

EMIL WOLF

EDITOR



PROGRESS IN OPTICS

VOLUME 49

CONTRIBUTORS

H. Benisty, C. Brosseau, N.J. Cerf, A. Dogariu, M. Dušek, J. Fiurášek,
M. Hendrych, A. Joshi, N. Lütkenhaus, V.N. Mahajan, C. Weisbuch, M. Xiao

PROGRESS IN OPTICS

VOLUME 49

EDITED BY

E. Wolf

University of Rochester, N.Y., U.S.A.

Contributors

H. Benisty, C. Brosseau, N.J. Cerf, A. Dogariu, M. Dušek, J. Fiurášek,
M. Hendrych, A. Joshi, N. Lütkenhaus, V.N. Mahajan, C. Weisbuch, M. Xiao



ELSEVIER

Amsterdam · Boston · Heidelberg · London · New York · Oxford · Paris
San Diego · San Francisco · Singapore · Sydney · Tokyo

Elsevier
Radarweg 29, PO Box 211, 1000 AE Amsterdam, The Netherlands
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK

First edition 2006

Copyright © 2006 Elsevier B.V. All rights reserved

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means electronic, mechanical, photocopying, recording or otherwise without the prior written permission of the publisher

Permissions may be sought directly from Elsevier's Science & Technology Rights Department in Oxford, UK: phone (+44) (0) 1865 843830; fax (+44) (0) 1865 853333; e-mail: permissions@elsevier.com. Alternatively you can submit your request online by visiting the Elsevier web site at <http://elsevier.com/locate/permissions>, and selecting Obtaining permission to use Elsevier material

Notice

No responsibility is assumed by the publisher for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions or ideas contained in the material herein. Because of rapid advances in the medical sciences, in particular, independent verification of diagnoses and drug dosages should be made

Library of Congress Catalog Card number: 61-19297

ISBN-13: 978-0-444-52732-5

ISBN-10: 0-444-52732-X

ISSN: 0079-6638

| |
|--|
| For information on all Elsevier publications visit our web site at books.elsevier.com |
|--|

06 07 08 09 10 10 9 8 7 6 5 4 3 2 1

Printed and bound in The Netherlands

Preface

This volume of *Progress in Optics* contains six review articles on a wide range of topics.

The first article by V. Mahajan deals with Gaussian apodization and beam propagation. The point-spread functions and the optical transfer functions of optical systems with Gaussian pupils are discussed and are compared with those for a uniformly illuminated pupil. The results are also applicable to the propagation of Gaussian beams, such as are encountered in laser transmitters. The analytical results are illustrated by numerical examples.

The next article by A. Joshi and M. Xio reviews recent investigations regarding the use of electromagnetically-induced transparency to manipulate and to control linear and nonlinear optical properties of atomic systems near resonance. Emphasis is given to enhanced four-wave mixing in three- and four-level atomic systems and to controlling nonlinear optical processes with three-level atoms inside an optical cavity.

The third article by H. Benisty and C. Weisbuch is concerned with photonic crystals and covers a broad range of topics, from physical properties of such crystals to some of their uses, for example in integrated optics.

In the article which follows, C. Brosseau and A. Dogariu discuss some basic mathematical aspects of three-dimensional electromagnetic fields, especially with regards to their polarization properties. The traditional theory of polarization is restricted to planar wavefields, but recent developments in optics, particularly in near-field optics, require broader theory. This article discusses the mathematical basis for such a generalization.

The fifth article by M. Dušek, N. Lütkenhaus and M. Hendrych deals with the relatively new field of quantum cryptography. This is a technique for secure communications based on quantum mechanics. The article explains the underlying principle of quantum cryptography, discusses the security of realistic systems and presents reviews of different experimental methods for practical implementation of this new technique.

The concluding article by N. Cerf and J. Fiurášek gives an account of researches on optical quantum cloning. After a brief introduction of the so-called “no-cloning

theorem” and its relationship with the linearity and causality of quantum mechanics, the concept of quantum cloning machine is explained. In particular state-independent and state-dependent cloning machines are discussed.

It is clear that this volume covers a broad range of subjects, some rather practical, other somewhat abstract. It seems, therefore, likely that the reader will find in this volume some reviews which will be of special interest to him.

Emil Wolf

Department of Physics and Astronomy
and
The Institute of Optics
University of Rochester
Rochester, NY 14627, USA

April 2006

Contents

| | |
|-------------------|---|
| Preface | v |
|-------------------|---|

| | |
|---|----------|
| Chapter 1. Gaussian apodization and beam propagation, Virendra N. Mahajan (El Segundo, CA and Tucson, AZ, USA) | 1 |
| § 1. Introduction | 3 |
| § 2. Theory | 5 |
| 2.1. Pupil function | 5 |
| 2.2. Point-spread function | 5 |
| 2.3. Optical transfer function | 5 |
| § 3. Aberration-free Gaussian pupil | 7 |
| 3.1. Pupil function | 7 |
| 3.2. Point-spread function | 10 |
| 3.3. Optical transfer function | 15 |
| § 4. Defocused Gaussian pupil | 16 |
| 4.1. Pupil function | 16 |
| 4.2. Point-spread function | 17 |
| 4.3. Axial irradiance | 21 |
| 4.4. Optical transfer function | 31 |
| § 5. Strehl ratio, aberration tolerance, and Zernike–Gauss polynomials | 34 |
| 5.1. Strehl ratio | 34 |
| 5.2. Depth of focus | 35 |
| 5.3. Balanced aberrations | 39 |
| 5.4. Zernike–Gauss polynomials | 40 |
| 5.5. Strehl ratio for primary aberrations | 43 |
| § 6. Balancing of defocus aberration with spherical aberration or astigmatism | 54 |
| 6.1. Focused beam | 54 |
| 6.2. Collimated beam | 62 |
| § 7. Aberrated Gaussian pupil | 66 |
| 7.1. Spherical aberration | 66 |
| 7.2. Symmetry properties | 66 |
| 7.3. Line of sight | 70 |
| § 8. Weakly-truncated Gaussian pupils and beams | 71 |
| 8.1. Pupil function | 71 |
| 8.2. Point-spread function | 72 |
| 8.3. Radius of curvature of the propagating wavefront | 77 |
| 8.4. Collimated beam | 78 |
| 8.5. Beam focusing and waist imaging by a lens | 80 |
| 8.6. Optical transfer function | 84 |
| 8.7. Strehl ratio, aberration balancing, and orthogonal polynomials | 85 |

| | |
|--|----|
| 8.8. Beam characterization and measurement | 88 |
| 8.9. Nonparaxial Gaussian beams | 90 |
| § 9. Conclusions | 91 |
| Acknowledgements | 92 |
| Appendix | 92 |
| References | 93 |

Chapter 2. Controlling nonlinear optical processes in multi-level atomic systems, Amitabh Joshi and Min Xiao (Fayetteville, AR, USA) 97

| | |
|--|-----|
| § 1. Introduction | 99 |
| § 2. Modified linear and nonlinear optical properties in multi-level atomic systems | 104 |
| 2.1. Absorption reduction and dispersion enhancement | 106 |
| 2.2. Enhanced nonlinearity in EIT systems | 116 |
| § 3. Enhanced four-wave mixing processes with induced atomic coherence | 126 |
| 3.1. Three-level atomic systems | 127 |
| 3.2. Four-level atomic systems | 130 |
| § 4. Controlled optical bistability and optical multistability with three-level atoms inside an optical cavity | 135 |
| 4.1. Theoretical calculation | 137 |
| 4.2. Controllable optical bistability | 140 |
| 4.3. Controllable optical multistability | 143 |
| 4.4. Controllable direction of hysteresis cycle in optical bistability/optical multistability | 146 |
| 4.5. Dynamic hysteresis in optical bistability | 149 |
| § 5. Controlled optical switching in three-level atomic systems | 152 |
| 5.1. Controlled switching between bistable states | 154 |
| 5.2. All-optical switching controlled by coupling laser beam | 155 |
| 5.3. Controlled optical switching in four-level N-type atomic system | 159 |
| § 6. Controlled optical instability with three-level atoms inside an optical cavity | 162 |
| 6.1. Theoretical calculation | 164 |
| 6.2. Experimental measurements | 167 |
| § 7. Summary and outlook | 170 |
| Acknowledgement | 172 |
| References | 172 |

Chapter 3. Photonic crystals, Henri Benisty (Orsay, France) and Claude Weisbuch (Palaiseau, France and Santa Barbara, CA, USA) 177

| | |
|--|-----|
| Nomenclature | 179 |
| Main variables and notations | 180 |
| § 1. Introduction | 180 |
| § 2. Basics of periodic dielectric media and theoretical tools | 184 |
| 2.1. 1D Bragg structure | 184 |
| 2.2. Density of states, local DOS | 191 |
| 2.3. Generalizing to 2D and 3D periodic crystals | 194 |
| 2.4. Basics of band calculation | 199 |
| 2.5. The discovery of PBG | 201 |
| 2.6. Bloch waves | 208 |
| 2.7. Theoretical tools: a brief overview | 209 |

| | |
|---|-----|
| § 3. Three-dimensional photonic crystals | 214 |
| 3.1. Introduction | 214 |
| 3.2. Fabrication of 3D photonic crystals | 216 |
| 3.3. Measurements | 222 |
| 3.4. Metallo-dielectric systems and microwave regime | 225 |
| § 4. Two-dimensional photonic crystals | 229 |
| 4.1. Theoretical aspects | 229 |
| 4.2. 2D photonic crystal with vertical guidance | 230 |
| 4.3. 2D realizations | 238 |
| 4.4. Applications and characterization methods | 246 |
| § 5. Photonic crystal fibers | 271 |
| Issues of fibre world | 271 |
| The playing field of photonic crystal fibers | 271 |
| Single-mode fiber | 275 |
| True PC guidance | 276 |
| High-Delta fiber | 278 |
| Special dispersion | 278 |
| Special applications: nonlinear optics, supercontinuum generation, etc. | 280 |
| Brief overview of PCF modeling tools | 283 |
| § 6. Conclusion and perspectives | 283 |
| Acknowledgements | 288 |
| References and special issues | 288 |
| Special issues of regular journals on photonic crystals, in chronological order | 288 |
| References | 289 |

Chapter 4. Symmetry properties and polarization descriptors for an arbitrary electromagnetic wavefield, *Christian Brosseau (Brest, France) and Aristide Dogariu (Orlando, FL, USA)*

315

| | |
|--|-----|
| § 1. Introduction | 317 |
| § 2. A brief survey of the interplay between polarization concepts and geometry | 320 |
| 2.1. From Bartholinus (1669) to Stokes (1852): polarization is a geometric property of light | 321 |
| 2.2. From Stokes (1852) to Poincaré (1892): polarization is an electromagnetic property of light | 323 |
| 2.3. From Poincaré (1892) to Wolf (1954): polarization is a statistical property of light | 325 |
| 2.4. And now: geometric algebra is back | 329 |
| § 3. Density matrix and the convexity property of the states of polarization | 332 |
| 3.1. Polarization and the density matrix | 333 |
| 3.2. Scalar invariants | 335 |
| 3.3. The convex set of polarization states | 335 |
| 3.4. Summary of geometric algebra | 337 |
| § 4. Polarization of a plane wave | 338 |
| 4.1. Density matrix and degree of polarization | 339 |
| 4.2. Some important consequences and concrete applications | 344 |
| § 5. Polarization of an arbitrary wave | 349 |
| 5.1. Density matrix and polarization descriptors | 350 |
| 5.2. Some applications | 361 |
| § 6. Postscript | 363 |

| | |
|---|-----|
| § 7. Summary and prospects | 364 |
| Acknowledgements | 367 |
| Appendix A: Lie groups in polarization optics | 368 |
| A.1. $SU(2)$ | 368 |
| A.2. $SU(3)$ | 370 |
| Appendix B: Madison convention for the density matrix of massive spin-1 particles | 371 |
| Appendix C: Degree of polarization of a field consisting of a superposition of an ensemble of evanescent waves of random amplitude and of black-body radiation | 374 |
| References | 376 |

Chapter 5. Quantum cryptography, Miloslav Dušek (Olomouc, Czech Republic), Norbert Lütkenhaus (Erlangen, Germany) and Martin Hendrych (Castelldefels, Barcelona, Spain) 381

| | |
|---|-----|
| § 1. Ciphering | 383 |
| 1.1. Introduction, cryptographic tasks | 383 |
| 1.2. Asymmetrical ciphers (public-key cryptography) | 385 |
| 1.3. Symmetrical ciphers (secret-key cryptography) | 387 |
| 1.4. Vernam cipher, key distribution problem | 389 |
| § 2. Quantum key distribution | 391 |
| 2.1. The principle, eavesdropping can be detected | 391 |
| 2.2. Quantum measurement | 391 |
| 2.3. Quantum states cannot be cloned | 392 |
| 2.4. Protocol BB84 | 393 |
| 2.5. Eavesdropping, intercept–resend attack | 395 |
| § 3. Some other discrete protocols for QKD | 397 |
| 3.1. Two-state protocol, B92 | 397 |
| 3.2. B92 protocol with a strong reference pulse | 397 |
| 3.3. Six-state protocol | 398 |
| 3.4. SARG protocol | 399 |
| 3.5. Decoy-state protocols | 399 |
| 3.6. Entanglement-based protocols | 400 |
| § 4. Experiments | 403 |
| 4.1. QKD with weak laser pulses | 403 |
| 4.2. Entanglement-based protocols | 409 |
| § 5. Technology | 413 |
| 5.1. Light sources | 413 |
| 5.2. Detectors | 418 |
| 5.3. Quantum channels | 422 |
| § 6. Limitations | 423 |
| 6.1. Transmission rate | 423 |
| 6.2. Limit on the distance | 424 |
| 6.3. Quantum repeaters | 424 |
| § 7. Supporting procedures | 425 |
| 7.1. Estimation of leaked information | 425 |
| 7.2. Error correction for classical bit strings | 425 |
| 7.3. Privacy amplification for classical bit strings | 426 |
| 7.4. Advantage distillation for classical bit strings | 428 |
| 7.5. Authentication of public discussion | 428 |

| | |
|---|-----|
| § 8. Security | 429 |
| 8.1. Attacks on ideal protocols | 430 |
| 8.2. Secure key rates from classical three-party correlations | 432 |
| 8.3. Bounds on quantum key distribution | 434 |
| 8.4. Security proofs | 436 |
| 8.5. Specific attacks | 439 |
| 8.6. Results | 442 |
| 8.7. Side channels and other imperfections | 445 |
| § 9. Prospects | 446 |
| Acknowledgements | 446 |
| References | 446 |

Chapter 6. Optical quantum cloning, *Nicolas J. Cerf (Bruxelles, Belgium) and Jaromír Fiurášek (Olomouc, Czech Republic)* 455

| | |
|---|-----|
| § 1. Introduction and history | 457 |
| 1.1. The no-cloning theorem | 457 |
| 1.2. Beyond the no-cloning theorem | 459 |
| 1.3. Quantum cloning without signaling | 463 |
| 1.4. Content of this review | 464 |
| § 2. Overview of quantum cloning machines | 466 |
| 2.1. Universal cloning machines | 466 |
| 2.2. Pauli and Heisenberg cloning machines | 468 |
| 2.3. Phase- and Fourier-covariant cloning machines | 469 |
| 2.4. Group-covariant cloning machines | 471 |
| 2.5. High- d state-dependent cloning machines | 472 |
| 2.6. Cloning a pair of orthogonal qubits | 473 |
| 2.7. Entanglement cloning machines | 474 |
| 2.8. Real cloning machines | 474 |
| 2.9. Highly-asymmetric cloning machines | 475 |
| 2.10. Continuous-variable cloning machines | 476 |
| 2.11. Probabilistic cloning machines | 478 |
| 2.12. Economical cloning machines | 479 |
| § 3. One-to-two quantum cloning as a CP map | 480 |
| 3.1. Isomorphism between CP maps and operators | 480 |
| 3.2. Covariance condition | 482 |
| 3.3. Cloning as a semidefinite programming problem | 483 |
| 3.4. Double-Bell ansatz | 485 |
| 3.5. Heisenberg cloning machines | 489 |
| 3.6. Three special cases of Heisenberg cloners | 492 |
| § 4. N -to- M universal quantum cloning | 495 |
| 4.1. Optimal cloning transformation | 495 |
| 4.2. Optimality proof for $1 \rightarrow M$ cloning of qubits | 500 |
| 4.3. Universal asymmetric quantum cloning | 501 |
| 4.4. Universal-NOT gate | 503 |
| § 5. Universal cloning of photons | 504 |
| 5.1. Amplification of light | 504 |
| 5.2. Symmetrization | 514 |
| 5.3. Universal asymmetric cloning of photons | 518 |
| 5.4. Cloning of orthogonally polarized photons | 522 |

| | |
|--|-----|
| § 6. Phase-covariant cloning of photons | 525 |
| 6.1. Phase-covariant cloning of qubits | 525 |
| 6.2. Phase-covariant cloning of qudits | 528 |
| 6.3. Optical phase-covariant cloning | 529 |
| 6.4. Experimental 1-to-3 phase-covariant cloning | 532 |
| § 7. Cloning of optical continuous variables | 533 |
| 7.1. Cloning of coherent states | 533 |
| 7.2. Cloning by phase-insensitive amplification | 535 |
| 7.3. Experimental cloning of coherent states | 538 |
| 7.4. Gaussian distribution with finite width | 539 |
| 7.5. Cloning of conjugate coherent states | 540 |
| § 8. Conclusions | 541 |
| Acknowledgements | 542 |
| References | 542 |
| Author index for Volume 49 | 547 |
| Subject index for Volume 49 | 571 |
| Contents of previous volumes | 575 |
| Cumulative index – Volumes 1–49 | 587 |

E. Wolf, Progress in Optics 49
© 2006 Elsevier B.V.
All rights reserved

1048781

Chapter 1

Gaussian apodization and beam propagation

by

Virendra N. Mahajan

The Aerospace Corporation, 2350 E. El Segundo Blvd., El Segundo, CA 90245, USA

and

College of Optical Sciences, University of Arizona, Tucson, AZ 85721, USA
e-mail: virendra.n.mahajan@aero.org

Contents

| | Page |
|---|------|
| § 1. Introduction | 3 |
| § 2. Theory | 5 |
| § 3. Aberration-free Gaussian pupil | 7 |
| § 4. Defocused Gaussian pupil | 16 |
| § 5. Strehl ratio, aberration tolerance, and Zernike–Gauss polynomials . . | 34 |
| § 6. Balancing of defocus aberration with spherical aberration or astigmatism | 54 |
| § 7. Aberrated Gaussian pupil | 66 |
| § 8. Weakly-truncated Gaussian pupils and beams | 71 |
| § 9. Conclusions | 91 |
| Acknowledgements | 92 |
| Appendix | 92 |
| References | 93 |

§ 1. Introduction

We consider optical systems with Gaussian apodization or Gaussian pupils, i.e., those with a Gaussian amplitude across the wavefront at their exit pupils. The mathematical treatment is applicable equally to an imaging system with Gaussian transmission (obtained, for example, by placing a Gaussian filter at its exit pupil) as well as a laser transmitter in which the laser beam has a Gaussian distribution across its exit pupil. In Section 2 we outline the general theory for obtaining the point-spread and optical transfer functions of a system from its pupil function.

A Gaussian illumination, which in principle extends to infinity, is truncated by the finite size of the pupil. An aberration-free system with a Gaussian pupil is considered in Section 3. We show that it yields a point-spread function (PSF) with a broader central bright spot but lower secondary maxima compared to the Airy pattern obtained for a uniform pupil (Jacquinot and Roizen-Dossier [1964], Buck [1967], Campbell and DeShazer [1969], Olaofe [1970], Dickson [1970], Schell and Tyras [1971], Williams [1973], Mahajan [1986]). Its central irradiance is smaller than that for a uniform pupil of the same total power (Mahajan [1980, 1986]). The corresponding optical transfer function (OTF) is higher for low spatial frequencies and lower for high spatial frequencies (Chung and Hopkins [1989], Mahajan [2004]). In Section 4 we discuss a defocused system and show that the principal maximum of axial irradiance of a focused beam with a small Fresnel number lies at a point that is closer to the pupil and not at the geometrical focus (Li and Wolf [1982], Carter [1982], Sucha and Carter [1984], Dementev and Domarkene [1984]). However, as in the case of a uniform pupil, the maximum central irradiance on a target at a fixed distance is obtained when the beam is focused on it (Mahajan [1986, 2004]).

The effect of aberrations on the central irradiance is considered in Section 5. It is shown that the Strehl ratio for a given amount of primary aberration is higher for a Gaussian pupil than that for a corresponding uniform pupil, or the aberration tolerance for a given Strehl ratio is higher for a Gaussian pupil (Lowenthal [1974], Mahajan [1995, 2003, 2005a]). Aberration balancing to reduce aberration variance and thus improve Strehl ratio for small aberrations is explained, and Zernike–Gauss polynomials that represent balanced aberrations are discussed. For

systems with small Fresnel numbers, it is shown in Section 6 that the axial irradiance closer to the pupil increases when the defocus aberration is balanced with spherical aberration (Yoshida and Asakura [1996], Jiang and Stamnes [1997], Mahajan [2005b]) or astigmatism (Mahajan [2005b]). Both focused and collimated beams are discussed.

The effect of an aberration on the PSF is considered in Section 7. It is shown that while apodization broadens the central bright spot but reduces the secondary maxima, balanced spherical aberration does not change the size of the bright spot but increases the secondary maxima, thus obliterating the positive attribute of apodization (Sklar [1975], Lowenthal [1975]). The line of sight of an aberrated system, defined as the centroid of its PSF, is shown to be affected by coma-type aberrations only, and lies farther from its center for a Gaussian pupil than for a uniform pupil (Mahajan [1985]).

Narrow or weakly-truncated Gaussian illumination is considered in Section 8. It is shown that a Gaussian beam exiting from the pupil remains Gaussian as it propagates (Siegman [1971], Gaskill [1978], Belland and Crenn [1982], Mahajan [2004]). The rings of the diffraction pattern disappear as the truncation decreases. Since the beam remains Gaussian as it propagates, the OTF of such a Gaussian pupil is also a Gaussian (Mahajan [2004]). It is shown that a focused beam has the smallest radius, called its waist, in a plane that is closer to the pupil than the focal plane. Considering the waist of a beam incident on a lens as an object, an imaging equation is developed in which the waist of the transmitted beam acts as the image (Williams [1973], Self [1983], Mahajan [1986]). It is shown that when the waist of the incident beam lies in the front focal plane, the waist of the transmitted beam lies in its back focal plane, and not at infinity as in conventional imaging. Similarly, whereas in conventional imaging of a real object by a positive lens forming a real image there is a minimum separation between an object and its image, there is no minimum separation between a real object waist and a real image waist. Whereas for small truncations the approximate expression $\exp(-\sigma_\phi^2)$ in terms of the phase aberration variance σ_ϕ^2 estimates the true value of the Strehl ratio quite well for wave aberration standard deviation $\sigma_w \lesssim \lambda/4$, it significantly underestimates it for large truncations, unless the Strehl ratio is greater than or equal to 0.9 or $\sigma_w \lesssim \lambda/20$ (Mahajan [2005a]). A beam quality factor of M^2 is defined that accounts for the difference in the divergence of a practical beam and that of an ideal Gaussian beam. Although most of our discussion is based on paraxial beams, i.e., for apertures or beam radii much larger than the optical wavelength, nonparaxial beams for which the beam radius is comparable to the wavelength are also discussed briefly.

§ 2. Theory

2.1. Pupil function

If $A(\vec{r}_p)$ is the amplitude and $\Phi(\vec{r}_p)$ is the phase aberration defined with respect to a reference sphere of radius of curvature R at a point \vec{r}_p in the plane of the exit pupil of the system, its pupil function can be written

$$P(\vec{r}_p) = A(\vec{r}_p) \exp[i\Phi(\vec{r}_p)]. \quad (2.1)$$

The total power in the exit pupil and, therefore, in the image is given by

$$P_{\text{ex}} = \int A^2(\vec{r}_p) d\vec{r}_p, \quad (2.2)$$

where $A^2(\vec{r}_p)$ is the irradiance at a pupil point.

2.2. Point-spread function

The point-spread function (PSF) of the system in a plane at a distance R from the plane of the pupil is given by (Goodman [1996], Born and Wolf [1999], Mahajan [2004])

$$PSF(\vec{r}_i) = \frac{1}{P_{\text{ex}}\lambda^2 R^2} \left| \int P(\vec{r}_p) \exp\left(-\frac{2\pi i}{\lambda R} \vec{r}_p \cdot \vec{r}_i\right) d\vec{r}_p \right|^2, \quad (2.3)$$

where \vec{r}_i is the position vector of a point in the image plane and λ is the wavelength of object radiation. The corresponding irradiance distribution of the image of a point object is given by

$$I(\vec{r}_i) = P_{\text{ex}} PSF(\vec{r}_i). \quad (2.4)$$

2.3. Optical transfer function

The optical transfer function (OTF) $\tau(\vec{v}_i)$ of a system is, by definition, the Fourier transform of its PSF,

$$\tau(\vec{v}_i) = \int PSF(\vec{r}_i) \exp(2\pi i \vec{v}_i \cdot \vec{r}_i) d\vec{r}_i, \quad (2.5)$$

where \vec{v}_i is a spatial frequency vector. Substituting eq. (2.3) into eq. (2.5), the OTF can also be written as the autocorrelation of its pupil function

$$\tau(\vec{v}_i) = \frac{\int P(\vec{r}_p) P^*(\vec{r}_p - \lambda R \vec{v}_i) d\vec{r}_p}{\int |P(\vec{r}_p)|^2 d\vec{r}_p}, \quad (2.6)$$

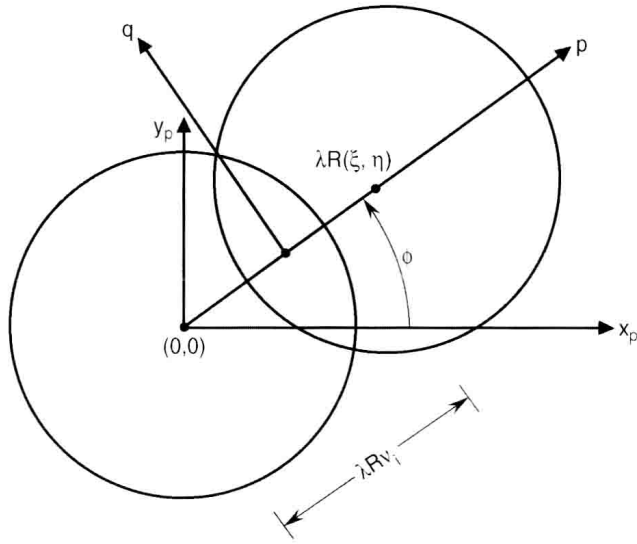


Fig. 1. Geometry for evaluating the OTF of a system with a circular pupil. The centers of the two pupils are located at $(0, 0)$ and $\lambda R(\xi, \eta)$ in the (x_p, y_p) coordinate system and $\mp(\lambda R/2)(v_i, 0)$ in the (p, q) coordinate system, where $v_i = (\xi^2 + \eta^2)^{1/2}$ and $\phi = \tan^{-1}(\eta/\xi)$. The shaded area is the overlap area of the two pupils. When normalized by the pupil radius a , the centers of the two pupils of unity radius lie at $\mp v$ along the p axis.

where the integration is across the overlap area of two intersecting pupils centered at $\vec{r}_p = 0$ and $\vec{r}_p = \lambda R \vec{v}_i$, shown shaded in fig. 1 for a circular pupil.

It is convenient to consider a (p, q) coordinate system whose origin lies at the midpoint of the line joining the centers of the two pupils, but whose axes are rotated by the polar angle ϕ of the spatial frequency vector with respect to those of the (x_p, y_p) coordinate system, as illustrated in fig. 1 for a system with a circular pupil of radius a or diameter $D = 2a$. The centers of the two pupils in this coordinate system are located at $\mp(\lambda R/2)(v_i, 0)$. The corresponding pupil function $P(p, q)$ may be obtained from the pupil function $P(x_p, y_p)$ by replacing x_p with $p \cos \phi - q \sin \phi$ and y_p with $p \sin \phi + q \cos \phi$. For a radially symmetric pupil function, it is obtained simply by replacing $x_p^2 + y_p^2$ by $p^2 + q^2$. It is evident from fig. 1 that the overlap of the two pupils reduces to zero as $\lambda R v_i \rightarrow D$. Accordingly, the OTF is zero for $v_i \geq v_c$, where $v_c = 1/(\lambda F)$ is the cutoff spatial frequency of the system. Here $F = R/D$ is the f -number of the image-forming light cone. Using normalized quantities $\vec{\rho} = \vec{r}_p/a$ and $\vec{v} = \vec{v}_i/v_c$, the OTF in polar coordinates (v, ϕ) may be written

$$\tau(v, \phi) = \frac{a^2}{P_{\text{ex}}} \iint P(p + v, q) P^*(p - v, q) dp dq, \quad 0 \leq v \leq 1. \quad (2.7)$$