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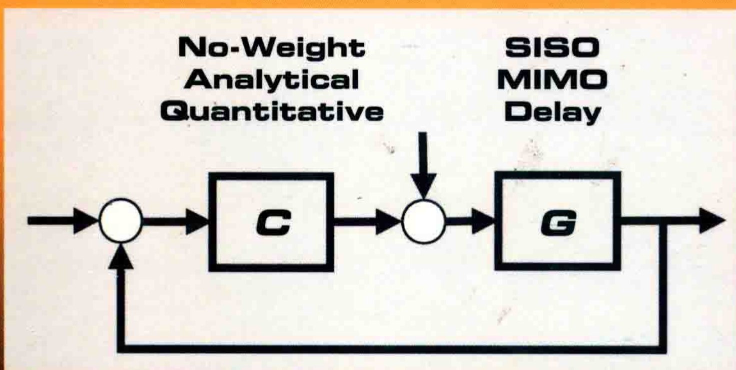
Quantitative Process Control Theory



Weidong Zhang

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Quantitative Process Control Theory

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**Dedicated
to My Parents**

Symbol Description

Abbreviations

DMC	Dynamic matrix control
DOF	Degree-of-freedom
IAE	Integral absolute error
IMC	Internal model control
ISE	Integral squared error
ITAE	Integral of time multiplied by absolute error
LFT	Linear fractional transformation
LHP	Left half-plane
LMI	Linear matrix inequality
LQ	Linear quadratic
LQG	Linear quadratic Gaussian
MAC	Matrix algorithmic control
MIMO	Multi-input/multi-output
MP	Minimum phase
MPC	Model predictive control
NMP	Non-minimum phase
PID	Proportional-integral-derivative
QPCT	Quantitative process control theory
RGA	Relative gain array
RHP	Right half-plane
SISO	Single-input/single-output
SSV	Structured singular value
SVD	Singular value decomposition

Symbols

$C(s)$ $(\boldsymbol{C}(s))$	Unity feedback loop controller
$C_1(s)$ $(\boldsymbol{C}_1(s))$	Controller of the reference loop
$C_2(s)$ $(\boldsymbol{C}_2(s))$	Controller of the disturbance loop
$d(s)$ $(\boldsymbol{d}(s))$	Disturbance at the plant output
$d'(s)$ $(\boldsymbol{d}'(s))$	Disturbance at the plant input
$e(s)$ $(\boldsymbol{e}(s))$	Tracking error
$G(s)$ $(\boldsymbol{G}(s))$	Nominal plant (plant model)
$\tilde{G}(s)$ $(\tilde{\boldsymbol{G}}(s))$	Real plant

$G_A(s)$ ($\boldsymbol{G_A}(s)$)	All-pass part of $G(s)$ ($\boldsymbol{G}(s)$)
$G_D(s)$ ($\boldsymbol{G_D}(s)$)	Time delay part of $G(s)$ ($\boldsymbol{G}(s)$)
$G_{MP}(s)$ ($\boldsymbol{G_{MP}}(s)$)	MP part of $G(s)$ ($\boldsymbol{G}(s)$)
$\boldsymbol{G_N}(s)$	All-pass part of $\boldsymbol{G_O}(s)$
$G_O(s)$ ($\boldsymbol{G_O}(s)$)	Rational part of $G(s)$ ($\boldsymbol{G}(s)$)
$H(s)$ ($\boldsymbol{H}(s)$)	Matrix for internal stability verification
H_2	Set of all stable strictly proper functions without poles on the imaginary axis
H_∞	Set of all stable proper functions without poles on the imaginary axis
$J(s)$ ($\boldsymbol{J}(s)$)	Filter
K	Gain of a plant
K_C	Gain of a PID controller
k_j	Multiplicity of a RHP zero z_j
k_{ij}	Largest multiplicity of z_j in the ith column of $\boldsymbol{G}^{-1}(s)$
K_u	Ultimate gain
l_j	Multiplicity of a RHP pole p_j
l_{ij}	Largest multiplicity of p_j in the ith row of $\boldsymbol{G}(s)$
$L(s)$ ($\boldsymbol{L}(s)$)	Open-loop transfer function
$\boldsymbol{M}(s)$	Matrix for robustness verification
p_j	jth pole
$Q(s)$ ($\boldsymbol{Q}(s)$)	IMC controller
$Q_{opt}(s)$ ($\boldsymbol{Q_{opt}}(s)$)	Optimal IMC controller
$r(s)$ ($\boldsymbol{r}(s)$)	Reference
$R(s)(\boldsymbol{R}(s))$	Controller of the Smith predictor
r_p	Number of RHP poles of a plant
r_z	Number of RHP zeros of a plant
$S(s)$ ($\boldsymbol{S}(s)$)	Nominal sensitivity transfer function
$\tilde{S}(s)$	Real sensitivity transfer function
$T(s)$ ($\boldsymbol{T}(s)$)	Complementary sensitivity transfer function (closed-loop transfer function)
T_D	Derivative constant of a PID controller
T_F	Filtering constant of a PID controller
T_I	Integral constant of a PID controller
T_p	Resonance peak
t_r	Rise time
T_u	Ultimate period
$u(s)$ ($\boldsymbol{u}(s)$)	Controller output
$\hat{u}(t)$	Constrained controller output
$\boldsymbol{v_j}$, $\boldsymbol{v_{jk}}$	Direction of the zero z_j
$W(s)$ ($\boldsymbol{W_{p1}}(s), \boldsymbol{W_{p2}}(s)$)	Performance weighting functions
$\boldsymbol{W_1}(s), \boldsymbol{W_2}(s)$	Uncertainty weighting functions
$y(s)$ ($\boldsymbol{y}(s)$)	Plant output
z_j	jth zero

Greek Characters

α_i	Smallest relative degree of all elements in the ith column of $\boldsymbol{Q_{opt}}(s)$
γ	Closed contour contained in Ω
$\delta_m(s)$ ($\boldsymbol{\delta_m}(s)$)	Uncertainty
$\Delta(s)$ ($\boldsymbol{\Delta}(s)$)	Normalized uncertainty
$\Delta_m(s)$ ($\boldsymbol{\Delta_m}(s)$)	Uncertainty profile
$\boldsymbol{\Delta_p}(s)$	Performance block in $\boldsymbol{\Delta}(s)$
$\boldsymbol{\Delta_u}(s)$	Uncertainty block in $\boldsymbol{\Delta}(s)$
θ	Time delay of a plant
θ_{ij}	Time delay of the ijth element of $\boldsymbol{G}(s)$
θ^{ij}	Prediction of the ijth element of $\boldsymbol{G}^{-1}(s)$
θ_{li}	Largest prediction of the ith column of $\boldsymbol{G}^{-1}(s)$
θ_{si}	Smallest time delay of the ith row of $\boldsymbol{G}(s)$
λ, λ_i	Performance degree
λ_{ij}	ijth relative gain
$\lambda_{ei}[\boldsymbol{T}(j\omega)]$	ith eigenvalue of $\boldsymbol{T}(j\omega)$
$\mu[\boldsymbol{M}(j\omega)]$	Structured singular value of $\boldsymbol{M}(j\omega)$
$\rho[\boldsymbol{T}(j\omega)]$	Spectral radius of $\boldsymbol{T}(j\omega)$
σ	Overshoot
$\sigma_i(\boldsymbol{T}(j\omega))$	ith singular value of $\boldsymbol{T}(j\omega)$
$\bar{\sigma}[\boldsymbol{T}(j\omega)]$	Maximum singular value of $\boldsymbol{T}(j\omega)$
$\underline{\sigma}[\boldsymbol{T}(j\omega)]$	Minimum singular value of $\boldsymbol{T}(j\omega)$
τ	Time constant of a plant
Ω	Simply connected open subset of the complex plane

Special Notation

$\|\cdot\|_1$	1-norm
$\|\cdot\|_2$	2-norm
$\|\cdot\|_\infty$	∞-norm
$:=$	Is defined as
$\forall$	For all
$\in$	Belong to
$\otimes$	Element-by-element product
$\bar{z}_r(\bar{\boldsymbol{A}})$	Complex conjugate
adj	Adjoint
deg	Degree of a polynomial
det	Determinant of a matrix
diag	Diagonal matrix
$\boldsymbol{T}^T(s)$	Transpose of a matrix or vector
$\boldsymbol{T}^H(j\omega)$	Complex conjugate transpose of a matrix: $\boldsymbol{T}^H(j\omega) = \bar{\boldsymbol{T}}^T(j\omega)$
$\boldsymbol{T}^*(s)$	Conjugate transpose of a system: $\boldsymbol{T}^*(s) = \boldsymbol{T}^T(-s)$

Im	Imaginary part of a complex number
$N_+(s)$	Polynomials with roots in the closed RHP
$N_-(s)$	Polynomials with roots in the open LHP
Re	Real part of a complex number
sup	Supremum
Trace	Trace of a matrix

Preface

Since the Industrial Revolution, control systems have played important roles in improving product quality, saving energy, reducing emissions, and relieving the drudgery of routine repetitive manual operations. In the past hundred years, many theories have been proposed for control system design. However, there are three main problems when some of these advanced control theories are applied to industrial systems:

1. These theories depend on empirical methods or trial-and-error methods in choosing weighting functions.
2. Both the design procedures and results are complicated for understanding and using.
3. The controllers cannot be designed or tuned for quantitative engineering performance indices (such as overshoot or stability margin).

In this book, an improved theory called the Quantitative Process Control Theory is introduced to solve these problems. This new theory has three features:

1. When using the theory, the designer is not required to choose a weighting function.
2. The design is suboptimal and analytical. It is easy to understand and use.
3. The controller can be designed or tuned for quantitative engineering performance indices.

These features enable the controller to be designed efficiently and quickly.

Mathematical proofs are provided in this book for almost all results, especially when they contribute to the understanding of the subjects presented. This will, I believe, enhance the educational value of this book. As few concepts as possible are introduced and as few mathematical tools as possible are employed, so as to make the book accessible. Examples are presented at strategic points to help readers understand the subjects discussed. Chapter summaries are included to highlight the main problems and results. At the end of each chapter, exercises are provided to test the reader's ability to apply the theory he/she has studied. They are an integral part of the book. There is no doubt that a serious attempt to solve these exercises will greatly improve one's understanding.

The methods developed here are not confined to process control. They are equally applicable to aeronautical, mechanical, and electrical engineering. To stress this point, examples with different backgrounds are adopted. With a few exceptions, these examples are based on real plants, including

- Paper-making machine
- Heat exchanger
- Hot strip mill
- Maglev
- Nuclear reactor
- Distillation column/Heavy oil fractionator
- Jacket-cooled reactor
- Missile
- Helicopter/Plane
- Anesthesia

The book is divided into 14 chapters. Important topics that are covered include

1. Introduction and review of classical analysis methods (Chapter 2)
2. Essentials of the robust control theory (Chapter 3)
3. H_∞ and H_2 proportional-integral-derivative controllers for stable plants with time delay (Chapters 4 and 5)
4. Quasi-H_∞ and H_2 controllers for stable plants with time delay (Chapter 6)
5. Quasi-H_∞ and H_2 controllers for integrating plants with time delay (Chapter 7)
6. Quasi-H_∞ and H_2 controllers for unstable plants with time delay (Chapter 8)
7. Complex control strategies, including two degrees-of-freedom control, cascade control, anti-windup control, and feedforward control (Chapter 9)
8. Analysis of multi-input/multi-output control systems (Chapter 10)
9. Classical multi-input/multi-output system design, including decentralized control and decoupling control (Chapter 11)
10. Quasi-H_∞ decoupling control for plants with time delay (Chapter 12)

11. H_2 optimal decoupling control for plants with time delay (Chapter 13)
12. Multivariable H_2 optimal control (Chapter 14)

This book is intended for a wide variety of readers. It is appropriate for higher level undergraduates and graduates in engineering, beginners in the research area of robust control, and engineers who want to learn new design techniques. It is assumed that readers have had an undergraduate course in classical control theory. A prior course on optimal control or process control would be helpful but is not a requirement.

This book has grown out of 15 years of research. The procedure is always much harder than anyone anticipates. I received financial support from the National Science Foundation of China, the Alexander von Humboldt Foundation, Germany, and the National Science Fund for Distinguished Young Scholars, China, which enabled me to pursue the research. I am vastly indebted to many people who have helped and inspired me to start, continue, and complete this book.

My first thanks goes to Professor Shengxun Zhang and Professor Youxian Sun, Zhejiang University. They brought me into the area of process control. I am grateful for the continuing help and support from Professor Xiaoming Xu, Professor Yugeng Xi, Professor Songjiao Shi, Professor Zuohua Tian, and Professor Xinping Guan at Shanghai Jiaotong University. I am also greatly indebted to Professor F. Allgöwer and Professor C.A. Floudas, who hosted me at the University of Stuttgart and Princeton University, respectively, as a visiting professor during the writing of this book.

The first six chapters of this book have been classroom tested for several years at Shanghai Jiaotong University. Many students have contributed their time to the book. I would like to thank my PhD students F. S. Alcántara Cano, Danying Gu, Daxiao Wang, and Mingming Ji for particularly helpful suggestions.

The book makes limited use of the material from several books. In particular, I want to express my sincere appreciation to Morari and Zafiriou (1989), Doyle et al. (1992), and Dorf and Bishop (2001).

Family members are a source of special encouragement in a job of this magnitude, and I send love and thanks to my parents and my son in this regard.

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Weidong Zhang received his BS, MS, and PhD degrees from Zhejiang University, China, in 1990, 1993, and 1996, respectively, and then worked as a postdoctoral fellow at Shanghai Jiaotong University. He joined Shanghai Jiaotong University in 1998 as an associate professor and has been a full professor since 1999. From 2003 to 2004 he worked at the University of Stuttgart, Germany, as an Alexander von Humboldt Fellow. From 2007 to 2008 he held a visiting position at Princeton University. In 2011 he was appointed chair professor at Shanghai Jiaotong University.

Dr. Zhang's research interests include control theory and its applications, embedded systems, and wireless sensor networks. He is probably the earliest researcher on the automotive reversing ultrasonic radar in China. He has many years of industry experience and was a control engineering consultant at Atmel Corporation (Shanghai R&D Center) in 2005.

Dr. Zhang is the author of more than 200 refereed papers and holds 15 patents. He is a recipient of National Science Fund for Distinguished Young Scholars of China.

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