



Yingxin Guo

Existence and stability of solutions to nonlinear dynamical systems

Yingxin Guo

Existence and stability of solutions to nonlinear dynamical systems



LAP LAMBERT Academic Publishing

Impressum / Imprint

Bibliografische Information der Deutschen Nationalbibliothek: Die Deutsche Nationalbibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über <http://dnb.d-nb.de> abrufbar.

Alle in diesem Buch genannten Marken und Produktnamen unterliegen warenzeichen-, marken- oder patentrechtlichem Schutz bzw. sind Warenzeichen oder eingetragene Warenzeichen der jeweiligen Inhaber. Die Wiedergabe von Marken, Produktnamen, Gebrauchsnamen, Handelsnamen, Warenbezeichnungen u.s.w. in diesem Werk berechtigt auch ohne besondere Kennzeichnung nicht zu der Annahme, dass solche Namen im Sinne der Warenzeichen- und Markenschutzgesetzgebung als frei zu betrachten wären und daher von jedermann benutzt werden dürften.

Bibliographic information published by the Deutsche Nationalbibliothek: The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Any brand names and product names mentioned in this book are subject to trademark, brand or patent protection and are trademarks or registered trademarks of their respective holders. The use of brand names, product names, common names, trade names, product descriptions etc. even without a particular marking in this works is in no way to be construed to mean that such names may be regarded as unrestricted in respect of trademark and brand protection legislation and could thus be used by anyone.

Coverbild / Cover image: www.ingimage.com

Verlag / Publisher:

LAP LAMBERT Academic Publishing

ist ein Imprint der / is a trademark of

AV Akademikerverlag GmbH & Co. KG

Heinrich-Böcking-Str. 6-8, 66121 Saarbrücken, Deutschland / Germany

Email: info@lap-publishing.com

Herstellung: siehe letzte Seite /

Printed at: see last page

ISBN: 978-3-659-31848-1

Copyright © 2013 AV Akademikerverlag GmbH & Co. KG

Alle Rechte vorbehalten. / All rights reserved. Saarbrücken 2013

Yingxin Guo

Existence and stability of solutions to nonlinear dynamical
systems

Existence and stability of solutions to nonlinear dynamical systems ¹

YINGXIN GUO

School of Mathematical Sciences, Qufu Normal University, PR CHINA

¹ *Mathematics Subject Classifications:* 54E35, 54E99, 34K20, 34K13, 92B20, 34B15, 34B25.

Key words: Stability, Neural networks, Lyapunov functionals, Matrix inequality, Nonlinear differential equations, Impulse, Stochastic

Corresponding author: Supported financially by the NNSF of China (10801088).
E-mail: yxguo312@163.com, yxguo312@yahoo.com.cn

Preface

The concept of a dynamical system has its origins in Newtonian mechanics. There, as in other natural sciences and engineering disciplines, the evolution rule of dynamical systems is given implicitly by a relation that gives the state of the system only a short time into the future. The relation is either a differential equation, difference equation or other time scale. Solving a dynamical system required sophisticated mathematical techniques and could be accomplished only for a small class of dynamical systems. To determine the state for all future times requires iterating the relation many times each advancing time a small step. The iteration procedure is referred to as solving the system or integrating the system. Once the system can be solved, given an initial point it is possible to determine all its future positions, a collection of points known as a trajectory or orbit. Now, Due to the advent of fast computing machines, numerical methods implemented on electronic computing machines have simplified the task of determining the orbits of a dynamical system.

For simple dynamical systems, knowing the trajectory is often sufficient, but most dynamical systems are too complicated to be understood in terms of individual trajectories. The difficulties arise because:

The systems studied may only be known approximately the parameters of the system may not be known precisely or terms may be missing from the equations. To address these questions several notions of stability have been introduced in the study of dynamical systems, such as Lyapunov stability or structural stability. The stability of the dynamical system implies that there is a class of models or initial conditions for which the trajectories would be equivalent. The operation for comparing orbits to establish their equivalence changes with the different notions of stability. The type of trajectory may be more important than one particular trajectory. The behavior of trajectories as a function of a parameter may be what is needed for an application. As a parameter is varied, the dynamical systems may have bifurcation points where the qualitative behavior of the dynamical system changes.

Nonlinear problems are of interest to engineers, physicists and mathematicians because most physical systems are inherently nonlinear in nature. Nonlinear equations are difficult to solve and give rise to interesting phenomena such as chaos. Some aspects of the weather are seen to be chaotic, where simple changes in one part of the system produce complex effects throughout. A system of dynamical differential equations is said to be nonlinear if it is not a linear

system. Problems involving nonlinear dynamical differential equations are extremely diverse, and methods of solution or analysis are problem dependent on the system itself.

As is well known, time delays are often encountered unavoidably in many practical systems such as automatic control systems, population models, inferred grinding models, the AIDS epidemic, neural networks, and so on. They describe a kind of ubiquitous phenomenon present in real systems where the rate of change of the state depends on not only the current state of the system but also its state at some time in history. Moreover, the existence of time delays may lead to the instability or bad performance of systems. So, it is of prime importance to consider the delay effects on the dynamical behavior of systems. Time delay in the stabilizing negative feedback term has a tendency to destabilize a system. On the other hand, an impulsive phenomenon exists universally in a wide variety of evolutionary processes where the state is changed abruptly at certain moments of time, involving such fields as chemical technology, population dynamics, physics and economics. It has also been shown that an impulsive phenomenon exists likewise in neural networks. For instance, during the implementation of electronic networks, when a stimulus from the body or the external environment is received by receptors, the electrical impulses will be conveyed to the neural net and an impulsive phenomenon which is called impulsive perturbations arises naturally. Moreover, impulsive perturbations, as well as time delays, can affect the dynamical behavior of neural networks. However, in most cases, the encountered instantaneous perturbations depend on not only the current state of neurons at impulse times but also the state of neurons in recent history. So, many of the existing results on delayed neural networks with impulsive perturbations can only be regarded as an ideal situation and they contain certain errors.

Uncertainty presents significant challenges in the reasoning about and controlling of dynamical systems. To address this challenge, numerous researchers are developing improved methods for existence and stability of solutions to nonlinear dynamical systems. This book presents a diverse collection of some of the latest research in this important area. In particular, this book gives an overview of some of the theoretical methods and tools for existence, control and stability of solutions to nonlinear dynamical systems, stochastic systems, and impulsive systems, and it presents the applications of these methods to problems in systems theory, science, and economics.

This book is an outgrowth of our results on existence and stability of solutions to nonlinear dynamical systems, stochastic systems, and impulsive systems, over the last five years. It is addressed to beginning graduate students of mathematics, engineering, and the physical sciences. Thus, we have attempted to present it while presupposing a minimal background: the reader is assumed to have some prior acquaintance with the concepts of “dynamical system” and “nonlinear” and also to believe “Banach space” is complete. The formal prerequisite consists of a good advanced calculus course and a motivation to study the theory of differential equations. My PhD thesis and articles [126-140] are

the main references in this book.

We briefly describe the contents of the various chapters. Chapter 1 present mainly our new results of the Razumikhin-type exponential stability criteria for impulsive stochastic functional differential systems. These new results are employed to impulsive stochastic equations with bounded or unbounded time-varying delays and stochastically perturbed equations. Moreover, some useful corollaries of exponential stability are derived by means of Lyapunov function, inequality techniques and the impulsive condition. The obtained results do not need the strong condition of impulsive gain $|d_k| < 1$.

Chapter 2 is an exposition of stability analysis of neutral stochastic delay differential equations by a generalization of Banachs contraction principle. In this chapter we give sufficient conditions to ensure that the zero solution is asymptotically stable in the mean square by means of a generalization of Banachs contraction principle. These conditions do not require the boundedness of delays, nor do they ask for a fixed sign on the coefficient functions. The results are shown to improve the previous globally stable results derived in the literature.

Chapter 3, 6, 7 and 8 is a discussion of the globally asymptotical stability and the robust stabilization in the mean square for stochastic recurrent neural networksstochastic Cohen-Grossberg neural networks and high-order bidirectional neural networks with time-varying delays and fixed moments of impulsive effect. The rules are extend and improve the previous ones.

Chapter 10, 11 and 12 is a discussion of special class of nonlinear differential equations where the existences of the positive solutions or nontrivial solutions can be obtained.

Chapter 4 is the theory of exponential stability and solution bounds for systems with bounded nonlinearities; Chapter 5 and 9 is about oscillation criteria based on a new weighted function for linear matrix Hamiltonian systems and for second order matrix differential systems.

This book can be used either as a monographs or a reference; to this end, the chapters contain many detailed proofs of core results and also state possible problems of current interest, and it is our hope that the book will provide a stimulus for future research.

Contents

1	New Razumikhin-type exponential stability criteria for impulsive stochastic functional differential systems	7
1.1	Introduction	7
1.2	Preliminaries and lemmas	9
1.3	Stability Analysis	11
1.4	Examples	18
1.5	Conclusion	19
2	Stability analysis of neutral stochastic delay differential equations by a generalization of Banach's contraction principle	21
2.1	Introduction	21
2.2	Model description and preliminaries	22
2.3	Globally asymptotical Stability	30
2.4	Examples	35
3	Globally robust stability analysis for stochastic Cohen-Grossberg neural networks with impulse control and time-varying delays	37
3.1	Introduction	37
3.2	Preliminaries and lemmas	39
3.3	Impulsive Stability Analysis	42
3.4	Numerical example	46
3.5	Conclusion	47
4	Exponential stability and solution bounds for systems with bounded nonlinearities	48
4.1	Introduction	48
4.2	Nonlinear time-dependent systems	48
4.3	Nonlinear time-dependent systems with delays	53
5	Oscillation criteria based on a new weighted function for linear matrix Hamiltonian systems	56
5.1	Introduction	56
5.2	Main Results	58
5.3	Examples	62

6	Mean square global asymptotic stability of stochastic recurrent neural networks with distributed delays	64
6.1	Introduction	64
6.2	Preliminaries and lemmas	65
6.3	Stability Analysis	67
6.4	An example	73
6.5	Conclusion	73
7	Asymptotic and robust mean square stability analysis of impulsive high-order dynamical neural networks with time-varying delays	74
7.1	Introduction	74
7.2	Preliminaries and lemmas	75
7.3	Impulsively exponential stability	77
7.4	Numerical examples	81
8	Global stability analysis of neural networks with time-varying delays and impulsive delays	85
8.1	Introduction	85
8.2	Preliminaries and lemmas	86
8.3	Impulsively stability	88
9	Oscillation criteria for second order matrix differential systems	101
9.1	Introduction	101
9.2	Main Results	102
9.3	Examples	104
10	Nontrivial solutions of fourth-order two-point boundary value problems	105
10.1	Introduction	105
10.2	Preliminaries and lemmas	106
10.3	Main results	107
10.4	Examples	110
11	Fixed point and the existence of positive solutions of nonlinear third-order three-point boundary value problems	112
11.1	Introduction	112
11.2	Preliminaries and lemmas	113
11.3	Main results	118
12	Nontrivial solutions for a nonlinear third-order three-point boundary value problem	123
12.1	Introduction	123
12.2	Preliminaries and lemmas	124
12.3	Main results	124
12.4	Examples	129

13 Appendices	130
13.1 Complete metric spaces	130
13.1.1 Metric spaces	130
13.1.2 Normed vector spaces	131
13.1.3 Cone metric spaces	131
13.2 The contraction mapping principle	132
13.3 Dynamical systems	134
13.3.1 Continuous dynamical systems.	134
13.3.2 Arbitrary switched systems	135
13.3.3 Dini derivatives	137
13.3.4 Stability of arbitrary switched systems	139
13.3.5 Two useful lemmas	141

原 书 缺 页

原 书 缺 页

原 书 缺 页

原 书 缺 页

原 书 缺 页