fit functions to data. Thus, continual and immediate access to technology is absolutely

essential. Because of their low cost, portability, and ability to personalize the mathematics, the authors prefer graphing calculators. These materials have also been successfully taught using microcomputer software (such as Maple) and are appropriate for use with spreadsheets.

It is worth noting that different technologies may give different model coefficients than those given in this book. We used a TI-83 graphing calculator to generate the models in the text and the answer key. Other technologies may use different fit criteria for some models than the criteria used by the TI-83.

Modeling Approach

We believe that modeling is an important tool and introduce it at the outset. Both linear and nonlinear models of discrete data are used to obtain functional relationships between the variables of interest. The functions given by the models are the ones used by students to conduct their investigations of calculus concepts. It is the connection to real-life data that most students feel shows the relevance of the mathematics in this course to their lives and adds reality to the topics studied.

Interpretation Emphasis

This book is substantially different from traditional texts, not only in the philosophy but also in its overall focus, level of activities, development of topics and attention to details. Interpretation of results is a key feature of this text that allows students to make sense of the mathematical concepts and appreciate the usefulness of those concepts in their lives.

Informal Style

While we appreciate the formality and precision of mathematics, we also recognize that this alone can deter students from access to mathematics. Thus, we have sought to make our presentations as informal as possible by using non-technical terminology where appropriate and a conversational style of presentation.

Projects

Projects included after each chapter are intended to be group projects with oral and/or written presentations. We recognize the importance of helping students develop the ability to work in groups, as well as hone presentation skills. The projects also give opportunity for students to practice the kind of writing that they will likely have to do in their future careers.

Other Pedagogical Features

Chapter Opener Each chapter opens with a real-life situation and several questions about the situation that relate to the key concepts in the chapter.

Concept Inventory A Concept Inventory is listed at the end of each section, giving students a brief summary of the major ideas developed in that section.

Section Activities The Section Activities begin by cementing concepts followed by explorations of topics using, for the most part, actual data in a variety of real-world settings. Questions and interpretations pertinent to the data and the concepts are always included in these activities. The activities do not mimic the examples in the

chapter discussion and thus require more independent thinking on the part of the students. Possible answers to odd activities are given at the end of the book.

Chapter Summary A Chapter Summary connects the results of the chapter topics and further emphasizes the importance of knowing these results.

Chapter Review Test A Chapter Review Test at the end of each chapter provides practice with techniques and concepts. Answers to the Chapter Review Tests are included in the answer key.

Supplements

The Instructor's Guide gives practical suggestions for using the text in the manner intended by the authors. It contains sample tests, ideas for in-class group work, suggestions for implementing and grading projects, and complete activity solutions.

The technology supplements provide technology-specific instructions ordered to match the organization of the text chapters. An open-book icon appears at places in the text where a new concept or skill is presented in the technology supplements.

A Student Solutions Guide is also available. A Test Item File is available in PC and Macintosh formats.

Acknowledgments

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TO STUDENTS

What this Book is About

This book is written to help you understand the inner workings of how things change and to help you build systematic ways to use this understanding in everyday real-life situations that involve change. Indeed, a primary focus of the material is on change, since calculus is the mathematics of change.

Even if you have studied calculus before, this book is probably different from any other mathematics textbook that you have used. It is based on three premises:

1. Understanding is as important as the mastery of mathematical manipulations. Algebraic skill and the ability to manipulate expressions must be regularly practiced, or they will fade away. If you understand concepts, you will be able to explain some things in your life forever.
2. Mathematics is present in all sorts of real-life situations. It is not just an abstract subject in textbooks. In real life, mathematics is often messy and not at all like the tidy, neat equations that you were taught to factor and solve. Speaking of equations, where do they come from? Nature seldom whispers an equation into our ears.
3. The new graphics technology in today's calculators and computers is a powerful tool that can help you understand important mathematical connections. Like many tools in various fields, technology frees you from tedious, unproductive work; enables you to engage situations more realistically; and lets you focus on what you do best . . . think and reason.

How to Use this Book

- Begin by throwing away any preconceived notions that you may have about what calculus is and any notion that you are "not good" in mathematics.
- Make a commitment to learn the material: not just a good intention, but a genuine commitment.
- Study this book. Notice that we said "study", not "read". Reading is a part of study, but study involves much more. You should not only read (and re-read) the discussions, but work through each example to understand its development.
- Use paper, pencil and your graphing calculator or computer when you study. These are your basic tools, and you cannot study effectively without them.
- Find a study partner, if at all possible. Each of you will be able to help the other learn. Communicating within mathematics, and about mathematics, is important to your overall development toward understanding mathematics.
- Write your solutions clearly and legibly, being certain to interpret all of your answers with complete sentences using proper grammar. Careful writing will help you sort through your ideas and focus your learning.

- Make every effort not to fall behind. You know the dangers, of course, but we remind you nevertheless.
- Finally, remember that there is no substitute for effective study. You have your most valuable resource with you at all times—your mind. Use it.

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