Calculus with Applications

Fifth Edition

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TO THE STUDENT

If you want further help with this course, you may wish to buy a copy of the *Student's Solution Manual* (ISBN 0-673-46755-4) that accompanies this textbook. This manual provides detailed, step-by-step solutions to the odd-numbered exercises in the textbook and can help you study and understand the course material. Also included for each chapter is a practice test, with answers so you can check your work. Your college bookstore either has this manual or can order it for you.

Ask your instructor for the program *GraphExplorer* that will be of help in approaching some of the exercises marked with . This program contains a comprehensive graphing utility and is available in IBM and Macintosh formats.

If you would like to find out how spreadsheets can help you learn and explore some of the topics discussed in this text, you might want to obtain a copy of Sam Spero's *The Electronic Spreadsheet and Elementary Calculus* (ISBN 0-673-46595-0) from your college bookstore. This booklet will help you get started using spreadsheets to enhance your problem-solving skills. Knowledge of spreadsheets is not assumed, and the material is adaptable to any spreadsheet program that may be available to you.

TO THE INSTRUCTOR

We will make available as "Special Topic" supplements the following titles: *Sequences, Series, and Other Topics*; *Linear Programming: The Graphical Method* and *Linear Programming: The Simplex Method*; and *Introduction to the Graphing Calculator*. Please place your order with your HarperCollins sales representative. These supplements are free to you and your students.

PREFACE

Calculus with Applications is a solid, application-oriented text for students majoring in business, management, economics, or the life or social sciences. A prerequisite of three to four semesters of high school algebra is assumed. Many new features, including new exercises, new applications, and motivational section-opening questions, make using this edition of the text easier and more enjoyable. Application exercises are designed to be as believable and realistic as possible.

This edition continues to offer the many popular features of the fourth edition: applications grouped by subject matter with subheadings indicating the specific application; extended applications to motivate student interest; careful exposition; fully developed examples with side comments; carefully graded exercises; and an algebra reference, designed to be used either in class or individually as necessary. The index of appications shows the abundant variety of applications included in the text and allows direct reference to particular topics.

NEW AND ENHANCED FEATURES

Conceptual and Writing Exercises To complement the drill and application exercises, several exercises that strengthen conceptual understanding are included in almost every exercise set. Also included are exercises that require the student to respond by writing a few sentences. (Some writing exercises are also conceptual in nature.)

Connections Some exercise sets and many review sections include one or more exercises (labeled ''Connections'') that integrate concepts and skills introduced earlier with those just introduced in the chapter, or that integrate different concepts presented in the chapter.

Summaries A few chapters include a summary of rules or formulas designed to help the student sort out the different ideas introduced in the chapter. These are included in chapters where students traditionally have trouble and become confused about when to use one of several techniques.

Section Openers Most sections open with a thought-provoking question that is answered in an application within the section or in the exercises.

Margin Reviews This feature is designed to help students better understand the ideas being presented. These notes give short explanations or comments reminding students of skills or techniques learned earlier that are needed at this point. Some include a few practice exercises; some include reference to material presented earlier in the text.

Cautions and Notes Common student difficulties and errors are now highlighted graphically and identified with the heading "Caution." Important comments and asides are treated similarly and given the heading "Notes."

KEY CONTENT CHANGES

The use of a scientific calculator is assumed in this edition, and references to calculator use occur throughout as appropriate. Although a graphing calculator is not required, its usefulness is mentioned, and some clearly labeled computer or graphing calculator exercises are included in a few sections. These exercises showcase the programming capability of graphing calculators, in addition to their graphing capabilities.

This edition preserves the basic format and topic order of the last edition, with the following changes.

- A new section on the graphing techniques of translations and reflections has been added to Chapter 1 on Functions and Graphs.
- In Chapter 2, the material on continuity has been combined with the introduction of limits in Section 2.1. Limits at infinity are now included in Chapter 3 in a new section discussing curve sketching.
- Applications of the derivative, presented in one long chapter in earlier editions, are now covered in Chapters 3 and 4.
- Some discussion on math of finance has been included in the section on exponential functions.
- Growth and decay applications of exponential and logarithmic functions are presented in Section 5.3, before the derivatives are introduced. These first three sections require no calculus and can be presented earlier with the functions discussed in Chapter 1.
- In Chapter 9, the first section includes elementary and separable differential equations, combining the first two sections of the previous edition.

SUPPLEMENTS

For the Instructor The *Instructor's Guide and Solution Manual* contains solutions to even-numbered exercises; a multiple-choice version and a short-answer version of a final exam, with answers; at least 100 extra test questions per chapter, with answers; and a set of concise teaching tips.

The *Instructor's Answer Manual* provides answers to every exercise in the text.

The *HarperCollins Test Generator* is the foremost product of its kind, enabling the instructor to create multiple versions of tests, to randomly regenerate "variables," and to insert questions of his or her own devising. The Test Generator comes free to adopters.

Accompanying two-color *Overhead Transparencies*, given free to adopters, can help enhance lectures.

The supplements in the series "Special Topics to Accompany *Calculus with Applications*" will be of interest. *Sequences, Series, and Other Topics*; *Linear Programming: The Graphical Method* and *Linear Programming: The Simplex Method*; and *Introduction to the Graphing Calculator* will be offered to you and your students at no charge should you desire to cover these topics.

For the Student The *Student's Solution Manual* (ISBN 0-673-46755-4) provides solutions to odd-numbered exercises and sample chapter tests with answers.

The Electronic Spreadsheet and Elementary Calculus (ISBN 0-673-46595-0), by Sam Spero, Cuyahoga Community College, helps students get started controlling graphing and problem solving by means of the spreadsheet. Knowledge of spreadsheets is not assumed, and the approach is adaptable to all spreadsheet programs.

GraphExplorer provides students and instructors with a comprehensive graphing utility, and is available in IBM and Macintosh formats.

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Business and Economics

General Interest

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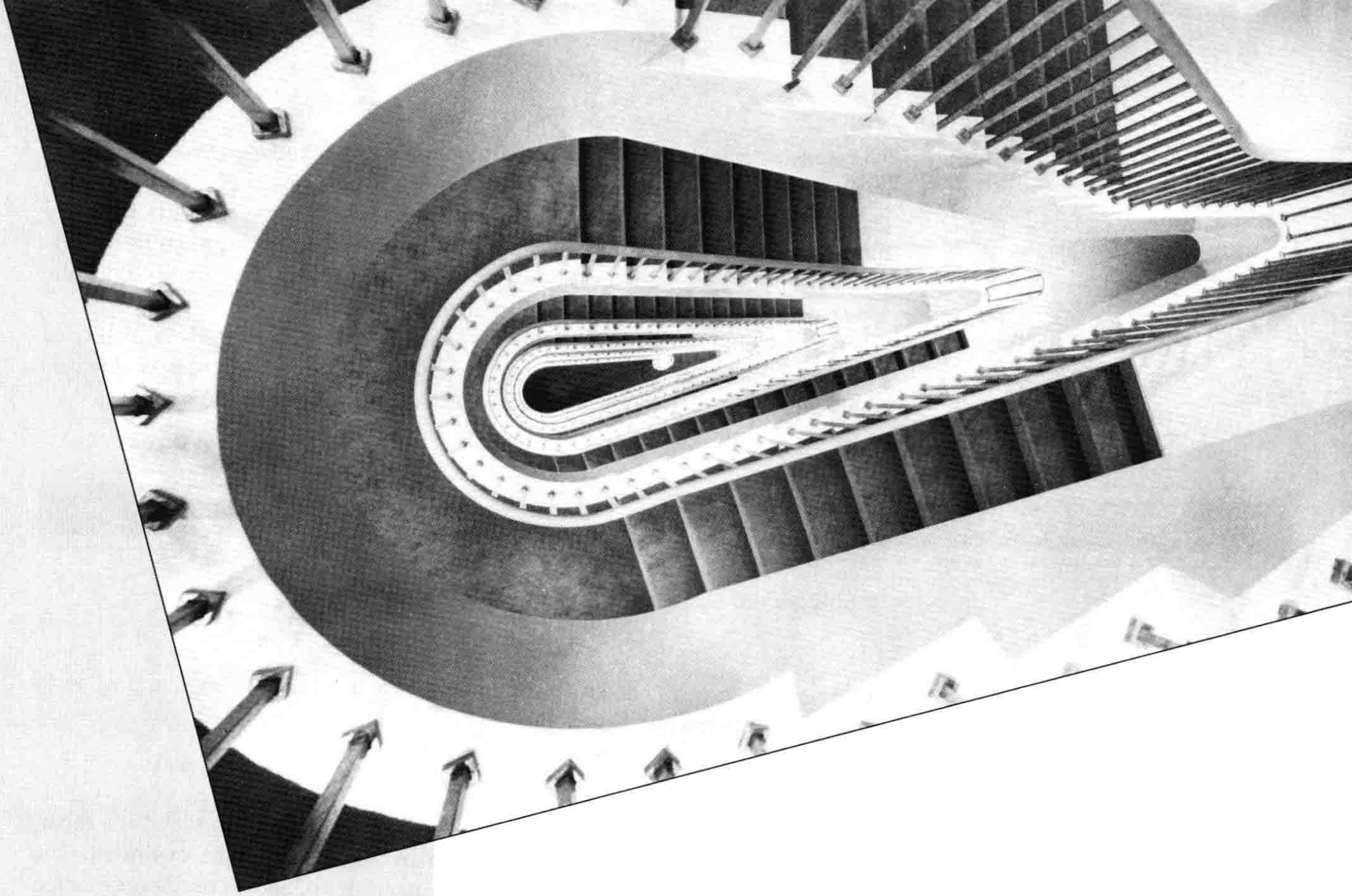

ALGEBRA REFERENCE

The study of calculus is impossible without a thorough knowledge of elementary algebra. The word *algebra* is derived from the Arabic word *al-jabr*, which appeared in the title of a book by Arab mathematician al-Khowarizmi in the early ninth century A.D. The study of algebra in modern times has grown to include many abstract ideas, but in this text we will be concerned primarily with the topics of elementary algebra found in al-Khowarizmi's book.

This algebra reference is designed for self-study; you can study it all at once or refer to it when needed throughout the course. Since this is a review, answers to all exercises are given in the answer section at the back of the book.

R.1 POLYNOMIALS

A **polynomial** is an expression of the form

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0,$$

where $a_0, a_1, a_2, \ldots, a_n$ are real numbers, n is a natural number, and $a_n \neq 0$. Examples of polynomials include

$$5x^4 + 2x^3 + 6x, \qquad 8m^3 + 9m^2 - 6m + 3, \qquad 10p, \qquad \text{and} \qquad -9.$$

Adding and Subtracting Polynomials An expression such as $9p^4$ is a **term;** the number 9 is the **coefficient**, p is the **variable,** and 4 is the **exponent.** The expression p^4 means $p \cdot p \cdot p \cdot p$, while p^2 means $p \cdot p$, and so on. Terms having the same variable and the same exponent, such as $9x^4$ and $-3x^4$, are **like terms.** Terms that do not have both the same variable and the same exponent, such as m^2 and m^4, are **unlike terms.**

Polynomials can be added or subtracted by using the **distributive property,** shown below.

If a, b, and c are real numbers, then

$$a(b + c) = ab + ac \qquad \text{and} \qquad (b + c)a = ba + ca.$$

Only like terms may be added or subtracted. For example,

$$12y^4 + 6y^4 = (12 + 6)y^4 = 18y^4,$$

and

$$-2m^2 + 8m^2 = (-2 + 8)m^2 = 6m^2,$$

but the polynomial $8y^4 + 2y^5$ cannot be further simplified. To subtract polynomials, use the fact that $-(a + b) = -a - b$. In the next example, we show how to add and subtract polynomials.

EXAMPLE 1 Add or subtract as indicated.

(a) $(8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8)$

Combine like terms.

$$(8x^3 - 4x^2 + 6x) + (3x^3 + 5x^2 - 9x + 8)$$
$$= (8x^3 + 3x^3) + (-4x^2 + 5x^2) + (6x - 9x) + 8$$
$$= 11x^3 + x^2 - 3x + 8$$

(b) $(-4x^4 + 6x^3 - 9x^2 - 12) + (-3x^3 + 8x^2 - 11x + 7)$
$$= -4x^4 + 3x^3 - x^2 - 11x - 5$$

(c) $(2x^2 - 11x + 8) - (7x^2 - 6x + 2)$
$$= (2x^2 - 11x + 8) + (-7x^2 + 6x - 2)$$
$$= -5x^2 - 5x + 6$$ ◀

Multiplying Polynomials The distributive property is used also when multiplying polynomials, as shown in the next example.

EXAMPLE 2 Multiply.

(a) $8x(6x - 4)$

$$8x(6x - 4) = 8x(6x) - 8x(4)$$
$$= 48x^2 - 32x$$

(b) $(3p - 2)(p^2 + 5p - 1)$

$$(3p - 2)(p^2 + 5p - 1)$$
$$= 3p(p^2) + 3p(5p) + 3p(-1) - 2(p^2) - 2(5p) - 2(-1)$$
$$= 3p^3 + 15p^2 - 3p - 2p^2 - 10p + 2$$
$$= 3p^3 + 13p^2 - 13p + 2$$ ◀

When two binomials are multiplied, the FOIL method (First, Outer, Inner, Last) is used as a shortcut. This method is shown below.

EXAMPLE 3 Find $(2m - 5)(m + 4)$ using the FOIL method.

$$(2m - 5)(m + 4) = \overset{\mathbf{F}}{(2m)(m)} + \overset{\mathbf{O}}{(2m)(4)} + \overset{\mathbf{I}}{(-5)(m)} + \overset{\mathbf{L}}{(-5)(4)}$$
$$= 2m^2 + 8m - 5m - 20$$
$$= 2m^2 + 3m - 20$$ ◀

EXAMPLE 4 Find $(2k - 5)^2$.
Use FOIL.

$$(2k - 5)^2 = (2k - 5)(2k - 5)$$
$$= 4k^2 - 10k - 10k + 25$$
$$= 4k^2 - 20k + 25$$

Notice that the product of the square of a binomial is the square of the first term, $(2k)^2$, plus twice the product of the two terms, $(2)(2k)(-5)$, plus the square of the last term, $(-5)^2$. ◀

Caution Avoid the common error of writing $(x + y)^2 = x^2 + y^2$. As Example 4 shows, the square of a binomial has three terms, so

$$(x + y)^2 = x^2 + 2xy + y^2.$$

Furthermore, higher powers of a binomial also result in more than two terms. For example, verify by multiplication that

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3.$$

R.1 Exercises

Perform the indicated operations.

1. $(2x^2 - 6x + 11) + (-3x^2 + 7x - 2)$

2. $(-4y^2 - 3y + 8) - (2y^2 - 6y - 2)$

3. $-3(4q^2 - 3q + 2) + 2(-q^2 + q - 4)$

4. $2(3r^2 + 4r + 2) - 3(-r^2 + 4r - 5)$

5. $(.613x^2 - 4.215x + .892) - .47(2x^2 - 3x + 5)$

6. $.83(5r^2 - 2r + 7) - (7.12r^2 + 6.423r - 2)$

7. $-9m(2m^2 + 3m - 1)$

8. $(6k - 1)(2k - 3)$

9. $(5r - 3s)(5r + 4s)$

10. $(9k + q)(2k - q)$

11. $\left(\frac{2}{5}y + \frac{1}{8}z\right)\left(\frac{3}{5}y + \frac{1}{2}z\right)$

12. $\left(\frac{3}{4}r - \frac{2}{3}s\right)\left(\frac{5}{4}r + \frac{1}{3}s\right)$

13. $(12x - 1)(12x + 1)$

14. $(6m + 5)(6m - 5)$

15. $(3p - 1)(9p^2 + 3p + 1)$

16. $(2p - 1)(3p^2 - 4p + 5)$

17. $(2m + 1)(4m^2 - 2m + 1)$

18. $(k + 2)(12k^3 - 3k^2 + k + 1)$

19. $(m - n + k)(m + 2n - 3k)$

20. $(r - 3s + t)(2r - s + t)$

R.2 FACTORING

Multiplication of polynomials relies on the distributive property. The reverse process, where a polynomial is written as a product of other polynomials, is called **factoring.** For example, one way to factor the number 18 is to write it as the product $9 \cdot 2$. When 18 is written as $9 \cdot 2$, both 9 and 2 are called **factors** of 18. It is true that $18 = 36 \cdot 1/2$, but 36 and 1/2 are not considered factors of 18; only integers are used as factors. The number 18 also can be written with three integer factors as $2 \cdot 3 \cdot 3$. The integer factors of 18 are ± 1, ± 2, ± 3, ± 6, ± 9, ± 18.

The Greatest Common Factor To factor the algebraic expression $15m + 45$, first note that both $15m$ and 45 can be divided by 15. In fact, $15m = 15 \cdot m$ and $45 = 15 \cdot 3$. Thus, the distributive property can be used to write

$$15m + 45 = 15 \cdot m + 15 \cdot 3 = 15(m + 3).$$

Both 15 and $m + 3$ are factors of $15m + 45$. Since 15 divides into all terms of $15m + 45$ (and is the largest number that will do so), 15 is the **greatest common factor** for the polynomial $15m + 45$. The process of writing $15m + 45$ as $15(m + 3)$ is often called **factoring out** the greatest common factor.

EXAMPLE 1 Factor out the greatest common factor.

(a) $12p - 18q$

Both $12p$ and $18q$ are divisible by 6. Therefore,

$$12p - 18q = 6 \cdot 2p - 6 \cdot 3q = 6(2p - 3q).$$

(b) $8x^3 - 9x^2 + 15x$

Each of these terms is divisible by x.

$$8x^3 - 9x^2 + 15x = (8x^2) \cdot x - (9x) \cdot x + 15 \cdot x$$
$$= x(8x^2 - 9x + 15) \quad \text{or} \quad (8x^2 - 9x + 15)x$$ ◀

Caution When factoring out the greatest common factor in an expression like $2x^2 + x$, be careful to remember the 1 in the second term.

$$2x^2 + x = 2x^2 + 1x = x(2x + 1), \text{ not } x(2x)$$

Factoring Trinomials A polynomial that has no greatest common factor (other than 1) may still be factorable. For example, the polynomial $x^2 + 5x + 6$ can be factored as $(x + 2)(x + 3)$. To see that this is correct, find the product $(x + 2)(x + 3)$; you should get $x^2 + 5x + 6$. To factor a polynomial of three terms such as $x^2 + 5x + 6$, where the coefficient of x^2 is 1, proceed as shown in the following example.

EXAMPLE 2 Factor $y^2 + 8y + 15$.

Since the coefficient of y^2 is 1, factor by finding two numbers whose *product* is 15 and whose *sum* is 8. Since the constant and the middle term are positive, the numbers must both be positive. Begin by listing all pairs of positive integers having a product of 15. As you do this, also form the sum of each pair of numbers.

Products	*Sums*
$15 \cdot 1 = 15$	$15 + 1 = 16$
$5 \cdot 3 = 15$	$5 + 3 = 8$

The numbers 5 and 3 have a product of 15 and a sum of 8. Thus, $y^2 + 8y + 15$ factors as

$$y^2 + 8y + 15 = (y + 5)(y + 3).$$

The answer also can be written as $(y + 3)(y + 5)$. ◀

If the coefficient of the squared term is *not* 1, work as shown below.

EXAMPLE 3 Factor $2x^2 + 9xy - 5y^2$.

The factors of $2x^2$ are $2x$ and x; the possible factors of $-5y^2$ are $-5y$ and y, or $5y$ and $-y$. Try various combinations of these factors until one works (if, indeed, any work). For example, try the product $(2x + 5y)(x - y)$.

$$(2x + 5y)(x - y) = 2x^2 - 2xy + 5xy - 5y^2$$
$$= 2x^2 + 3xy - 5y^2$$