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EDITOR

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Preface

This volume of *Progress in Optics* contains six review articles on a wide range of topics.

The first article by V. Mahajan deals with Gaussian apodization and beam propagation. The point-spread functions and the optical transfer functions of optical systems with Gaussian pupils are discussed and are compared with those for a uniformly illuminated pupil. The results are also applicable to the propagation of Gaussian beams, such as are encountered in laser transmitters. The analytical results are illustrated by numerical examples.

The next article by A. Joshi and M. Xio reviews recent investigations regarding the use of electromagnetically-induced transparency to manipulate and to control linear and nonlinear optical properties of atomic systems near resonance. Emphasis is given to enhanced four-wave mixing in three- and four-level atomic systems and to controlling nonlinear optical processes with three-level atoms inside an optical cavity.

The third article by H. Benisty and C. Weisbuch is concerned with photonic crystals and covers a broad range of topics, from physical properties of such crystals to some of their uses, for example in integrated optics.

In the article which follows, C. Brosseu and A. Dogariu discuss some basic mathematical aspects of three-dimensional electromagnetic fields, especially with regards to their polarization properties. The traditional theory of polarization is restricted to planar wavefields, but recent developments in optics, particularly in near-field optics, require broader theory. This article discusses the mathematical basis for such a generalization.

The fifth article by M. Dušek, N. Lütkenhaus and M. Hendrych deals with the relatively new field of quantum cryptography. This is a technique for secure communications based on quantum mechanics. The article explains the underlying principle of quantum cryptography, discusses the security of realistic systems and presents reviews of different experimental methods for practical implementation of this new technique.

The concluding article by N. Cerf and J. Fiurášek gives an account of researches on optical quantum cloning. After a brief introduction of the so-called "no-cloning theorem" and its relationship with the linearity and causality of quantum mechanics, the concept of quantum cloning machine is explained. In particular state-independent and state-dependent cloning machines are discussed.

It is clear that this volume covers a broad range of subjects, some rather practical, other somewhat abstract. It seems, therefore, likely that the reader will find in this volume some reviews which will be of special interest to him.

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Chapter 1

Gaussian apodization and beam propagation

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Contents

§ 1. Introduction

We consider optical systems with Gaussian apodization or Gaussian pupils, i.e., those with a Gaussian amplitude across the wavefront at their exit pupils. The mathematical treatment is applicable equally to an imaging system with Gaussian transmission (obtained, for example, by placing a Gaussian filter at its exit pupil) as well as a laser transmitter in which the laser beam has a Gaussian distribution across its exit pupil. In Section 2 we outline the general theory for obtaining the point-spread and optical transfer functions of a system from its pupil function.

A Gaussian illumination, which in principle extends to infinity, is truncated by the finite size of the pupil. An aberration-free system with a Gaussian pupil is considered in Section 3. We show that it yields a point-spread function (PSF) with a broader central bright spot but lower secondary maxima compared to the Airy pattern obtained for a uniform pupil (Jacquinot and Roizen-Dossier [1964], Buck [1967], Campbell and DeShazer [1969], Olaofe [1970], Dickson [1970], Schell and Tyras [1971], Williams [1973], Mahajan [1986]). Its central irradiance is smaller than that for a uniform pupil of the same total power (Mahajan [1980, 1986]). The corresponding optical transfer function (OTF) is higher for low spatial frequencies and lower for high spatial frequencies (Chung and Hopkins [1989], Mahajan [2004]). In Section 4 we discuss a defocused system and show that the principal maximum of axial irradiance of a focused beam with a small Fresnel number lies at a point that is closer to the pupil and not at the geometrical focus (Li and Wolf [1982], Carter [1982], Sucha and Carter [1984], Dementev and Domarkene [1984]). However, as in the case of a uniform pupil, the maximum central irradiance on a target at a fixed distance is obtained when the beam is focused on it (Mahajan [1986, 2004]).

The effect of aberrations on the central irradiance is considered in Section 5. It is shown that the Strehl ratio for a given amount of primary aberration is higher for a Gaussian pupil than that for a corresponding uniform pupil, or the aberration tolerance for a given Strehl ratio is higher for a Gaussian pupil (Lowenthal [1974], Mahajan [1995, 2003, 2005a]). Aberration balancing to reduce aberration variance and thus improve Strehl ratio for small aberrations is explained, and Zernike–Gauss polynomials that represent balanced aberrations are discussed. For systems with small Fresnel numbers, it is shown in Section 6 that the axial irradiance closer to the pupil increases when the defocus aberration is balanced with spherical aberration (Yoshida and Asakura [1996], Jiang and Stamnes [1997], Mahajan [2005b]) or astigmatism (Mahajan [2005b]). Both focused and collimated beams are discussed.

The effect of an aberration on the PSF is considered in Section 7. It is shown that while apodization broadens the central bright spot but reduces the secondary maxima, balanced spherical aberration does not change the size of the bright spot but increases the secondary maxima, thus obliterating the positive attribute of apodization (Sklar [1975], Lowenthal [1975]). The line of sight of an aberrated system, defined as the centroid of its PSF, is shown to be affected by coma-type aberrations only, and lies farther from its center for a Gaussian pupil than for a uniform pupil (Mahajan [1985]).

Narrow or weakly-truncated Gaussian illumination is considered in Section 8. It is shown that a Gaussian beam exiting from the pupil remains Gaussian as it propagates (Siegman [1971], Gaskill [1978], Belland and Crenn [1982], Mahajan [2004]). The rings of the diffraction pattern disappear as the truncation decreases. Since the beam remains Gaussian as it propagates, the OTF of such a Gaussian pupil is also a Gaussian (Mahajan [2004]). It is shown that a focused beam has the smallest radius, called its waist, in a plane that is closer to the pupil than the focal plane. Considering the waist of a beam incident on a lens as an object, an imaging equation is developed in which the waist of the transmitted beam acts as the image (Williams [1973], Self [1983], Mahajan [1986]). It is shown that when the waist of the incident beam lies in the front focal plane, the waist of the transmitted beam lies in its back focal plane, and not at infinity as in conventional imaging. Similarly, whereas in conventional imaging of a real object by a positive lens forming a real image there is a minimum separation between an object and its image, there is no minimum separation between a real object waist and a real image waist. Whereas for small truncations the approximate expression $\exp(-\sigma_\Phi^2)$ in terms of the phase aberration variance σ_Φ^2 estimates the true value of the Strehl ratio quite well for wave aberration standard deviation $\sigma_W \lesssim \lambda/4$, it significantly underestimates it for large truncations, unless the Strehl ratio is greater than or equal to 0.9 or $\sigma_W \lesssim \lambda/20$ (Mahajan [2005a]). A beam quality factor of M^2 is defined that accounts for the difference in the divergence of a practical beam and that of an ideal Gaussian beam. Although most of our discussion is based on paraxial beams, i.e., for apertures or beam radii much larger than the optical wavelength, nonparaxial beams for which the beam radius is comparable to the wavelength are also discussed briefly.

§ 2. Theory

2.1. *Pupil function*

If $A(\vec{r}_p)$ is the amplitude and $\Phi(\vec{r}_p)$ is the phase aberration defined with respect to a reference sphere of radius of curvature R at a point $\vec{r}_p$ in the plane of the exit pupil of the system, its pupil function can be written

$$P(\vec{r}_p) = A(\vec{r}_p)\exp\left[\mathrm{i}\Phi(\vec{r}_p)\right]. \tag{2.1}$$

The total power in the exit pupil and, therefore, in the image is given by

$$P_{\mathrm{ex}} = \int A^2(\vec{r}_p)\,\mathrm{d}\vec{r}_p, \tag{2.2}$$

where $A^2(\vec{r}_p)$ is the irradiance at a pupil point.

2.2. *Point-spread function*

The point-spread function (PSF) of the system in a plane at a distance R from the plane of the pupil is given by (Goodman [1996], Born and Wolf [1999], Mahajan [2004])

$$PSF(\vec{r}_i) = \frac{1}{P_{\mathrm{ex}}\lambda^2 R^2}\left|\int P(\vec{r}_p)\exp\left(-\frac{2\pi\mathrm{i}}{\lambda R}\vec{r}_p\cdot\vec{r}_i\right)\mathrm{d}\vec{r}_p\right|^2, \tag{2.3}$$

where $\vec{r}_i$ is the position vector of a point in the image plane and λ is the wavelength of object radiation. The corresponding irradiance distribution of the image of a point object is given by

$$I(\vec{r}_i) = P_{\mathrm{ex}}PSF(\vec{r}_i). \tag{2.4}$$

2.3. *Optical transfer function*

The optical transfer function (OTF) $\tau(\vec{v}_i)$ of a system is, by definition, the Fourier transform of its PSF,

$$\tau(\vec{v}_i) = \int PSF(\vec{r}_i)\exp(2\pi\mathrm{i}\vec{v}_i\cdot\vec{r}_i)\,\mathrm{d}\vec{r}_i, \tag{2.5}$$

where $\vec{v}_i$ is a spatial frequency vector. Substituting eq. (2.3) into eq. (2.5), the OTF can also be written as the autocorrelation of its pupil function

$$\tau(\vec{v}_i) = \frac{\int P(\vec{r}_p)P^*(\vec{r}_p - \lambda R\vec{v}_i)\,\mathrm{d}\vec{r}_p}{\int |P(\vec{r}_p)|^2\,\mathrm{d}\vec{r}_p}, \tag{2.6}$$

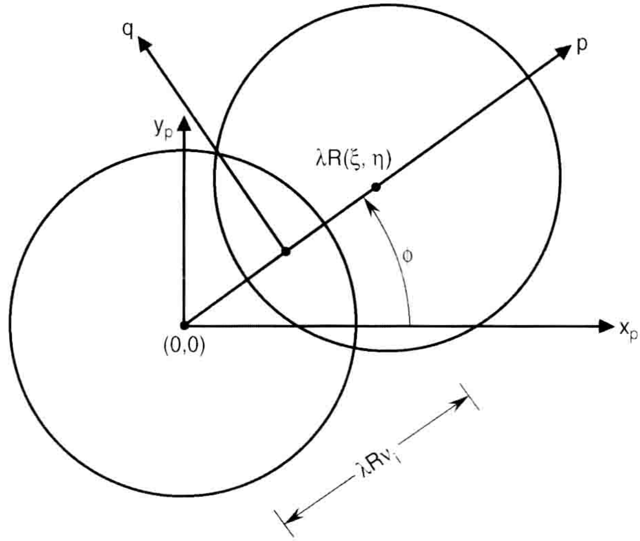

Fig. 1. Geometry for evaluating the OTF of a system with a circular pupil. The centers of the two pupils are located at $(0, 0)$ and $\lambda R(\xi, \eta)$ in the (x_p, y_p) coordinate system and $\mp(\lambda R/2)(v_i, 0)$ in the (p, q) coordinate system, where $v_i = (\xi^2 + \eta^2)^{1/2}$ and $\phi = \tan^{-1}(\eta/\xi)$. The shaded area is the overlap area of the two pupils. When normalized by the pupil radius a, the centers of the two pupils of unity radius lie at $\mp v$ along the p axis.

where the integration is across the overlap area of two intersecting pupils centered at $\vec{r}_p = 0$ and $\vec{r}_p = \lambda R \vec{v}_i$, shown shaded in fig.1 for a circular pupil.

It is convenient to consider a (p, q) coordinate system whose origin lies at the midpoint of the line joining the centers of the two pupils, but whose axes are rotated by the polar angle ϕ of the spatial frequency vector with respect to those of the (x_p, y_p) coordinate system, as illustrated in fig. 1 for a system with a circular pupil of radius a or diameter $D = 2a$. The centers of the two pupils in this coordinate system are located at $\mp(\lambda R/2)(v_i, 0)$. The corresponding pupil function $P(p, q)$ may be obtained from the pupil function $P(x_p, y_p)$ by replacing x_p with $p \cos\phi - q \sin\phi$ and y_p with $p \sin\phi + q \cos\phi$. For a radially symmetric pupil function, it is obtained simply by replacing $x_p^2 + y_p^2$ by $p^2 + q^2$. It is evident from fig. 1 that the overlap of the two pupils reduces to zero as $\lambda R v_i \to D$. Accordingly, the OTF is zero for $v_i \geqslant v_c$, where $v_c = 1/(\lambda F)$ is the cutoff spatial frequency of the system. Here $F = R/D$ is the f-number of the image-forming light cone. Using normalized quantities $\vec{\rho} = \vec{r}_p/a$ and $\vec{v} = \vec{v}_i/v_c$, the OTF in polar coordinates (v, ϕ) may be written

$$\tau(v, \phi) = \frac{a^2}{P_{ex}} \iint P(p + v, q) P^*(p - v, q)\, dp\, dq, \quad 0 \leqslant v \leqslant 1. \quad (2.7)$$