A Course in Simple Calculus

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Metallurgical Industry Press

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Preface

This book is designed for an introductory course in calculus. We hope to provide a solid foundation in the mathematics while at the same time lead the reader to the core mathematical vocabulary. In order to limit the length of text, a complete frame on calculus theory based on unitary function is presented only.

Recently, as we seen, more and more demand on English calculus come from non-engineering specialties, such as *Science English Specialty* and *Foreign Trade Specialty*. However, the suitable texts are lagged. The first aim of this book is to meet the request of these specialties. With PC's prevalence and non-expense, the popular mathematical software, such as *MathLab*, *Mathematica*, *MathCAD* and so on, have being used in engineering and technology subject progressively. Numerous successful applications of this popular mathematical software deduce such an incontestable fact that this popular mathematical software will play a more and more important role and be an absolutely necessary tool for our future engineers in the new century. Unfortunately, for Chinese students, the use of this software is inconvenient because all the software mentioned above is English edition and contains many mathematical vocabularies. In view of this, the second aim of this book is to present a bridge between applying calculus and using this software for our engineering students. Besides these, as we noted, for some science subject students such as *Mathematics*, *Informatics* and *Computer Science*, the special English text we used before is either too thick or too hard to read. We hope this book can help the reader master the core English in mathematics as soon as possible, so that they can understand the special material favoringly. This is the third aim of this book. No other than the above reason, we wish this book to be useful for the following specialty: *Science English*, *Foreign Trade*, *Mathematics*, *Informatics*, *Computer Science* and majority engineering specialties.

A Course in Simple Calculus has arisen from the revised lectures, which the authors have used in recent years for *Science English Specialty* and the mathematical experimentation of Engineering Specialties. The text is self-contained.

As one notable characteristic of this course, we present one chapter on the usage of *Mathematica* software at the end of this book, so that the reader can master this useful and powerful tool as quickly as possible.

It is a pleasure to acknowledge the help that made the book possible. We would particularly like to express our gratitude to *Metallurgical Industry Press*, for their earnest attitude and efficient work.

We would like to thank two respectable mathematicians. They are of mathematics department of Hebei University, Professor Zhixue Zhang(张知学), *the Secretary-general of Hebei Mathematical Association*, and Professor Yanbin Han (韩彦彬), *the Reviewer of USA Mathematical Review*, for their edification and urge.

We also wish to express our thanks to the leaders of Hebei Institute of Technology for their support and concern.

We owe a special thanks to Huiling Liu (刘会灵) for keyboarding the manuscript and for proofreading the book in manuscript.

Any errors that appear are the responsibility of the authors.

We will appreciate having these brought to our attention.

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CONTENTS

CHAPTER 4 DIFFERENTIATION AND ITS APPLICATIONS

CHAPTER 5 INTEGRATION

CHAPTER 6 RUNNING MATHEMATICA

CHAPTER 1

PRELUDE TO CALCULUS

The purpose of this introductory chapter is to establish the notation terminology that will be used throughout the book and to present a few diverse results from set theory and analysis that will be needed later. The style here is deliberately terse, since this chapter is intended as a reference rather than a systematic exposition.

1.1 Logic

It is useful to note the following logic that is often insufficiently appreciated by students. Let p and q be two statements, for example:

$$p = \text{"it is a dog"}$$
$$q = \text{"it has four legs"}$$

Then we may say:

(1) If p, then q.

(2) If q, then p.

(3) If not p, then not q.

(4) If not q, then not p.

Use the above example and say each sentence in words, they are:

(1) If it is a dog, then it has four legs.

(2) If it has four legs, then it is a dog.

(3) If it is not a dog, then it does not have four legs.

(4) If it does not have four legs, then it is not a dog.

Obviously, (1) and (4) are equivalent, that is, they have the same meaning. But (1) and (2) are not equivalent. Also, (1) and (3) are not equivalent. Therefore, in order to prove (1), we may prove equivalently (4).

It is a simple exercise to show that $\sqrt{2}\notin\mathbb{Q}$($\mathbb{Q}$ denotes the set of rational numbers). Recall that if $x\in\mathbb{Q}$ then by definition $x=a/b$ where $a,b\in\mathbb{Z}$ and $b\neq 0$. We shall prove that $\sqrt{2}\notin\mathbb{Q}$.

Suppose it is not true that $\sqrt{2}\notin\mathbb{Q}$. That is, suppose $\sqrt{2}\in\mathbb{Q}$. Then $\sqrt{2}=a/b$ where a, $b\in\mathbb{Z}$ and $b\neq 0$. We may assume that a and b have no common factors. Thus $\sqrt{2}b=a$ or $2b^2=a^2$. Since $2b^2$ is divisible by 2, so is a^2. That is, a is divisible by 2. In other words, a^2 is divisible by 4 or $2b^2$ is divisible by 4. It implies that b^2 is divisible by 2 or b is divisible by 2. Hence a and b have a common factor 2, which contradicts the previous assumption that they do not have any common factor, therefore the assumption that $\sqrt{2}\in\mathbb{Q}$ must be wrong. Hence $\sqrt{2}\notin\mathbb{Q}$. This is exactly what happens in the case of proving $\sqrt{2}\notin\mathbb{Q}$ where $p=$"$x=\sqrt{2}$"and $q=$"$x\notin\mathbb{Q}$".

1.2 Number Systems

Our notation for the fundamental number systems is as follows:

$\mathbb{N}$ = the set of positive integers. (not including zero)

$\mathbb{Z}$ = the set of integers

$\mathbb{Q}$ = the set of rational numbers

$\mathbb{R}$ = the set of real numbers

Here the words "family" and "collection" will be used synonymously with "set". Obviously:

$$\mathbb{N}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$$

1.3 Real Numbers

The real numbers are already familiar to you. Real numbers can be represented by terminating or non-terminating decimal expansions. Any terminating decimal can be written in non-terminating form by adding zeros:

$$\frac{3}{8}=0.375=0.3750000\cdots$$

Any repeating non-terminating decimal, such as:

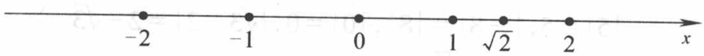
$$\frac{7}{22}=0.318181818\cdots\cdots$$

represents a rational number, is the quotient of two integers. Conversely, every rational number is represented by a repeating decimal expansion. The decimal expansion of an irrational number, such as:

$$\sqrt{2}=1.414213562\cdots\cdots$$

is non-terminating and non-repeating.

Each real number is represented by precisely one point of $\mathbb{R}$, and each point of $\mathbb{R}$ represents precisely one real number.



The following properties of inequalities of real numbers are fundamental and often used:

(1) If $a<b$ and $b<c$, then $a<c$;

(2) If $a>b$ then $a+c>b+c$;

(3) If $a>b$ then $a-c>b-c$.

Furthermore, we have:

(4) If $c>0$ and $a>b$ then $ac>bc$;

(5) If $c>0$ and $a>b$ then $\frac{a}{c}>\frac{b}{c}$.

But the inequality sign is reversed if $c>0$ in (4) and (5) is replaced by $c<0$. That is:

(6) If $c<0$ and $a>b$ then $ac<bc$;

(7) If $c<0$ and $a>b$ then $\frac{a}{c}<\frac{b}{c}$.

In particular, we have:

(8) If $a<b$ then $-a>-b$;

(9) If $0<a<b$ then $\frac{1}{a}>\frac{1}{b}$.

Remark: It is important for a, b to have the same sign in (9). Otherwise it is not true, for example, $-2<2$ and still $-\frac{1}{2}<\frac{1}{2}$.

The statement $a\geqslant b$ means: either $a>b$ or $a=b$. Hence it is correct to say that $2\geqslant 2$. The above rules (2)-(9) still hold with $>$ replaced by $\geqslant$ except when the division is involved. For example, in (5) we must have $c>0$ and then $a\geqslant b$ implies

$a/c \geqslant b/c$ Similarly, we must have $c<0$ in (7) and $0<a$ in (9).

1.4 Absolute Value

The (non-negative) distance along the real line between zero and real number a is the **absolute value** of a, written $|a|$. Equivalently:

$$|a| = \begin{cases} a & if \quad a \geqslant 0 \\ -a & if \quad a<0 \end{cases}$$

For example:

$$|5|=5, |-8|=|8|, |0|=0, |\sqrt{3}-2|=2-\sqrt{3}$$

the latter being true because $2>\sqrt{3}$.

The following properties of absolute values are frequently used:

(1) $|a|=|-a|=\sqrt{a^2} \geqslant 0$;

(2) $|a \cdot b|=|a| \cdot |b|$;

(3) $-|a| \leqslant a \leqslant |a|$;

(4) $|a|<b$ if and only if $-b \leqslant a \leqslant b$;

(5) $|a+b| \leqslant |a|+|b|$ for $a, b \in R$.

We can deduce the following inequalities from (5):

(6) $|a-b| \leqslant |a|+|b|$;

(7) $||a|-|b|| \leqslant |a-b|$.

The **distance** between the real number a and b is defined to be $|a-b|$ or $|b-a|$, this distance is simply the length of the line segment of the real line $\mathbb{R}$ with endpoints a and b.

1.5 Intervals

Suppose that S is a set (collection) of real numbers. Then we write $x \in S$ if and only if x is an **element** of S. The set S may be described by means of the notation:

$$S=\{x: \cdots\cdots\}$$

where the ellipsis represents a condition that the real number x satisfies exactly when x belongs to S.

The set S inclusion set T is denoted by $S \supset T$.

The **union** of the two sets S and T is the set $S \cup T$ given by:

$$S \cup T = \{x : \text{either } x \in S \text{ or } x \in T \text{ or both}\}$$

The **intersection** of the two sets S and T is the set $S \cap T$ defined as follows:

$$S \cap T = \{x : \text{both } x \in S \text{ and } x \in T\}$$

The most important sets of real numbers in calculus are **intervals**. If $a < b$, then the **open interval** (a, b) is defined to be the set:

$$(a, b) = \{x : a < x < b\}$$

and the **closed interval** $[a, b]$ is:

$$[a, b] = \{x : a \leqslant x \leqslant b\}$$

We also use the **half-open intervals** and **unbounded interval**, which have forms such as:

$$[a, b) = \{x : a \leqslant x < b\}$$
$$(a, b] = \{x : a < x \leqslant b\}$$

and:

$$[a, +\infty) = \{x : x \geqslant a\} \qquad (a, +\infty) = \{x : x > a\}$$
$$(-\infty, b] = \{x : x \leqslant b\} \qquad (-\infty, b) = \{x : x < b\}$$

1.6 The Least Upper Bound Axiom

Let $S \subset R$. Then S is said to be **bounded** if there is a real number M such that $|x| \leqslant M$ for every $x \in S$. If there is $M \in R$ such that $x \leqslant M$ for every $x \in S$ then we say: S is bounded above or M is an upper bound of S. Similarly, if there is $m \in R$ such that $m \leqslant x$ for every $x \in S$ then S is bounded below or m is a lower bound of S. Obviously, a set is bounded if and only if it is bounded above and below.

Definition: A number M is called the least upper bound of a set S and we write $M = \sup S$ if:

(1) M is an upper bound of S;

(2) for any upper bound M_1 of S, we have $M \leqslant M_1$.

That is, M is the least of all upper bounds of S. Similarly, a number m is called the **greatest lower bound** of a set S and we write $m = \inf S$ if:

(1) m is a lower bound of S, and;

(2) for any lower bound m_1 of S, we have $m \geqslant m_1$.

For example, the set $S = \{1, 1/2, 1/3, \cdots\}$ has the least upper bound 1 and the greatest lower bound 0. Note that $1 \in S$ whereas $0 \notin S$. In other words, sup S and inf S may or may not belong to the set S.

We give two more examples. Let:

$$S_1=\left\{(-1)^n \frac{n}{n-1}\ n=1,2,\cdots\right\}$$

and:

$$S_2=\{x:x^2<2\}$$

Then inf $S_1=-1$, sup $S_1=1$, inf $S_2=-\sqrt{2}$ and sup $S_2=\sqrt{2}$.
However sup $\{1,2,3\cdots\}$ does not exist.

The least upper bound axiom: if $S\subset R$ and S is bounded above then S has a least upper bound.

By an axiom we mean a statement, which we accept to be true without proof. Given a set $S\subset R$, denote:

$$-S=\{-x:x\in S\}$$

Then it is easy to verify that:

$$\sup S=-\inf(-S)$$

and also:

$$\inf S=-\sup(-S)$$

Hence it follows from the least upper bound axiom that if $S\subset R$ and S is bounded below then S has a greatest lower bound. In what follows, we shall assume the above least upper bound axiom and its consequence, and use them to prove the results involving limits.

1.7 Elementary Function

An **exponential function** is one of the forms $f(x)=a^x$. Note that x is the variable; the number a is a constant. Thus an exponential function is "a constant raised to a variable power", whereas the **power function** $p(x)=x^k$ is "a variable raised to a constant power."

The **logarithmic function** with base a is introduced as the inverse of the exponential function $f(x)=a^x$. That is, $\log_a x$ is "the power to which a must be raised to get x", so that:

$$y=\log_a x \text{ means that } a^y=x$$

The **logarithm function** $g(x)=\log_e x$ with base e is called the natural logarithm function. It is commonly denoted by the special logarithm symbol ln, so that:

$$\log_e x = \ln x$$

The natural logarithm function is the inverse of the **natural exponential function**, so:

$$e^{\ln x} = x \text{ for all } x > 0 \text{ and } \ln(e^x) = x \text{ for all } x$$

The graphs are sketched in Figurer 1.1.

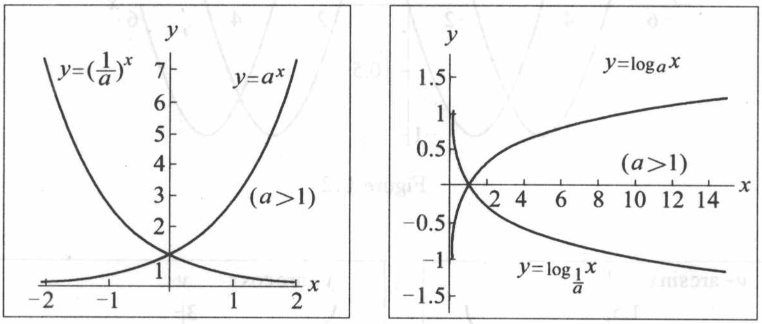


Figure 1.1

In elementary trigonometry, the sine and cosine function, which are shown in Figure 1.2 are most basic trigonometric functions. The **sine function** is denoted by $\sin x$, the domain is $(-\infty, +\infty)$, the range is $[0,1]$. The **cosine function** is denoted by $\cos x$, the domain is $(-\infty, +\infty)$, the range is $[0,1]$. The other standard trigonometric functions are defined by:

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

The inverse of the function:

$$f(x) = \sin x \qquad -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$$

is denoted by arcsin (Figure 1.3); the domain of arcsin is $[-1,1]$.

The inverses of the function:

$$g(x) = \cos x \qquad 0 \leqslant x \leqslant \pi$$

is denoted by arccos (Figure 1.3); the domain of arccos is $[-1,1]$.

Similarly, the inverses of the functions $\tan x$ and $\cot x$ are denoted by $\arctan x$ and $\operatorname{arccot} x$ (Figure 1.4).

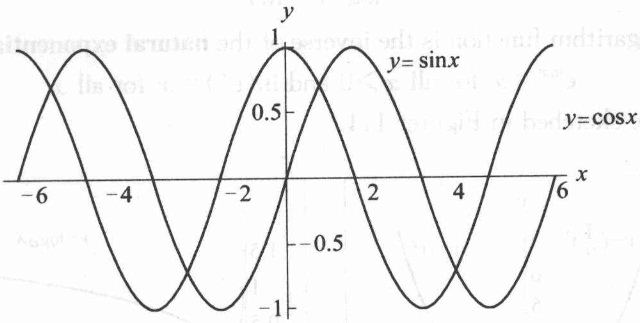


Figure 1.2

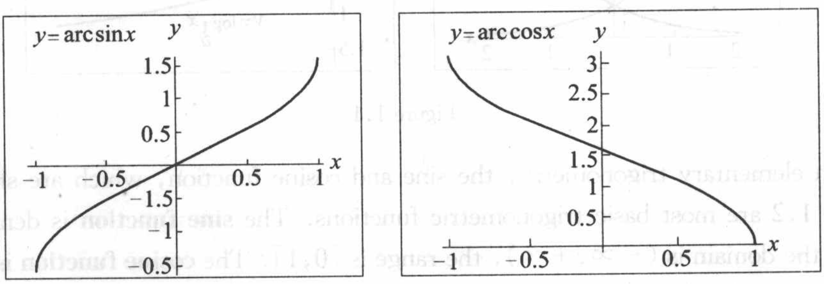


Figure 1.3

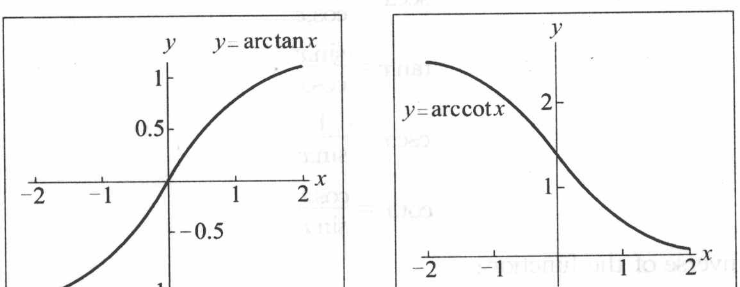


Figure 1.4

CHAPTER 2

INFINITE SEQUENCES AND INFINITE SERIES

2.1 Concept of Sequence

An array of real numbers $\{a_1, a_2, a_3, \cdots\}$ is called a sequence. Sometimes we write $\{a_n\}$. For example, the following are all sequences:

$$\{1,2,3,\cdots\}$$
$$\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},\cdots\right\}$$
$$\left\{1,1+\frac{1}{2},1+\frac{1}{2}+\frac{1}{3},1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4},\cdots\right\}$$
$$\left\{1,-\frac{1}{2},\left(\frac{1}{2}\right)^2,-\left(\frac{1}{2}\right)^3,\cdots\right\}$$

A sequence $\{a_n\}$ is said to be **increasing** if:

$$a_1 \leqslant a_2 \leqslant a_3 \leqslant \cdots$$

Similarly, $\{a_n\}$ is **decreasing** if:

$$a_1 \geqslant a_2 \geqslant a_3 \geqslant \cdots$$

or $\{-a_n\}$ is increasing. Sometimes we want to be more specific and say: $\{a_n\}$ is **strictly increasing** if:

$$a_1 < a_2 < a_3 < \cdots$$

and $\{a_n\}$ is **strictly decreasing** if:

$$a_1 > a_2 > a_3 > \cdots$$

Also, we say: $\{a_n\}$ is non-decreasing if some equalities in $a_1 \leqslant a_2 \leqslant a_3 \leqslant \cdots$ may actually occur, and $\{a_n\}$ is non-increasing if some equalities in $a_1 \geqslant a_2 \geqslant a_3 \geqslant \cdots$ may actually occur. A sequence $\{a_n\}$ is said to be **monotone** if it is either increasing or decreasing.

A sequence $\{a_n\}$ is said to be **bounded** or bounded by M if :

$$|a_n| \leqslant M \quad \text{for all } n$$

That is, as a set $\{a_1, a_2, \cdots\}$, it is bounded. It is easy to give examples of sequences, which are monotone and bounded.

2.2 The Limit of a Sequence

We shall define the limit of a sequence $\{a_n\}$. This is the first important concept in calculus. Let us motivate by computing $\sqrt{2}$. Take $a_1 = 1$ to be the first approximation of $\sqrt{2}$. We observe that:

$$a_1 \cdot \frac{2}{a_1} = 2$$

Since $a_1^2 < 2$ or $a_1 < \sqrt{2}$, a_1 is too small and $\frac{2}{a_1}$ is too large. A better approximation of $\sqrt{2}$ would be:

$$a_2 = \frac{1}{2}\left(a_1 + \frac{2}{a_1}\right)$$

Again, we have:

$$a_2 \cdot \frac{2}{a_2} = 2$$