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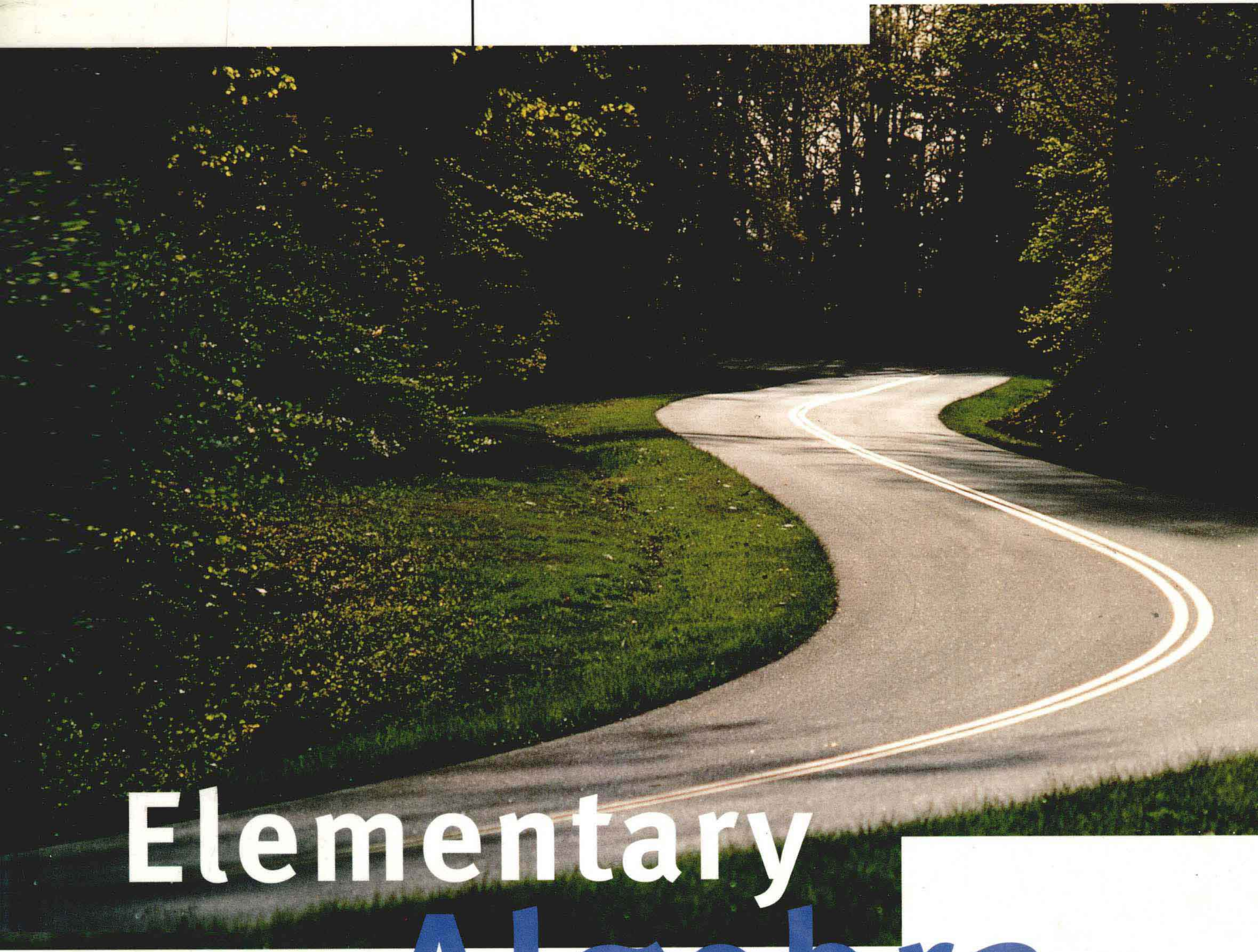

Elementary Algebra

SECOND EDITION

ELEMENTARY ALGEBRA

Second Edition

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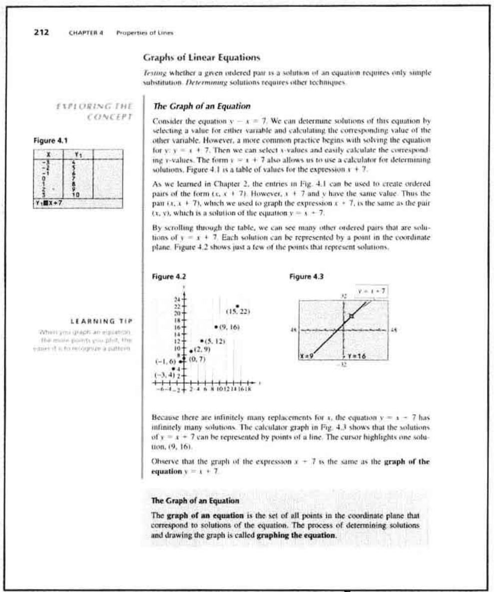

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Graphs of Linear Equations

Testing whether a given ordered pair is a solution of an equation requires only simple substitution. *Determining* solutions requires other techniques.

EXPLORING THE CONCEPT

The Graph of an Equation

Consider the equation $y - x = 7$. We can determine solutions of this equation by selecting a value for either variable and calculating the corresponding value of the other variable. However, a more common practice begins with solving the equation for y: $y = x + 7$. Then we can select x-values and easily calculate the corresponding y-values. The form $y = x + 7$ also allows us to use a calculator for determining solutions. Figure 4.1 is a table of values for the expression $x + 7$.

Figure 4.1

X	Y1	
-3	4	
-2	5	
-1	6	
0	7	
1	8	
2	9	
3	10	
Y1■X+7		

As we learned in Chapter 2, the entries in Fig. 4.1 can be used to create ordered pairs of the form $(x, x + 7)$. However, $x + 7$ and y have the same value. Thus the pair $(x, x + 7)$, which we used to graph the expression $x + 7$, is the same as the pair (x, y), which is a solution of the equation $y = x + 7$.

By scrolling through the table, we can see many other ordered pairs that are solutions of $y = x + 7$. Each solution can be represented by a point in the coordinate plane. Figure 4.2 shows just a few of the points that represent solutions.

LEARNING TIP

Figure 4.2

Figure 4.3

Because there are infinitely many replacements for x, the equation $y = x + 7$ has infinitely many solutions. The calculator graph in Fig. 4.3 shows that the solutions of $y = x + 7$ can be represented by points of a line. The cursor highlights one solution, (9, 16).

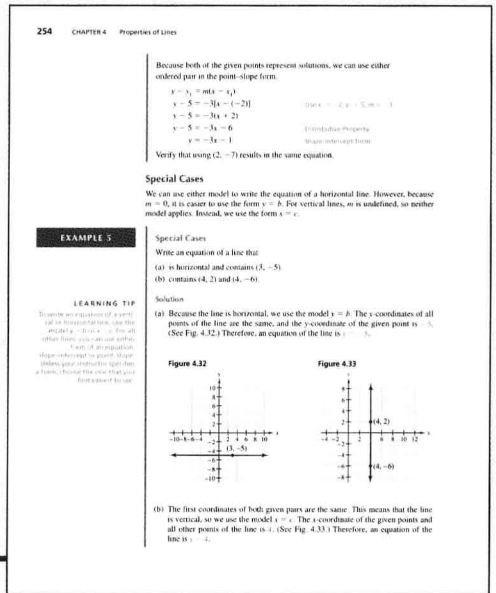
Observe that the graph of the expression $x + 7$ is the same as the **graph of the equation** $y = x + 7$.

The Graph of an Equation

The **graph of an equation** is the set of all points in the coordinate plane that correspond to solutions of the equation. The process of determining solutions and drawing the graph is called **graphing the equation.**

Explorations

The Exploration/Discovery format in the first edition has been revised to Exploring the Concept. This new pedagogical device guides students from concrete experiences to generalizations and formal rules. Students become active participants in the learning process by experimenting and asking "What if" questions.



254 CHAPTER 4 Properties of Lines

Because both of the given points represent solutions, we can use either ordered pair in the point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = -3[x - (-2)]$$
$$y - 5 = -3(x + 2)$$
$$y - 5 = -3x - 6$$
$$y = -3x - 1$$

Verify that using (2, −7) results in the same equation.

Special Cases

We can use either model to write the equation of a horizontal line. However, because $m = 0$, it is easier to use the form $y = b$. For vertical lines, m is undefined, so neither model applies. Instead, we use the form $x = c$.

EXAMPLE 5

Special Cases

Write an equation of a line that

(a) is horizontal and contains (3, −5).

(b) contains (4, 2) and (4, −6).

LEARNING TIP

Solution

(a) Because the line is horizontal, we use the model $y = b$. The y-coordinates of all points of the line are the same, and the y-coordinate of the given point is −5. (See Fig. 4.32.) Therefore, an equation of the line is $y = -5$.

Figure 4.32

Figure 4.33

(b) The first coordinates of both given pairs are the same. This means that the line is vertical, so we use the model $x = c$. The x-coordinate of the given points and all other points of the line is 4. (See Fig. 4.33.) Therefore, an equation of the line is $x = 4$.

Features for Discovery, Visualization, and Support

Examples

All sections contain numerous, titled Examples, many with multiple parts graded by difficulty. These Examples illustrate concepts, procedures, and techniques, and they reinforce the reasoning and critical thinking needed for problem solving. Detailed solutions include helpful comments that justify the steps taken and explain their purpose.

Figure 2.16

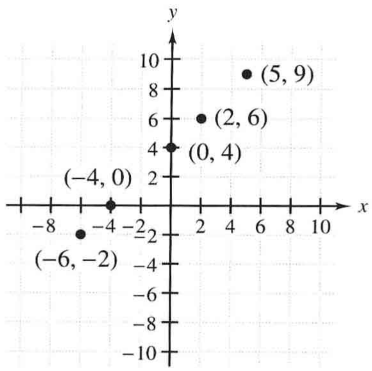


Figure 2.28

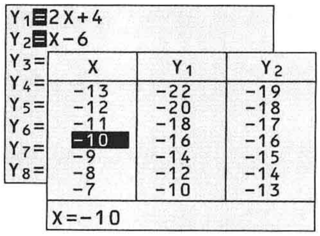
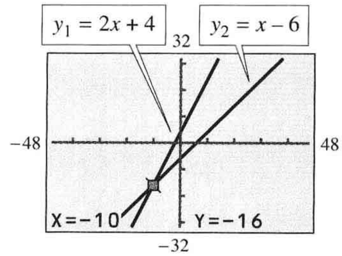


Figure 2.29



Features for Discovery, Visualization, and Support

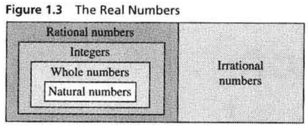
Graphs

Both traditional and calculator graphs are used throughout the exposition and exercises to assist students in visualizing concepts. Calculator displays are representative and are intended to resemble what students typically obtain on their own calculators.

Square Root Most calculators have a key for calculating square roots.

If we join together the set of rational numbers and the set of irrational numbers, the combined set is called the set **R** of **real numbers.** (See Fig. 1.3.) We can describe the real numbers as all numbers that have decimal names.

Figure 1.3 The Real Numbers



In parts (a) and (d) of Example 1, the decimal names can be written with a finite number of nonzero digits. Such decimals are called **terminating decimals.** In parts (b) and (c), the decimal names have repeating patterns and are called **repeating decimals.** A convenient way to represent a repeating decimal is with a line above the repeating block. Thus $\frac{1}{3} = 0.\overline{3}$ and $\frac{8}{11} = 0.\overline{72}$.

Decimal Usually you can set the number of decimal places that you want your calculator to display.

Key Words

We indicate the appropriate use of a calculator with a Key Word and a short description of the pertinent calculator function. These Key Words appear at the initial point of use. Each Key Word references the accompanying *Graphing Calculator Keystroke Guide,* where specific keystroke information for several popular calculator models can be found, including the TI-83. Selected keys from typical graphing calculators are inside the back cover.

The Commutative Properties

From their experience with arithmetic, most students readily accept these properties when they are restricted to the positive integers. It usually takes little persuasion to extend the properties to all real numbers.

Some properties are self-evident from our experience with arithmetic. For example, we know that numbers can be added or multiplied in any order and the results will be the same.

$5 + 3 = 3 + 5 = 8 \qquad 7(2) = 2(7) = 14$

The Commutative Properties of Addition and Multiplication

For any real numbers a and b,

1. $a + b = b + a$ Commutative Property of Addition
2. $ab = ba$ Commutative Property of Multiplication

Note: Subtraction and division are *not* commutative operations. For example, $12 \div 6 = 2$, but $6 \div 12 = 0.5$. In Section 1.6, we will see that $8 - 5 \neq 5 - 8$.

Notes

Special remarks and cautionary notes that offer additional insight appear throughout the text.

These observations suggest the following definition of subtraction.

Definition of Subtraction

For any real numbers a and b, $a - b = a + (-b)$.

This definition provides the method for performing subtraction.

When introduced to this rule, students often begin to confuse the methods for adding and subtracting, so keep repeating the pertinent rules as you do examples.

Performing Subtraction with Real Numbers

To subtract any two real numbers,

1. change the minus sign to a plus sign, and
2. change the subtrahend to its opposite.

Then evaluate the sum.

An obvious benefit of converting a difference into a sum is that we already know the rules of addition.

...ns, you should be able to apply the definition mentally ...lator. (When you do use a calculator, you do not need to ... calculator does it for you.)

We introduce this property of subtraction now because it is particularly useful in rational expressions.

Property of the Opposite of a Difference

For any real numbers a and b, $-(a - b) = b - a$.

As we will see in our later work, this is an important rule for simplifying expressions and for reversing the order of subtraction when we want to do so.

EXAMPLE 2

Using the Property of the Opposite of a Difference

Use the Property of the Opposite of a Difference to rewrite each expression without grouping symbols. In part (a), evaluate the expression.

(a) $-(3 - 11)$ (b) $-(5 - x)$

Solution

(a) $-(3 - 11) = 11 - 3 = 8$ (b) $-(5 - x) = x - 5$

Definitions, Properties, and Procedures

Important definitions, properties, and procedures are shaded and titled for easy reference.

EXAMPLE 4

Increasing and Decreasing a Number by a Given Percent

(a) What is 30 increased by 150%?
(b) What is 80 decreased by 25%?

LEARNING TIP

Doubling an amount means a 100% increase; decreasing an amount by 50% results in half the original amount. Use familiar benchmarks such as these to think about the reasonableness of your answers.

Solution

(a) 150% of 30 is $(1.50)(30) = 45$. Multiply the number by the percent.
$30 + 45 = 75$ Add the result to the original number.
When 30 is increased by 150%, the result is 75.

(b) 25% of 80 is $(0.25)(80) = 20$. Multiply the number by the percent.
$80 - 20 = 60$ Subtract the result from the original number.
When 80 is decreased by 25%, the result is 60.

To calculate the percent increase or decrease from an original number to a new number, we begin by determining the positive difference between the two numbers. Then the percent increase or decrease is the ratio of this difference to the original number.

Learning Tip

Every section has at least one Learning Tip that offers students helpful strategies and alternative ways of thinking about concepts.

End-of-Section Features

A typical section ends with three features—Class Discussion, Quick Reference, and Speaking the Language. Two new features — Class Discussion and Speaking the Language have been added to help students develop their Critical Thinking and Communication Skills.

88 CHAPTER 2 Algebra Basics, Equations, and Inequalities

2.2 CLASS DISCUSSION

Do you think that the latitude and longitude system for the earth is the same as a rectangular coordinate system?

2.2 QUICK REFERENCE

Familiar Graphs

- Bar graphs and line graphs show the relationship between two sets of data. Such graphs can be read from either axis to the other axis.
- Data from one set can be paired with data from another set, and the pairings can be represented with points.

Graphs in Mathematics

- The **rectangular coordinate system** (or **Cartesian coordinate system** or **coordinate plane**) consists of a horizontal number line called the **x-axis** and a vertical number line called the **y-axis.** The axes intersect at a point called the **origin.**
- In a coordinate plane, positive numbers are indicated with *tick marks* to the right of the origin on the x-axis and above the origin on the y-axis. Negative numbers are indicated with tick marks to the left of the origin on the x-axis and below the origin on the y-axis.
- The axes divide the plane into four **quadrants** that are numbered counterclockwise.
- The location of each point in a coordinate plane can be described with a pair of numbers called an **ordered pair.** The first number of the pair is the **x-coordinate,** and the second number is the **y-coordinate.**
- Highlighting the point corresponding to an ordered pair is called **plotting** the point. Plotting the points corresponding to the ordered pairs of a set is called **graphing** the set.

Coordinate Systems on a Calculator

- When we produce a coordinate system on a calculator, we must select a *window* (the minimum and maximum x- and y-values for the axes) and a *scale* (the distance between tick marks on the axes).
- The sign patterns for the coordinates in each of the quadrants are as follows:

Quadrant I	*Quadrant II*	*Quadrant III*	*Quadrant IV*
$(+, +)$	$(-, +)$	$(-, -)$	$(+, -)$

- Every point of the x-axis has a y-coordinate of 0. Every point of the y-axis has an x-coordinate of 0.

2.2 SPEAKING THE LANGUAGE

1. In a rectangular coordinate system, the point at which the x-axis and the y-axis intersect is called the ______.
2. Each point in the coordinate plane is associated with a(n) ______ pair.
3. The x-axis and the y-axis divide the coordinate plane into regions called ______.
4. Every point of the ______ has an x-coordinate of 0.

Class Discussion

Every section ends with a topic for Class Discussion. Building on concepts and procedures developed within the section, this new feature fosters student interaction and critical thinking. Answers are listed in the Instructor's Annotated Edition.

Quick Reference

Quick Reference appears at the end of all sections except those dealing exclusively with applications. These detailed summaries of the important rules, properties, and procedures are grouped by subsection for a handy reference and review tool.

Speaking the Language

Speaking the Language now appears at the end of all sections except those dealing exclusively with applications. This new feature helps students to think and communicate in the language of mathematics by reinforcing vocabulary and contextual meanings. Answers are listed in the Instructor's Annotated Edition.

SECTION 2.1 Algebraic Expressions 81

57. What is the procedure for simplifying an expression?

58. Describe two ways to remove the parentheses from the expression $-(a + b)$.

In Exercises 59–64, remove the grouping symbols and simplify the product.

59. $-5(3x)$ 60. $9(-2y)$

61. $-\frac{1}{4}(4y)$ 62. $\frac{1}{3}(6a)$

63. $-\frac{5}{6}\cdot\left(-\frac{9}{10}t\right)$ 64. $\frac{2}{3}\left(\frac{3}{2}y\right)$

In Exercises 65–74, use the Distributive Property to simplify each expression.

65. $5(4 + 3b)$ 66. $-2(3a - 7)$

67. $-3(5 - 2x)$ 68. $5(2x - 3)$

69. $2(3x + y)$ 70. $-2(x + 5)$

71. $-3(-x + y - 5)$ 72. $4(2x + y - 6)$

73. $-\frac{3}{5}(5x + 20)$ 74. $\frac{2}{3}(6x - 15)$

In Exercises 75–80, write the given expression without grouping symbols.

75. $-(x - 4)$ 76. $-(3 + 5x)$

77. $-(3 + m)$ 78. $-(2a - 3b + 7d)$

79. $-(2x + 5x - 3)$ 80. $-(-4x - y + 7)$

In Exercises 81–92, simplify the expression.

81. $2(x - 3) + 4$ 82. $4(x - 2) - 3x$

83. $5 - 3(x - 1)$ 84. $3 - 2(1 - 2y)$

85. $-2(x - 3y) + y + 2x$

86. $2(4x - 1) + 3(1 - 2x)$

87. $(2a + 7) + (5 - a)$

88. $-(x + 3) + (4 - 2x)$

89. $2 + (2a - b) + a - b$

90. $2a - 7(1 + a) + 9$

91. $4(1 - 3x) - (3x + 4)$

92. $9 - 3(1 + 3t) + 3(3t - 2)$

In Exercises 93–96, simplify the expression.

93. $2[5 - (2x - 3)] + 3x$

94. $-[2x + (4 - 3x)] - x$

95. $4 - 5[x - 3(x + 2)]$

96. $8x - 7[6 + (-5 + 2x)]$

Real-Life Applications

97. It is 8 A.M., and a student has a final exam at 6 P.M. The exam grade is predicted by $8t + 20$, where t is the number of study hours. What is the predicted grade if the student studies from now until 4 P.M.?

98. The expression $5x - 5 + 4(1 - 2x)$ describes the temperature drop (in °F) x hours after sunset. Write the expression in simplified form and determine the temperature drop 4 hours after sunset.

Modeling with Real Data

99. From 599 sites in 1985, the number of cellular phone antenna sites is expected to reach 117,920 by 2005. (Source: Cellular Telecommunications Industry Association.)

The expression $\frac{11}{2}t^3$ models the number of cellular phone antenna sites, where t represents the number of years since 1980.

Year	Cellular Phone Antenna Sites
1985	599
1995	21,000
2005	117,920

(a) What values of t represent the years 1985 and 2005?

(b) For which of these two years is the model more accurate?

100. (a) Write an expression with decimal coefficients that is equivalent to the model expression in Exercise 99.

(b) Use the equivalent model to estimate the number of sites in 1999. Round your answer to the nearest whole number.

Challenge

In Exercises 101 and 102, simplify.

101. $2(3n + 5) - 7 - [3(n + 1) - 2(n - 4)]$

102. $a - [b - (a - b)] - (a - b)$

Exercises

A typical exercise set in each section includes exercises from each of the following groups: Concepts and Skills (including Writing and Concept Extension), Real-Life Applications and Modeling with Real Data, Group Project, and Challenge. Geometric Models exercises appear in selected sections throughout the text.

2.7 EXERCISES

Concepts and Skills

1. In your own words, explain what we mean when we write $x + 3 \le 7$.

2. What do we mean by the *solution set* of an inequality?

In Exercises 3–8, for the given inequality determine which of the given numbers is a solution.

3. $2 + x > 7$; 3, 5, 9, 11

4. $5 - x \le 3$; $-8, -1, 2, 5$

5. $4 \le 1 - 3x$; $-4, -2, -1, 0$

6. $2x - 7 > 5$; 0, 2, 6, 10

7. $-3 \le 2x - 7 < 5$; $-1, 2, 5, 6, 9$

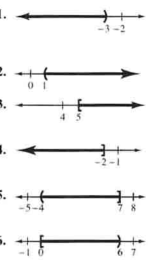
8. $5 < 2 - 3x \le 14$; $-6, -4, -1, 1$

9. When we draw a number line graph of an inequality, what symbols do we use to indicate whether an endpoint is included in the graph?

10. If $-3 < x \le 7$, describe the possible values of x.

In Exercises 11–16, write an inequality that corresponds to the given number line graph. Use x for your variable.

11. 12. 13. 14. 15. 16.

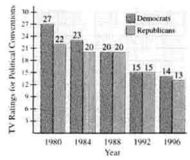


Concepts and Skills

Most exercise sets begin with the basic skills and concepts discussed within the text. These include Writing Exercises, designed to help students gain confidence in their ability to communicate, and Concept Extension exercises, which go slightly beyond the text examples. The notation CE in the Instructor's edition identifies Concept Extension exercises.

Group Project: Political Convention TV Ratings

As shown in the bar graph, the TV ratings for the Democratic and Republican conventions have declined. (Source: Neilsen Media Research.)



The TV ratings for the Democratic conventions can be modeled by the expression $-0.85x + 26.6$, and the model for the Republican conventions is $-0.575x + 22.6$. In both models, x represents the number of years since 1980.

89. (a) Write an inequality indicating that the TV rating for the Democratic convention was at least 20.

(b) Estimate the solution of the inequality in part (a).

90. Repeat Exercise 89 for the Republican convention.

91. (a) Write an inequality that indicates that the ratings for the Republican convention exceeded the ratings for the Democratic convention.

(b) Estimate and interpret the solution of the inequality in part (a).

92. What are some factors that might reverse the downward trend in the ratings?

Group Project

Group Projects now appear in the exercises for many sections. These series of exercises focus on real data and allow students to work together to solve problems. An index of Group Projects is inside the back cover.

73. Suppose you were to depict the information as ordered pairs with the year as the first coordinate and then plot the ordered pairs. In which quadrant would they lie?

74. Write the information as ordered pairs with the number of years since 1985 as the first coordinate.

75. To the nearest integer, compute the average number A of resolutions during the 9 years shown in the table on page 91. Suppose that you plot the points in Exercise 74 and draw a horizontal line A units above the x-axis. What would be the significance of the ordered pairs in Exercise 74 that lie

(a) above the line? (b) below the line?

76. What is the purpose of such resolutions and what is your opinion of the effectiveness of these Security Council actions?

Challenge

In Exercises 77–80, the coordinates of the endpoints of a line segment are given. Determine the coordinates of the midpoint of the segment.

77. $(-2, 3), (6, 3)$ 78. $(-5, -2), (-1, -2)$

79. $(-4, 1), (-4, 3)$ 80. $(3, -3), (3, 7)$

81. One side of a rectangle has vertices $(-2, -3)$ and $(7, -3)$. Another side is 8 units long. Point P is a third vertex in Quadrant I. Determine the coordinates of point P.

82. A square whose sides are 8 units long is drawn with its center at the origin. What are the coordinates of the four vertices?

Graphing Calculator Icons

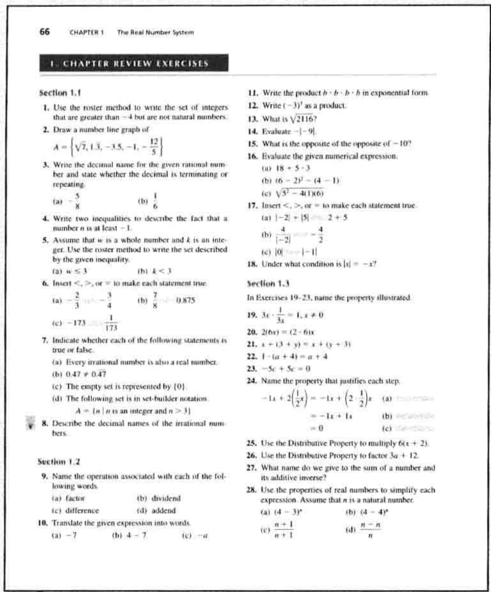
Exercises best completed with a graphing calculator now have icons, which are in only the Instructor's Annotated Edition so that instructors may determine appropriate use.

Challenge

These problems appear at the end of most exercise sets and offer more challenging work than the standard and Concept Extension problems.

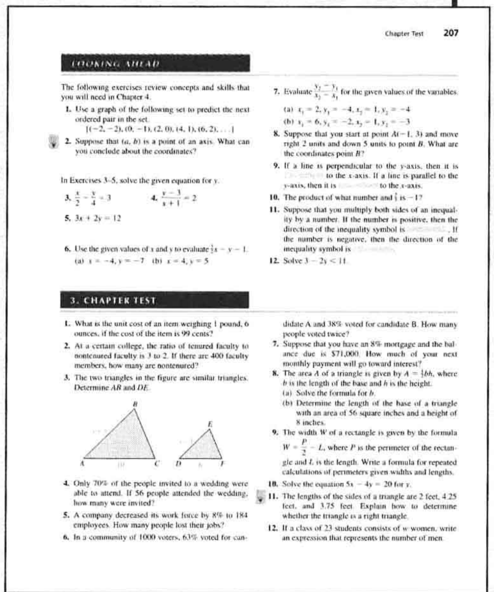
Chapter Review Exercises

Each chapter ends with a set of review exercises. These exercises include helpful section references that direct students to the appropriate sections for review. The answers to the odd-numbered review exercises are included at the back of the text.



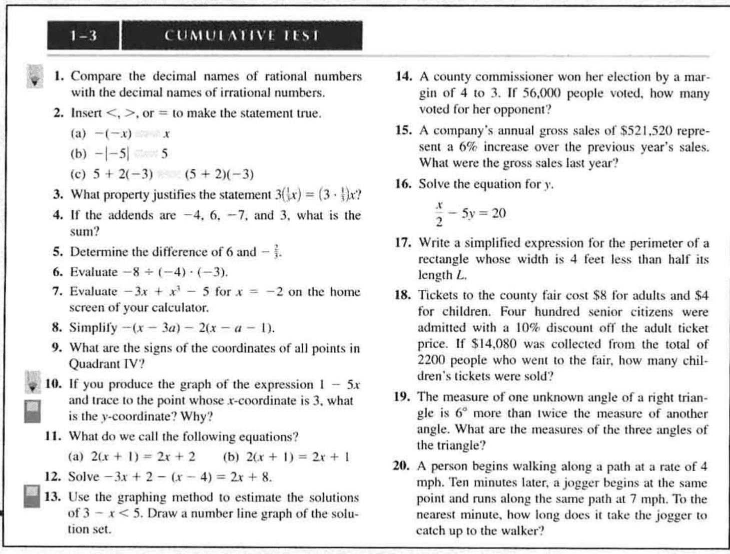
End-of-Chapter Features

At the end of each chapter, these features appear in the following order: Chapter Review Exercises, Looking Ahead (except Chapter 10), Chapter Test, Cumulative Test (at the end of selected chapters).



Looking Ahead

New to the second edition, this short list of review exercises focuses on previously discussed skills and concepts that will be needed in the upcoming chapter. Answers are in the Instructor's Annotated Edition.



1–3 CUMULATIVE TEST

1. Compare the decimal names of rational numbers with the decimal names of irrational numbers.
2. Insert <, >, or = to make the statement true.
   (a) $-(-x)$ ___ x
   (b) $-|-5|$ ___ 5
   (c) $5 + 2(-3)$ ___ $(5 + 2)(-3)$
3. What property justifies the statement $3(\frac{1}{3}x) = (3 \cdot \frac{1}{3})x$?
4. If the addends are -4, 6, -7, and 3, what is the sum?
5. Determine the difference of 6 and $-\frac{2}{3}$.
6. Evaluate $-8 \div (-4) \cdot (-3)$.
7. Evaluate $-3x + x^3 - 5$ for $x = -2$ on the home screen of your calculator.
8. Simplify $-(x - 3a) - 2(x - a - 1)$.
9. What are the signs of the coordinates of all points in Quadrant IV?
10. If you produce the graph of the expression $1 - 5x$ and trace to the point whose x-coordinate is 3, what is the y-coordinate? Why?
11. What do we call the following equations?
    (a) $2(x + 1) = 2x + 2$ (b) $2(x + 1) = 2x + 1$
12. Solve $-3x + 2 - (x - 4) = 2x + 8$.
13. Use the graphing method to estimate the solutions of $3 - x < 5$. Draw a number line graph of the solution set.
14. A county commissioner won her election by a margin of 4 to 3. If 56,000 people voted, how many voted for her opponent?
15. A company's annual gross sales of $521,520 represent a 6% increase over the previous year's sales. What were the gross sales last year?
16. Solve the equation for y.
    $\frac{x}{2} - 5y = 20$
17. Write a simplified expression for the perimeter of a rectangle whose width is 4 feet less than half its length L.
18. Tickets to the county fair cost $8 for adults and $4 for children. Four hundred senior citizens were admitted with a 10% discount off the adult ticket price. If $14,080 was collected from the total of 2200 people who went to the fair, how many children's tickets were sold?
19. The measure of one unknown angle of a right triangle is 6° more than twice the measure of another angle. What are the measures of the three angles of the triangle?
20. A person begins walking along a path at a rate of 4 mph. Ten minutes later, a jogger begins at the same point and runs along the same path at 7 mph. To the nearest minute, how long does it take the jogger to catch up to the walker?

Chapter Test

A Chapter Test follows each chapter review. The answers to all the test questions, with the appropriate section references, are included at the back of the text.

Cumulative Test

A Cumulative Test appears at the end of Chapters 3, 5, 8, and 10. The answers to all the test questions, with section references, are included at the back of the text.

ELEMENTARY

ALGEBRA

Contents

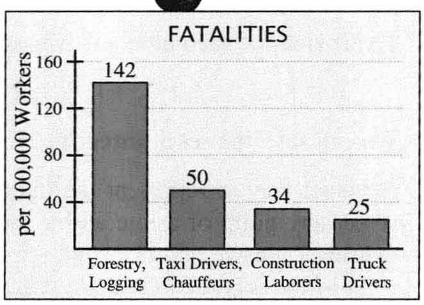
Chapter 1

The Real Number System

The accompanying bar chart shows the four occupations with the highest fatality rates per 100,000 workers.

The data in the bar chart can be written in the form of **rational numbers,** which are **real numbers** of a specific form. Operations can be performed with these numbers to draw conclusions about the hazards of these occupations. For more on this real-data problem, see Exercises 79–82 at the end of Section 1.5.

We begin our study with an examination of the structure, order, and properties of the real number system. We then turn to the rules and methods for adding, subtracting, multiplying, and dividing real numbers. These basic skills are essential to your success in this and all future courses in mathematics.



(Source: Bureau of Labor Statistics.)

1.1 THE REAL NUMBERS

The Integers • The Rational Numbers • The Real Numbers • Order of the Real Numbers

The Integers

It is reasonable to believe that the earliest use of numbers was for counting. We call the numbers 1, 2, 3, . . . the **counting numbers** or **natural numbers.**

In mathematics it is convenient to organize a collection of objects, such as numbers, into a **set** and to name the set for easy reference. For example, we can write the set N of natural numbers as $N = \{1, 2, 3, \dots\}$. Braces are used to enclose the set of numbers, and the three dots indicate that the numbers continue without end. The numbers in the set are called the **elements** of the set.

Adding just one additional element 0 to set N results in a new set W, which is called the set of **whole numbers.**

$$W = \{0, 1, 2, 3, \dots\}$$

Because every natural number is also a whole number, we say that set N is a **subset** of set W.

The method of writing a set by listing its elements is called the **roster method.** If the number of elements in a set is 0, we call the set the **empty set,** which is represented by the symbol Ø.

Note: The set {0}, which has one element, is not the same as the set Ø, which has no elements.

In everyday life we encounter numbers that describe measurements that are less than 0, such as a temperature that is below zero or a bank account that is overdrawn, or that represent decreases or losses, such as a decline in stock value or a loss of yardage in a football game.

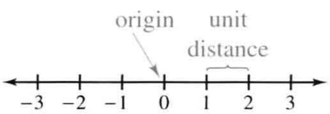
To provide for such numbers, we expand the set W of whole numbers into the set

$$J = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

We call set J the set of **integers.** Note that sets N and W are subsets of set J.

A visual way to represent the integers is a **number line.** To draw a number line, we select any point of a line and associate it with the number 0. This point is called the **origin.** Then we associate the remaining integers with points that are to the left and right of the origin and that are spaced one unit apart. We call this distance the **unit distance.** (See Fig. 1.1.) Numbers to the right of the origin are called **positive numbers.** Numbers to the left of the origin are called **negative numbers** and they are identified with the symbol −. The number 0 is neither positive nor negative.

Figure 1.1



Note: For emphasis, we sometimes identify positive numbers with the symbol +, but usually this symbol is omitted. We read +9 as *positive* 9, not plus 9. Similarly, the symbol −6 is read *negative* 6, not minus 6.

The Rational Numbers

For positive integers, the basic operations of addition, subtraction, multiplication, and division are familiar to us from arithmetic. In particular, division can be indicated in the form of a fraction such as $\frac{3}{5}$ or $\frac{12}{7}$. The number above the fraction bar is called the **numerator,** and the number below the fraction bar is called the **denominator.**

A special kind of fraction is one in which the numerator and denominator are both integers. Such fractions are called **rational numbers.**

Definition of a Rational Number

A **rational number** is a number that can be written in the form $\frac{p}{q}$, where p and q are integers and q is not 0.

A letter (or any other symbol) used to represent an unknown number is called a **variable.** In the definition of a rational number, the numerator is represented by the variable p, and p can be replaced with any integer. The denominator is represented by the variable q, and q can be replaced with any integer except 0.

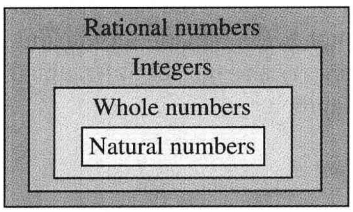
We used the roster method to write the sets of natural numbers, whole numbers, and integers. Another method for writing a set is with **set-builder notation.** For example, the set Q of rational numbers can be written

$$Q = \left\{ \frac{p}{q} \mid p \text{ and } q \text{ are integers and } q \text{ is not } 0 \right\}$$

The vertical line is read *such that.* Thus the set Q of rational numbers is the set of all numbers of the form $\frac{p}{q}$ such that p and q are integers and q is not 0. According to the definition, each of the following is a rational number.

$$\frac{5}{8}, \quad \frac{-10}{3}, \quad \frac{7}{1}, \quad \frac{0}{2}, \quad \frac{-9}{-16}$$

Figure 1.2



From arithmetic, we know that if p is a positive integer, then p can be written as the rational number $\frac{p}{1}$. Later we will see that this is also true if p is a negative integer. In short, every integer is also a rational number. Thus the set J of integers is a subset of set Q. Figure 1.2 shows the relationship among the sets N, W, J, and Q.

Every rational number $\frac{p}{q}$ has a decimal name that can be determined by dividing p by q.

Divide

Although the ÷ key may be used for division, the symbol / may be displayed on the screen.

EXAMPLE 1

Decimal Names for Rational Numbers

Use your calculator to determine the decimal names for the following rational numbers.

(a) $\frac{5}{8}$ (b) $\frac{1}{3}$ (c) $\frac{8}{11}$ (d) 6