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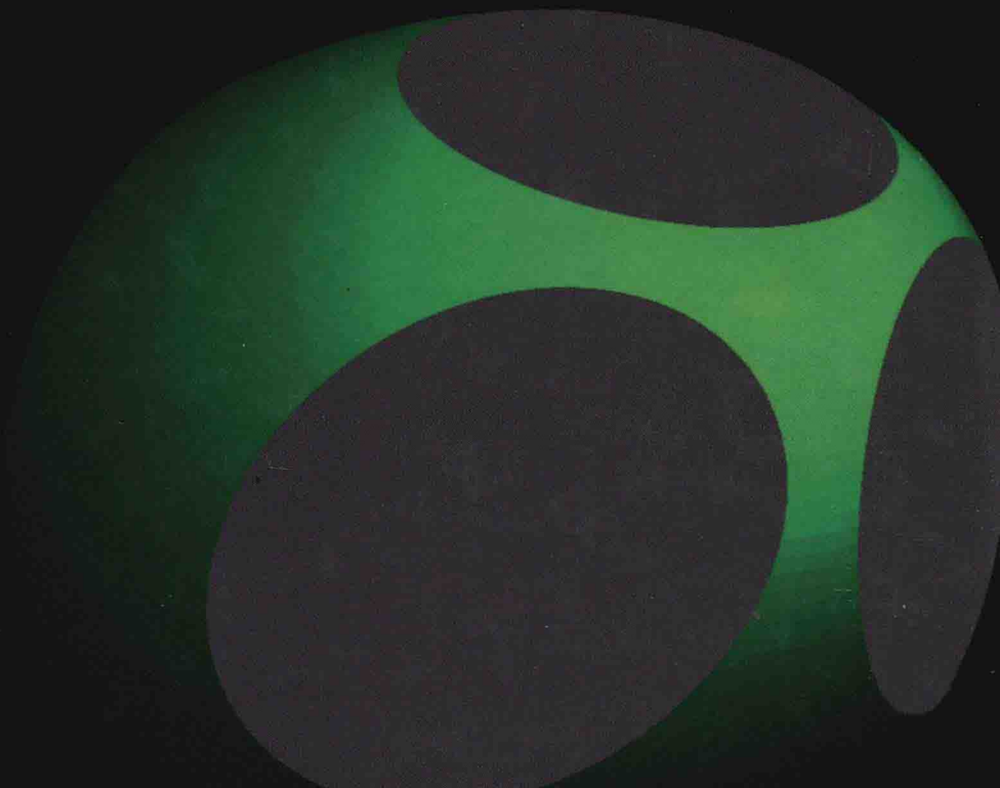

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Preface

The technical and scholarly interest in materials failure goes back almost to the beginnings of classical mechanics and deformable body mechanics. The effort to put order and organization into the field of failure characterization and failure criteria has been unflagging over the ensuing time span, measured in decades and even centuries. Despite the high level of sustained activity, the long time rate of progress was agonizingly slow.

By many measures of difficulty, the treatment of failure in solids (materials) is comparable to that of turbulence in fluids, both being controlled by non-linear physical effects. It is only in the modern era that the elements needed for constructing a complete, three-dimensional theory of failure for homogeneous materials have coalesced into meaningful forms. This book presents the derivation and a detailed examination of the resultant general theory of failure for materials science and materials engineering.

Chapter 1 outlines the materials failure problem and completely sets the course for all that follows. The coverage spans the full range, starting with the efforts from some of history's greatest scientists and ultimately leading to the most recent developments, such as the failure of anisotropic fiber composite materials and an examination of microscale and nanoscale failure. Many interrelated areas of the materials failure discipline are included. Although the coverage is broad, there is no compromise in quality or rigor.

The world of materials synthesis and materials applications offers many opportunities and many challenges. Few are of higher priority or in greater need than that of understanding materials failure.

Recognition

This book could not have been written without the supporting foundations of long-established elasticity theory and modern plasticity theory. In this connection, a special note of remembrance and appreciation is due to

Rodney Hill
and
Daniel C. Drucker

Their individual efforts (along with others) gave substance and clarity to the plasticity formalism, and greatly helped to solidify it as a major discipline alongside that of elasticity. With elasticity theory and plasticity theory firmly in place, failure theory could complete the trilogy. Perhaps this would have pleased Professor Drucker and Professor Hill, since each of them took initial steps in that direction.

Technical Status and Challenges

A complete and comprehensive theory of failure for homogeneous materials is developed. The resulting general failure theory for isotropic materials and its related failure criteria are calibrated by two properties: the uniaxial tensile and compressive strengths, T and C. From such readily available data for most materials, the entire family of failure envelopes can be generated for any and all states of stress in any isotropic material. It is not just a coincidence that the number of independent failure properties being at two is the same as the number of independent elastic properties for isotropy. This relationship will be of considerable consequence.

It will require a long and involved derivation to establish the operational capability and results mentioned briefly above. First, however, a historical survey and evaluation must be conducted. Insofar as general applications are concerned, the complete and absolute unsuitabilities are detailed for the Mises, Tresca, Drucker–Prager, and Coulomb–Mohr failure criteria. Only the Mises form is satisfactory for a specific class of materials—that being only for ductile metals. A general failure theory must cover not only ductile metals but also brittle metals, glassy and crystalline polymers, ceramics, glasses, and a variety of other isotropic materials types, just as does elasticity theory. Treating failure criteria with the appropriate and necessary generality has always remained an unsolved and historically formidable problem, and significant progress has been greatly impeded—even blocked.

The failure problem and its need for resolution provides the impetus for the account and developments presented here. The end result is the long-missing general theory of materials failure, contained herein. All of this is made possible by a new and transparently clear physical insight or recognition. It is that of an organizing principle whereby the entire spectrum of isotropic materials can be characterized by and classified by their uniaxial strengths ratio, T/C. The limits on T/C define the ductile and brittle limits of physical behavior. Two coordinated but competitive failure criteria can then be derived: the polynomial-invariants criterion and the fracture criterion—both expressed in terms of T/C and with stress non-dimensionalized by C. This formalism identifies universal failure forms applicable to all materials at the same T/C specification. It is a complete and self-contained theory, secured by the two most basic strength properties conceivable. It is the field theory of failure.

One of several unique features of the book is a thorough treatment of ductile versus brittle behavior for isotropic materials. The fundamental form for the ductile/brittle transition is derived as part of the failure theory. A means for gauging ductility levels is a further outcome from the derivation. Along with the experimental evaluation and verification of the failure theory, many examples of failure behavior and applications of failure criteria are presented.

The relationship of failure criteria (for homogeneous materials) to fracture mechanics (for structures) is included. Rigorous definitions of yield stress and strength are developed. Reasonably general treatments are presented for anisotropic fiber composites failure as well as investigations into microscale and nanoscale aspects of failure. Last, but still very important, there is a probe into damage models leading to failure and a fairly extensive derivation of probabilistic failure and probabilistic life prediction to complete the book. All of these technical areas are difficult and important failure-related problems in their own right.

The intended use for the book is as follows. Although it is at a quite high level, it fits in with mainstream mechanics of materials curricula in materials science and in mechanical, aerospace, and civil engineering departments. It is thus seen to be appropriate for both classroom use (upper undergraduate or graduate) and for related research. Despite the advanced level, the final forms of the failure criteria are totally practical and adaptable for use by engineers having design responsibilities.

I am deeply appreciative of Stanford University and the Office of Naval Research for creating and sustaining an environment that encourages the best possible work. To a lifetime of friends and colleagues, I am grateful for the opportunity to have interacted with them in such important, exciting, and exacting fields.

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The Perspective on Failure and Direction of Approach

This book is concerned with the means and methods of three-dimensional analysis for predicting materials failure. It is the first book on theoretically derived but physically based approaches for predicting the envelopes of failure for all the major classes of materials. Such problems have almost always been treated by empirical means. The approach uses the intrinsic static strength properties for homogeneous materials to predict their failure behavior in the complex multi-dimensional stress conditions of physical applications.

1.1 Materials Failure Problem

The use of failure criteria has the same motivation as that when fracture mechanics is used to predict the failure of structures due to extreme stress concentration conditions at flaws in their load-bearing elements. In fact, the failure criteria of concern here for homogeneous materials and fracture mechanics comprise complementary disciplines. Each is intended to treat a distinct and extremely broad class of problems.

The title of this book, *The Theory of Materials Failure*, has a meaningful relationship with the titles and contents of the longstanding books [1.1]–[1.3]:

- *Theory of Elasticity*, Timoshenko and Goodier
- *Mathematical Theory of Elasticity*, Sokolnikoff
- *The Mathematical Theory of Plasticity*, Hill

This book presumes familiarity with the notation and general coverage of the above books, or other more recent but largely equivalent books. More than that, it is the logical continuation and conclusion of these and other similar books, in the following sense. Elasticity and plasticity provide part of the constitutive formulation for materials behavior. The other part is the missing part; namely, what is the permissible range for these behaviors? The range of behavior is restricted by failure, and failure criteria are the related governing constitutive forms. In briefest, broadest terms, this book is intended to supply the missing physical and mathematical complement to all elasticity and plasticity books, especially including the three classics noted above.

Specific to the coordinated opportunity represented by failure theory is the example of elasticity theory itself. The tensor-based theory of elasticity has had a revolutionary effect on the way materials are made into load-bearing structures. Elasticity is one of the oldest and most valued of all field theories, and is applied as a standardized and required procedure in virtually all applications. A tensor-based theory of failure would be the full partner to elasticity theory. Failure characterization can be seen as the three-dimensional completion and terminus of linear elastic behavior. Plasticity theory and behavior does not contradict this; rather, it represents a more complex transition from the elastic state to failure in the case of ductile materials. All of this is symptomatic of the physical problem itself: the failure of materials due to excessive states of loading.

With or without plasticity, failure still essentially represents the terminus of the elastic behavior. This will turn out to be a pivotal key to the problem. In general, elasticity, plasticity, and failure theories together comprise the continuum of materials responses to states of imposed stress. Each requires its own constitutive formulation. Two of the three corresponding knowledge bases are complete and standardized. The third—failure—is strongly needed and long overdue.

1.2 Direction and Scope

The continuing problem and enduring challenge is to predict materials failure behavior in general situations based upon knowledge of the loads and minimal failure data obtained from testing. The data that are usually found in comprehensive handbooks and in textbooks are those of the uniaxial tensile and compressive strengths. The historic activity for isotropic materials has always been and still is to attempt to theoretically prescribe the general, multi-dimensional envelope that discriminates between a safe and stable state of stress, and that of the total loss of the load-bearing capability. Ideally, this failure envelope or failure criterion