Mechanics of Materials S. I. EDITION

S. P. Timoshenko
James M. Gere

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Stephen P. Timoshenko

Professor Emeritus of Applied Mechanics
Stanford University

James M. Gere

Professor of Civil Engineering
Stanford University

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PREFACE

It should be possible to write a textbook on mechanics of materials that meets the needs of undergraduate students, who are learning about the subject for the first time, and graduate engineers who require a dependable reference. The goal of the authors has been to satisfy both of these requirements. We have tried to present the theories and methods in a teachable and easy-to-learn manner, with ample discussions and illustrative examples, so that undergraduate students can readily master the fundamentals. However, the text often goes beyond the elementary stages, so that both more advanced subjects and more specialized subjects are included. Thus, the graduate engineer, whether he is engaged in design or research, or whether he is extending his studies on his own initiative, will find that there is much additional material of interest to him.

A glance through the table of contents will show the topics covered in this book. These topics include the analysis of structural members subjected to axial load, torsion, and bending, as well as all of the basic concepts of mechanics of materials, such as strain energy, stress and strain transformations, inelastic behaviour, and so forth. Special topics of interest to engineers include thermal effects, nonprismatic beams, large deflections of beams, bending of unsymmetrical beams, the shear centre, and many others. Finally, the last chapter gives an introduction to structural analysis and energy methods, including the unit-load method, reciprocal theorems, flexibility and stiffness methods, strain energy theorems, potential energy theorems, Rayleigh-Ritz method, and complementary energy theorems. This chapter can serve the reader as a foundation for the study of modern structural theory.

There is clearly more material in this book than would be covered in a typical undergraduate course, hence each teacher has the opportunity to select the material that he feels is the most fundamental and important. Teachers will also appreciate the hundreds of new problems in the book (over 600 problems total) that are available for homework assignments or for use in class discussions.

The reader will soon discover the extensive references that are collected at the back of the book. These references give the historical development and the original sources of the subject matter. Furthermore, because there is much interest in the pioneers who developed the subject, we have also included biographical notes in many places in the references.

This book is "new" in the sense that it is a completely new presentation of mechanics of materials, covering subject matter of current interest. But in another sense it is "old," because it has evolved from the well-known two-volume series entitled *Strength of Materials* by Professor Timoshenko. *Strength of Materials* was last revised in 1955 and 1956, when a third edition was published. The second edition was published in 1940 and 1941, and the first edition was published in 1930. Moreover, the first edition was actually based upon several earlier versions published in Russia and extending as far back as 1908. A listing of those early Russian editions can be found in Timoshenko's bibliography, which appears in his autobiography, "As I Remember" (D. Van Nostrand Co., Inc., 1968). The authors are hopeful that this book, accompanied by a later volume titled *Advanced Mechanics of Materials*, will have brought this long line of textbooks up to date.

To acknowledge all of the people who have contributed to this book in some manner would clearly be impossible, but a major debt is owed to Professor D. H. Young, who read the entire manuscript and offered many valuable suggestions. We are indebted also to another colleague, Professor William Weaver, Jr., for advice on the chapter on structural analysis and energy methods. A different type of debt is owed to the many students who studied from earlier versions of this book and from whom the authors have learned how to write a better textbook! And, of course, no book could be written without the help of devoted secretaries—Mrs. Mark F. Nelson, Jeanne Mackenzie, Mrs. Richard E. Platt, and Susan Bennett. To these persons and many others, the authors are pleased to express their gratitude.

Stanford, California S. P. Timoshenko
July, 1971 J. M. Gere

PREFACE TO S.I. EDITION

In the conversion of this textbook into SI units the endeavour has been to use the recommended metric units as far as possible. The most appropriate unit for stress is still under debate amongst engineers and the main alternatives are Newtons per square millimetre (N/mm^2), mega-Newtons per square metre (MN/m^2), the Pascal (one mega-Pascal = 1 MN/m^2 = 1 N/mm^2) and the bar (0.1 N/mm^2). The adoption of N/mm^2 as the basic unit for stress in the recently published British Standard Code of Practice 110 for the structural use of concrete and in the BCSA/CONSTRADO Handbook on Structural Steelwork has been taken as a precedent and followed in this book. For length, metres and millimetres have been used, except where areas, second moments of area and section moduli have been quoted from the Handbook on Structural Steelwork in centimetre units.

All examples have been recast to maintain ease of numerical working, so that dimensions do not correspond precisely with those used in the original American edition of the book.

In the conversion of this book it its hoped that nothing has been done to obscure or detract from the clarity of the original text.

Dundee, Scotland A. R. Cusens
March, 1973 R. A. Estañero

LIST OF SYMBOLS

A	area, action (force or couple), constant
$a, b, c,$	dimensions, distances, constants
C	constant of integration, centroid
c	distance from neutral axis to outer surface of a beam
D	displacement, kinematic unknown
d	diameter, dimension, distance
E	modulus of elasticity, elliptic integral of the second kind
E_r	reduced modulus of elasticity
e	eccentricity, dimension, distance
F	force, elliptic integral of the first kind, flexibility coefficient
f	shear flow, shape factor for plastic bending
f_s	form factor for shear
G	modulus of elasticity in shear
g	acceleration of gravity
H	distance, force, reaction, horsepower
h	height, dimension
I	moment of inertia (or second moment) of a plane area
I_x, I_y, I_z	moments of inertia with respect to x, y, and z axes
I_1, I_2	principal moments of inertia
I_{xy}	product of inertia of a plane area with respect to the x and y axes
J	polar moment of inertia, torsion constant
K	bulk modulus of elasticity, effective length factor for a column
k	symbol for $\sqrt{P/EI}$
L	length, span
M	bending moment, reactive couple
M_p	plastic moment for a beam
M_y	yield moment for a beam
N	axial force
n	factor of safety, number, ratio, integer, revolutions per minute
O	origin of coordinates
P	concentrated force, load, axial force
P_{cr}	critical load for a column
P_u	ultimate load
P_w	working load or allowable load
P_y	yield load
p	pressure
Q	concentrated force, first moment (or static moment) of a plane area

q	intensity of distributed load (load per unit distance)
q_u	ultimate load
q_y	yield load
R	reaction, radius
r	radius, distance, radius of gyration ($r = \sqrt{I/A}$)
S	force, section modulus for a beam, shear centre, stiffness coefficient
s	distance, length along a curved line
T	temperature, twisting couple or torque
T_u	ultimate torque
T_y	yield torque
t	thickness
U	strain energy
u	strain energy per unit volume
$U^\star$	complementary energy
$u^\star$	complementary energy per unit volume
V	shear force, volume
v	deflection, velocity
v', v'', etc.	dv/dx, d^2v/dx^2, etc.
W	weight, work
$W^\star$	complementary work
X	statical redundant
x, y, z	rectangular coordinates, distances
$\bar{x}, \bar{y}, \bar{z}$	coordinates of centroid
Z	plastic modulus for a beam
α	angle, coefficient of thermal expansion, ratio
α_s	shear coefficient
β	angle
γ	shear strain, weight per unit volume
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	shear strains in the xy, yz, and zx planes
γ_θ	shear strain for inclined axes
δ, Δ	deflection, displacement, elongation
ϵ, ϵ	normal strain
$\epsilon_x, \epsilon_y, \epsilon_z$	normal strains in the x, y, and z directions
$\epsilon_1, \epsilon_2, \epsilon_3$	principal normal strains
ϵ_y	yield strain
ϵ_θ	normal strain for inclined axes
θ	angle, angle of twist per unit length, angle of rotation of beam axis
θ_p	angle to a principal plane or a principal axis
θ_s	angle to a plane of maximum shear stress
κ	curvature ($\kappa = 1/\rho$)
κ_y	yield curvature
λ	distance
ρ	radius, radius of curvature, radial distance in polar coordinates
ν	Poisson's ratio
σ	normal stress

$\sigma_x, \sigma_y, \sigma_z$ normal stresses on planes perpendicular to the x, y, and z axes
σ_θ normal stress on inclined plane
$\sigma_1, \sigma_2, \sigma_3$ principal stresses
σ_{cr} critical stress for a column ($\sigma_{cr} = P_{cr}/A$)
σ_r residual stress
σ_u ultimate stress
σ_w working stress or allowable stress
σ_y yield stress
τ, τ shear stress
$\tau_{xy}, \tau_{yz}, \tau_{zx}$ shear stresses on planes perpendicular to the x, y, and z axes and parallel to the y, z, and x axes
τ_θ shear stress on inclined plane
τ_u ultimate stress in shear
τ_w working stress or allowable stress in shear
τ_y yield stress in shear
ϕ angle, angle of twist
ψ nondimensional factor
ω angular velocity

CONTENTS

CHAPTER 1

TENSION, COMPRESSION, AND SHEAR

1.1 INTRODUCTION

Mechanics of materials is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading. It is a field of study that is known by a variety of names, including "strength of materials" and "mechanics of deformable bodies." The solid bodies considered in this book include axially-loaded bars, shafts, beams, and columns, as well as structures that are assemblies of these components. Usually the objective of our analysis will be the determination of the stresses, strains, and deformations produced by the loads; if these quantities can be found for all values of load up to the failure load, then we will have obtained a complete picture of the mechanical behaviour of the body.

Theoretical analyses and experimental results have equally important roles in the study of mechanics of materials. On many occasions we will make logical derivations to obtain formulas and equations for predicting mechanical behaviour, but at the same time we must recognize that these formulas cannot be used in a realistic way unless certain properties of the material are known. These properties are available to us only after suitable experiments have been made in the laboratory. Also, many problems of importance in engineering cannot be handled efficiently by theoretical means, and experimental measurements become a practical necessity. The historical development of mechanics of materials is a fascinating blend of both theory and experiment, with experiments pointing the way to useful results in some instances and with theory doing so in others. Such famous men as Leonardo da Vinci (1452–1519) and Galileo Galilei (1564–1642) made experiments to determine the strength of wires, bars, and beams, although they did not develop any adequate theories (by today's standards) to explain their test results. By contrast, the famous mathematician Leonhard Euler (1707–1783) developed the mathematical theory of columns and calculated the critical load of a column in 1744, long before any experimental evidence existed to show the significance of his results. Thus, Euler's theoretical results remained unused for many years, although today they form the basis of column theory.*

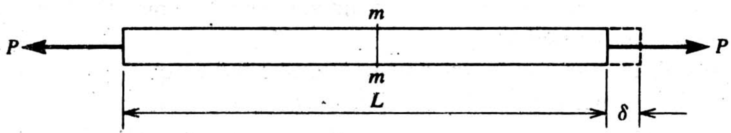
The importance of combining theoretical derivations with experimentally determined properties of materials will be evident as we proceed with our study

* The history of mechanics of materials, beginning with da Vinci and Galileo, is given in Refs. 1-1, 1-2, and 1-3.

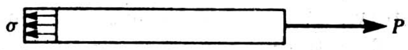
of the subject. In this chapter we will begin by discussing some fundamental concepts, such as stress and strain, and then we will investigate the behaviour of simple structural elements subjected to tension, compression, and shear.

1.2 STRESS AND STRAIN

The concepts of stress and strain can be illustrated in an elementary way by considering the extension of a *prismatic bar* (see Fig. 1-1a). A prismatic bar is one that has constant cross section throughout its length and a straight axis. In this illustration the bar is assumed to be loaded at its ends by axial forces P that produce a uniform stretching, or *tension*, of the bar. By making an artificial cut (section mm) through the bar at right angles to its axis, we can isolate part of the bar as a free body (Fig. 1-1b). At the right-hand end the tensile force P is applied, and at the other end there are forces representing the action of the removed portion of the bar upon the part that remains. These forces will be continuously distributed over the cross section, analogous to the continuous



(a)



(b)

Fig. 1-1. Prismatic bar in tension.

distribution of hydrostatic pressure over a submerged surface. The intensity of force, that is, the force per unit area, is called the *stress* and is commonly denoted by the Greek letter σ. Assuming that the stress has a uniform distribution over the cross section (see Fig. 1-1b), we can readily see that its resultant is equal to the intensity σ times the cross-sectional area A of the bar. Furthermore, from the equilibrium of the body shown in Fig. 1-1b, we can also see that this resultant must be equal in magnitude and opposite in direction to the force P. Hence, we obtain

$$\sigma = \frac{P}{A} \tag{1-1}$$

as the equation for the uniform stress in a prismatic bar. This equation shows that stress has units of force divided by area—for example, Newtons per square millimetre (N/mm^2)* or pounds per square inch (psi). When the bar is being stretched by the forces P, as shown in the figure, the resulting stress is a *tensile stress*; if the forces are reversed in direction, causing the bar to be compressed, they are called *compressive stresses*.

A necessary condition for Eq. (1-1) to be valid is that the stress σ must be uniform over the cross section of the bar. This condition will be realized if the axial force P acts through the centroid of the cross section, as can be demonstrated by statics (see Prob. 1.2-1). When the load P does not act at the centroid, bending of the bar will result, and a more complicated analysis is necessary (see Art. 5.10). Throughout this book, however, it is assumed that all axial forces are applied at the centroid of the cross section unless specifically stated to the contrary. Also, unless stated otherwise, it is generally assumed that the weight of the object itself is neglected, as was done when discussing the bar in Fig. 1-1.

The total elongation of a bar carrying an axial force will be denoted by the Greek letter δ (see Fig. 1-1a), and the elongation per unit length, or *strain*, is then determined by the equation

$$\epsilon = \frac{\delta}{L} \tag{1-2}$$

where L is the total length of the bar. Note that the strain ϵ is a nondimensional quantity. It can be obtained accurately from Eq. (1-2) as long as the strain is uniform throughout the length of the bar. If the bar is in tension, the strain is a *tensile strain*, representing an elongation or stretching of the material; if the bar is in compression, the strain is a *compressive strain*, which means that adjacent cross sections of the bar move closer to one another.

1.3 THE TENSILE TEST

The relationship between stress and strain in a particular material is determined by means of a *tensile test*. A specimen of the material, usually in the form of a round bar, is placed in a testing machine and subjected to tension. The force on the bar and the elongation of the bar are measured as the load is increased. The stress in the bar is found by dividing the force by the cross-sectional area, and the strain is found by dividing the elongation by the length along which the elongation occurs. In this manner a complete *stress-strain diagram* can be obtained for the material.

The typical shape of the stress-strain diagram for structural steel is shown in Fig. 1-2a, where the axial strains are plotted on the horizontal axis and the corresponding stresses are given by the ordinates to the curve *OABCDE*. From O to A the stress and strain are directly proportional to one another and the diagram is *linear*. Beyond point A the linear relationship between stress and strain no longer exists; hence the stress at A is called the *proportional limit*. For

* Alternative units (identically equivalent to N/mm^2) are MN/m^2 and MPascal.

low-carbon (structural) steels, this limit is usually between 200 N/mm² and 250 N/mm², but for high-strength steels it may be much greater. With an increase in loading, the strain increases more rapidly than the stress, until at point B a considerable elongation begins to occur with no appreciable increase in the tensile force. This phenomenon is known as *yielding* of the material, and the stress at point B is called the *yield point* or *yield stress*. In the region BC the material is said to have become *plastic* and the bar may actually elongate plastically by

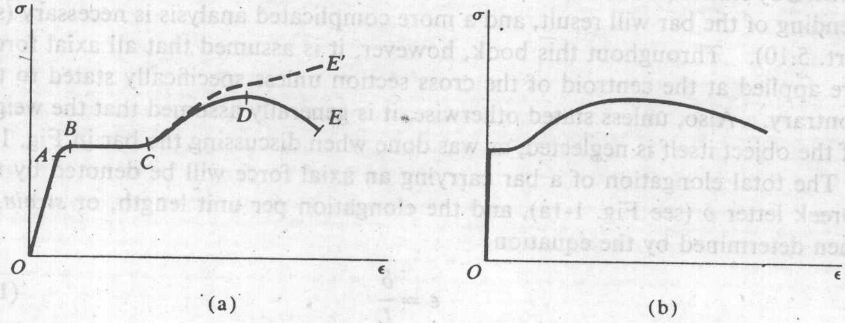


Fig. 1-2. Typical stress-strain curve for structural steel: (a) pictorial diagram (not to scale); (b) diagram to scale.

an amount which is 10 or 15 times the elongation which occurs up to the proportional limit. At point C the material begins to *strain harden* and to offer additional resistance to increase in load. Thus, with further elongation the stress increases, and it reaches its maximum value, or *ultimate stress*, at point D. Beyond this point fur her stretching of the bar is accompanied by a reduction in the load, and fracture of the specimen finally occurs at point E on the diagram.

During elongation of the bar a lateral contraction occurs, resulting in a decrease in the cross-sectional area of the bar. This phenomenon has no effect on the stress-strain diagram up to about point C, but beyond that point the decrease in area will have a noticeable effect upon the calculated value of stress. A pronounced *necking* of the bar occurs (see Fig. 1-3), and if the actual

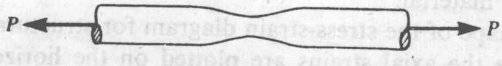


Fig. 1-3. Necking of a bar in tension.

cross-sectional area at the narrow part of the neck is used in calculating σ, it will be found that the *true stress-strain curve* follows the dashed line CE'. Whereas the total load the bar can carry does indeed diminish after the ultimate stress is