THEORY OF ELASTICITY AND PLASTICITY

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Professor of Mechanics

ELASTICITY
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China Building Materials Industry Press

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Preface

This book is based on my previous version in Chinese published 20 years ago. It follows the same style as the original book, i.e., elasticity and plasticity are combined in one textbook. To organize the materials in this way reflects the actual process in the real world. It is a continuous process that when a solid body or structure subjects to a gradually increased loading, it may cause the deformation from elastic state to plastic state, and then finally lose its designed functions. To teach students those two theories together may help to compare those two processes, for the understanding of the entire deformation process, and therefore help students to build solid knowledge and systematic concepts about elasticity and plasticity. It will also benefit the research of the theory as well as engineering designs.

Comparing with the Chinese version of *Theory of Elasticity and Plasticity*, a lot of changes have been made in this version. Almost every chapter has added new contents. Theory of plates and shells has been removed due to a recent trend of teaching this topic in a separate course. Meanwhile, dynamic problems have been included as a new chapter. The major additions reflect the developments and extensions of interest and practical applicability that have occurred since the appearance of the Chinese version in 1979.

This book is still easy to read and understand. Several efforts have been made to achieve this result by introducing the most recent reference articles, using less complicated formulas and avoiding abstract concepts in mathematics. It should help the students to overcome the difficulties while firstly entering this field, and it should also help them to focus on the most important concepts rather than immerse in complicated mathematics formulas.

I wish that this book would serve as a practical textbook for senior engineering students or graduates studies. It should also be used as a convenient reference book for engineers to solve the problems in their daily design work.

In the new period of development of science and technology, more and more engineering students will face new challenges when they join work force in the next few years. To adapt English textbooks will definitely help them more efficiently reading and writing technical articles in English.

I wish to express my gratitude to Professor Xu Bingye and Professor Yang Huizhu for their frequent help and valuable suggestions. I also acknowledge the assistance of my students and colleagues of the Institute of Applied Mechanics of Taiyuan University of Technology Dr. Chen Weiyi, Dr. Shu Xuefeng, Dr. Ma Hongwei, Dr. Zhang Nianmei and Professor Cai Zhongmin, for their support and help in completing this work.

杨桂通

Yang Guitong

Jan. 27, 2002

CONTENTS

1 Introduction

1.1 Elasticity and Plasticity

Essential properties of deformable bodies subjected to external force or other external action are elastic and plastic behaviors. As discussed in the discipline of **mechanics of materials**, that is, if the external forces producing deformation do not exceed a certain limit, that is so called **yield criteria**, the deformation disappears with the removal of the forces, and then we consider this property as elasticity. Otherwise, the deformation does not disappear after removal of the forces, and then we consider the property as plasticity. Another main difference between perfect elasticity and plasticity, in mathematical view, is a linear problem and a nonlinear problem, respectively.

The atom forces in the material internal structure determine the mechanism of these two kinds of deformations. In fact, the internal structure of solid materials is always stable, on the basis of balance forces between atoms in a solid. The suction force makes the atoms tend to close upon to each other, and the repulsion force makes the atoms maintain some reasonable distance. In normal cases, these two forces are in equilibrium state. Atomic structure will not be considered here. We will be interested in the macroscopical response only. When a solid body is subjected to external loading, there are two different responses: elastic response and plastic response.

Elastic deformation is a simple case easy to be understood. Plastic deformation is a more complex case. Fig. 1.1 shows the typical curve for a simple tension specimen of metal. The initial elastic region generally appears as a straight line OA, where A

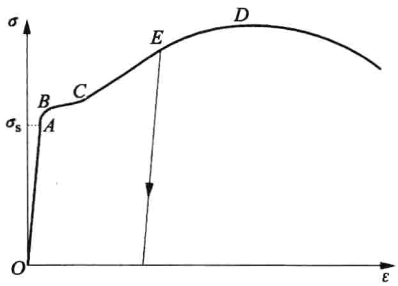

Fig. 1.1 Stress-strain diagram for an annealed cast-steel specimen

defines as the **limit of proportionality.** On further straining, the relation between stress and strain is no longer linear but the material is still elastic, and with release of the load, the specimen reverts to its original length. The maximum stress point B at which the load can be applied without causing any permanent deformation defines as the **elastic limit**. The point B is also called the **yield point**, for it marks the initiation of plastic or irreversible deformation. Usually, there is very little difference between the proportional limit, A, and the elastic limit, B. The behavior in the flat region BC is generally referer to as **plastic flow**. After C the material is exhibited strain hardening or also known as work hardening. Over some point D the material may be exhibited strain softening, as shown in Fig. 1.1.

Now, consider the unloading from some point E beyond the yield point. The behavior is as indicated in Fig. 1.1. That is, when the stress is reduced, the strain decreases along an almost elastic unloading line parallel to OA. So we say that the unloading obeys the elastic rule.

Fig. 1.2 is the typical graph of stresses versus relative elongation (compression) for four kinds of materials.

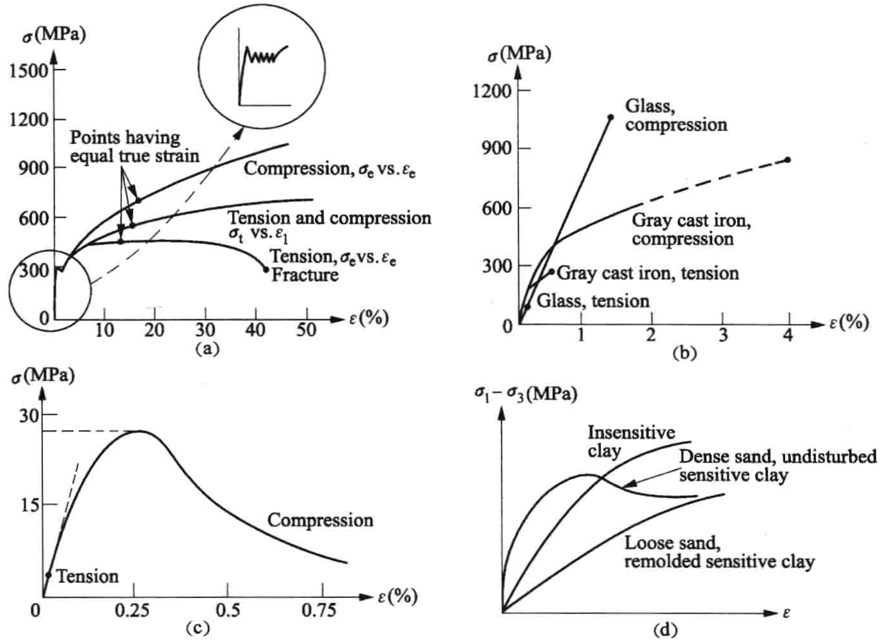

Fig. 1.2 Stress-strain diagrams: (a) Ductile metal, (b) Cast iron and glass, (c) Typical concrete or rock, (d) Soils, triaxial compression. (Experimental data are taken from reference[15].)

1.2 Basic Hypothesis

The subject of theory of elasticity and plasticity is concerned with the deformation and motion of elastic-plastic bodies or structures under the action of applied load or other disturbances. The general assumptions employed in the study of theory of elasticity and plasticity are the same as those used in the mechanics of continuous medium. Therefore, throughout this book, we have: (a) **continuum hypothesis**, we shall suppose that the macroscopic behavior of the solid bodies is the same as if they were perfectly continuous in structure; and physical quantities such as the mass and momentum associated with the matter contained within a given small volume will be regarded as being spread uniformly and without any caves, cracks and discontinuous. (b) **uniform hypothesis and isotropic hypothesis**, that are, the materials of a elastic-plastic body are homogeneous and uniformly distributed over its volume so that the smallest element cut from the body possesses the same specific physical properties as the body. The elastic properties are the same in all directions. (c) **small deformation hypothesis**, in this book, we discuss small deformation only.

1.3 Historical Remarks

Before the engineering design of structures, one must not only know the internal force field acting on the structural materials, but also know the material response. It means that we need give an analysis of the stresses, deformation and displacement of structural elements. Therefore we have to know the constitutive relation of the materials. Seeking some methods to solve these problems, many researchers have continually studied for over 2000 years.

The pioneering works of theory of elasticity and plasticity were given by Augustin Cauchy (1789—1857), Marie-Henri Navier (1785—1836), Leonard Euler (1707—1783), Simon Denis Poisson (1781—1840), Barre de Saint-Venant (1797—1886), Nikolai Ivanobich Mushihailishibili (1891—1976), Ludwig Prandtl (1875—1953), Thomas Young (1773—1829), Richard von Mises (1883—1953), and many others.

The general principles employed in the study of theory of elasticity and plasticity are the same as those used in studying the mechanics of continuous medium. Their basic formulations can be attributed primarily to the work of Euler and Cauchy. Euler first brought forward the general principles of linear and angular momentum balance for continuous media upon which rest all continuum mechanics, including theory of elasticity and plasticity. Cauchy first gave the concept of the stress and strain at a point and also found the general differential equations of motion or equilibrium of a continuum in term of the stress. Cauchy's work on elasticity provided a detailed kinematical theory of strain and deformation. The extension of the mathematical theory to more general solids was first made by Navier in 1821 using special assumption concerning the molecular forces of elastic solids. Technical application began earliest in 1855, when Saint-Venant solved the problem of the twisting of prismatic bars and worked out detailed numerical results. Saint-Venant also took up the problem of plastic flow and developed two-dimensional governing equations which were subsequently generalized to three dimensions by M. Levy in 1871. In 1864 H. Tresca reported experiments to the French Academy, which suggested that the plastic yielding of a metal occured when the maximum shear stress reached to a critical value. After Tresca, in 1913 R. von Mises published his yield condition theory based on theory of distortional energy.

In the last century (1901—2000), the theory of elasticity and plasticity had been rapidly developed in theory and engineering practice. Many great contributors should be mentioned, such as B. G. Galerkin, G. R. Kirchhoff, S. P. Timoshenko, J. L. Lagrange, A. Nadai, A. A. Il'yushin, W. W. Sokolovsky, W. Prager, R. Hill, Kh. A. Rakhmatulin, G. I. Taylor, P. Perzyna, and many others.

In this period, especially in last 50 years, theory of elasticity and plasticity rapidly developed in China too. Qian Xuesen, Qian Weichang, Hu Haichang, Wang Ren, Huang Kezhi, Xu Bingye, Wu Jike, Huang Zhuping, Gao Yuchen, Wang Ziqiang, and many others developed the theory of elasticity and plasticity, specially in the engineering applications. In this period many valuable books about elasticity and plasticity on theoretical and engineering applications were published.

2 Stress

2.1 Force and Stress

There are two kinds of forces: body force and surface force. The force acting on each internal particle of body or structural members is so called **body force**. For example, the gravity force, inertia force, the electromagnetic force and mass force, etc. are all the body forces. The force acting on the surface of the body or structure is so called **surface force**. For example, the wind load, the fluid pressure, the contact force between two bodies, etc. are all the surface forces.

Now we discuss the magnitude and direction of body force and surface force of a body in a coordinate $Oxyz$. Let us take a volume element ΔV at a point C, if the body force of ΔV is ΔF, then the mean density of body force in this volume element ΔV is $\Delta F/\Delta V$. When ΔV approaches to point C, the $\Delta F/\Delta V$ will approach to a limit vector F_b

$$\lim_{\Delta V \to 0} \frac{\Delta F}{\Delta V} = F_b \tag{2.1}$$

Obviously, the direction of the body force vector F_b coincides with the limit direction of body force in ΔV. The unit of the body force is N/m^3. Suppose F_b is the mass force of unit mass at a neighborhood of point C, and let $m = \rho dV$ is the mass of volume element dV, ρ is the density of mass, mF_b is the force acting on the mass of dV. Therefore the body force of unit volume is ρF_b.

Similarly, we can define the surface force vector F_s

$$\lim_{\Delta S \to 0} \frac{\Delta F}{\Delta S} = F_s \tag{2.2}$$

and the surface force acting on surface element dS is $F_s dS$.

We now give the concept of stress and denote the vector of internal force Δp acting on the element of area ΔS, cut out from the cross section C at any point P

(Fig. 2. 1). The outward normal vector is represent by n. We observe that the force acting on this elemental area, due to the action of material of the part B (which we throw aside) on the material of part A, can be reduced to a resultant Δp. If we continuously contract the elemental area ΔS, the limiting value of the ratio $\Delta p / \Delta S$ gives us the magnitude of the stress acting on the cross section C at any point P. The limit direction of the resultant Δp is the direction of the stress. In the general case the direction of the stress is inclined to the area ΔS on which it acts and we can resolve it into two components: a normal stress perpendicular to the area and a shearing stress acting in the plane of the area ΔS. The stress vector σ is defined in terms of these quantities by the following equation

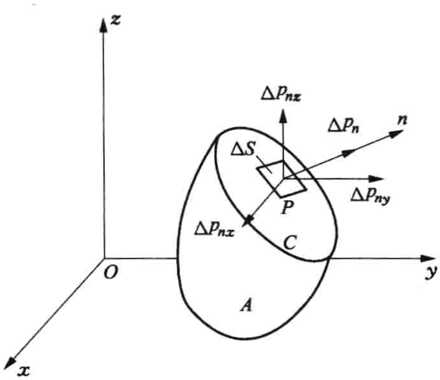

Fig. 2. 1 Stress vector at a point P

$$\sigma = \lim_{\Delta S \to 0} \frac{\Delta p}{\Delta S} \tag{2.3}$$

We now introduce the concept of stress tensor. To do this we first consider a small hexahedral element of material about some point P in the body. Let the faces of this element be parallel to the coordinate planes and on each of the six positive faces, whose outward normals in the positive directions, resolve the associated stress vectors into components along the coordinate directions as shown in Fig. 2.2. The notations used in this Figure in denoting the components of the stress vectors acting on each of the positive faces are as follows: The first subscript denotes the coordinate axis along which the outward normal of the considered face points and the second subscript denotes the direction in which the component acts. Thus for

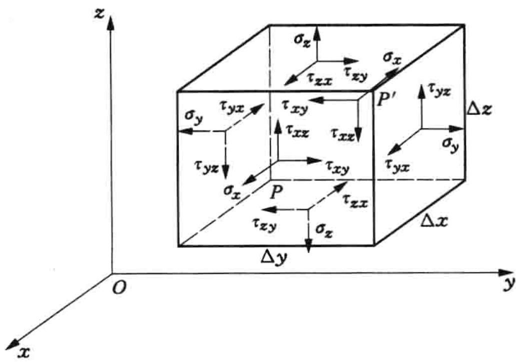

Fig. 2. 2 Components of stress vector

example σ_{xy} denotes the stress component of the stress vector acting on the faces with normal along axis x, its direction is toward axis y. As a rule, the tension stress is considered as positive stress, a compression stress is considered as a negative stress. The components of stress tensor can be written by using a common rule, which σ represents a normal component of stress, and τ represents shearing components of stress. From Fig. 2.2, it is easy to understand that when the small hexahedral approaches to a limit as 'infinitesimal', the stresses on the hexahedral represent the stress state at point P. Therefore the stress state at point P has all total 9 components, in which 3 normal stresses and 6 shear stresses (in fact, according to the **shear stress reciprocal theorem**, there are 3 shear stresses only).

2.2 Stress Tensor and Stress Deviation Tensor

The 9 components of stress may be represented by a second-order tensor, in which every row has 3 components of stress acting on one face at point P as follows

$$\begin{matrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{matrix}$$

These 9 components of stress define a new physical quantity $\sum$, and it describes just the stress state at point P. When we make a coordinate transformation, every component of stress may change its quantities. But the new physical quantity $\sum$ is unchanged at the same point P. As we know from mathematics, when coordinate transforms of elements obey a given coordinate transformation rule, then they determine a second-order tensor, as is so called **stress tensor**. Later we will see that the stress tensor is a symmetrical second-order tensor

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix} \tag{2.4}$$

Here i, j $(=1, 2, 3)$ associate with the axes x, y, z, and i represents rows and j represents columns, and the normal stress is represented by σ, the shear stress is represented by τ. Therefore, it is clear that **stress tensor perfectly determines the stress state at a given point.**

Now we discuss the principal stresses. Let that the direction n at a point in a body is so oriented that the resultant stress vector p^n, associated with direction is