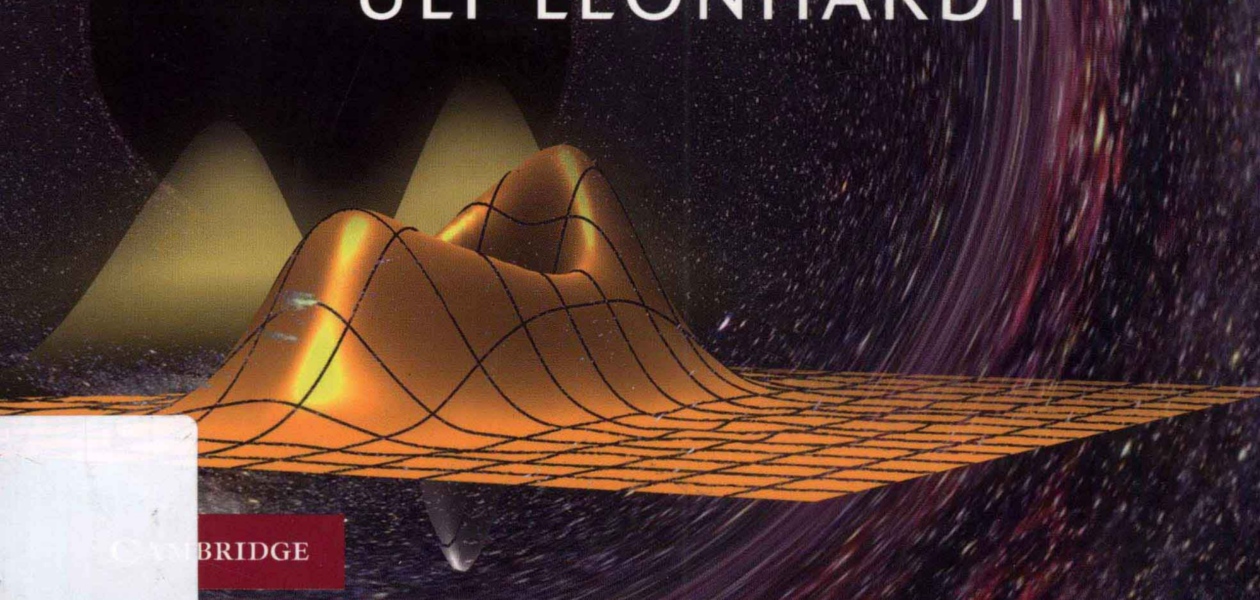
ESSENTIAL Quantum Optics

From Quantum Measurements to Black Holes

ULF LEONHARDT



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Essential Quantum Optics

From Quantum Measurements to Black Holes

Covering some of the most exciting trends in quantum optics – quantum entanglement, teleportation, and levitation – this textbook is ideal for advanced undergraduate and graduate students. The book journeys through the vast field of quantum optics following a single theme: light in media. A wide range of subjects are covered, from the force of the quantum vacuum to astrophysics, from quantum measurements to black holes.

Ideas are explained in detail and formulated so that students with little prior knowledge of the subject can follow them. Each chapter ends with several short questions followed by a more detailed homework problem, designed to test the reader and show how the ideas discussed can be applied. Solutions to homework problems are available at www.cambridge.org/9780521145043.

ULF LEONHARDT is Professor of Theoretical Physics at the University of St Andrews. His research interests include quantum electrodynamics in media and state reconstruction in quantum mechanics. He is one of the inventors of invisibility devices and artificial black holes.

For Irina

Acknowledgements

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Many people have contributed to awakening my interest in quantum optics and to begin the research expedition from quantum measurements to black holes which I have tried to describe in this book. As a student in Jena I became fascinated by quantum mechanics listening to Dirk-Gunnar Welsch's lectures on quantum theory. I did not mind the mathematics raining down on the audience, but enjoyed his clear thoughts that I tried to formulate in my own way from the sketchy notes I took. At that time I felt I understood quantum mechanics. The quantum theory of light was already the subject of my Diploma thesis in Jena. I learned the craft of quantum optics from Ludwig Knöll who always understood my questions and was most kind and helpful. I learned the art of quantum optics from Harry Paul in Berlin, during my PhD. I am very grateful for the many conversations that led me to appreciate that quantum mechanics happens in reality and not in Hilbert space, and how astonishing and mysterious it is. Since then I know that I don't understand quantum mechanics. This book is an attempt to convey both the clarity of quantum theory and the sense of profoundly not understanding quantum physics. I was deeply impressed by Harry Paul's style of doing physics in conversations, without focusing on technicalities, and I hope that some of this shines through the technicalities of this book. I also learned to appreciate experiments and to discuss them, to the extent that I joined Michael Raymer's experimental group in Eugene as theorist in the laboratory. From him I learned to respect the dedication and patience it takes to turn ideas into reality. Wolfgang Schleich in Ulm gave me the freedom to give the lecture course that made me fall in love with general relativity. I never learned general relativity from attending lectures, I learned it from giving lectures, fittingly in Ulm, Albert Einstein's place of birth. Since then I have been trying to invent optical applications of the ideas of general relativity, from optical black holes to invisibility. Stig Stenholm was most generous in giving me a haven in Stockholm where I benefited from his broad mind and wide research interests that, I hope, have broadened my horizons. I learned the quantum physics of horizons from Renaud Parentani in many conversations where I enjoyed his wit and clarity, and also from making many errors of my own – according to the classic definition of the expert: an expert is someone who made all possible errors (and learned from them). In this book, I hope to have conveyed some of the lessons learned.

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Chapter 1
Introduction

1.1 A note to the reader

Quantum optics has grown from a sub-discipline in atomic, molecular, and optical physics to a broad research area that bridges several branches of physics and that captures the imagination of the public. Quantum information science has put quantum optics into the spotlight of modern physics, as has the physics of ultracold quantum gases with its many spectacular connections to condensed-matter physics. Yet through all these exciting developments quantum optics has maintained a characteristic core of ideas that I try to explain in this slim volume.

Quantum optics focuses on the simplest quantum objects, usually light and few-level atoms, where quantum mechanics appears in its purest form without the complications of more complex systems, often demonstrating in the laboratory the thought experiments that the founders of quantum mechanics dreamed of. Quantum optics has been, and will be for the foreseeable future, quintessential quantum mechanics, the quantum mechanics of simple systems, based on a core of simple yet subtle ideas and experiments.

One of the strengths of quantum optics is the close connection between theory and experiment. Although this book necessarily is theoretical, many of the theoretical ideas I describe are guided by experiments or, in turn, have inspired experiments themselves. Another strength of quantum optics is that it is done by individuals or small teams. One single person or a small group can build and perform an entire experiment. One theorist can do all the calculations for an important problem, often with just pencil and paper. As John A. Wheeler said, "It is nice to know that the computer understands the problem, but I want to understand it, too."

This book does not attempt to cover the entire field of quantum optics. The book is focused and selective in the material it applies and expounds, for three good reasons. One of these clearly is that students or other readers do not need to know much in advance to understand this book. You should have experience in working with quantum mechanics at the level of British senior honours students or American junior postgraduate students, and you should know the basics of classical electromagnetism – that is all. Everything else follows and is deduced and explained without any need of further reference. But you should have an inquisitive mind and be able to do mathematics. I use mathematics as a tool, but I have tried to derive the main results with as little technical effort as possible. Most of the eight chapters of this book make a one semester course, the senior honours course on quantum optics that I have taught at St Andrews.

Another reason for being focused is that understanding is more valuable than knowledge. It is better to study one subject in depth than many on the surface. Understanding one subject well gives you confidence to tackle many more and to anticipate their workings by analogy and using your imagination. As Albert Einstein put it, "Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there ever will be to know and understand."

The third reason is the subject this book focuses on: light in media. Media are transparent materials like glass, water or air, but space itself may be regarded a medium for light, in particular the curved space of gravity. Quantum electrodynamics in media has been the backbone of quantum optics for the last 50 years and is likely to remain so for the next. Another vital ingredient of quantum optics is the theory of irreversible quantum processes that has been highly developed in the theory of the laser or in models of the quantum measurement process. A historically important inspiration came from astronomy in the 1950s when quantum fluctuations were used to measure the size of stars, whereupon quantum fluctuations of light became a research subject in its own right. Thinking about quantum fluctuations eventually led to experimental tests of quantum nonlocality – testing the nature of reality itself – and applications in quantum cryptography. These days, when cosmologists no longer are "often in error but seldom in doubt" (Lev D. Landau) quantum optics is beginning to play a serious role in cosmology, as the observed fluctuations of the cosmic microwave background show. Ideas from astrophysics also inspired applications of quantum optics in laboratory analogues of the event horizon. Quantum optics returns to one of its roots. The quantum features of light in media even have technological implications, they appear as vacuum forces in micro- and nano machines. As I will describe, the subject of light in media contains all these themes, from quantum measurements to black holes.[1]

So although this book focuses on the "light side" of quantum optics, you will be amazed to see how many topics such a modest subject contains. Here is an alphabet of samples: **A**spect's experiment, **B**ell's Theorem, **C**asimir Forces, **D**ielectric Media, **E**instein's Relativity, **F**luctuations, **G**ibbs Ensembles, **H**awking Radiation, **I**rreversibility, **J**oint Measurements of Position and Momentum, **K**atzen (in German Cats, here Schrödinger Cats), **L**indblad's Theorem, **M**odels of the **M**easurement Process, **N**on-Classical Light, **O**ptical Homodyne Tomography, **P**olarization Correlations, **Q**uantum Communication, **R**eversible Dynamics, **S**queezed Light, **T**eleportation, **U**nruh Effect, **V**acuum Noise, **W**ave–Particle Dualism, $\boldsymbol{x}$ and $\boldsymbol{y}$ Coordinates, and **Z**ero Point Energy. I hope the story of this journey through quantum optics is told with sufficient clarity and occasional amusement for the reader. I also hope to have captured not only the sights and insights on the way, but also the sense of excitement and adventure.

1.2 Quantum theory

Let us recall the basic axioms of quantum theory, and let us try to motivate them. This book is of course not the place for a comprehensive development of the theory. We assume that the reader is already familiar with the basic formalism of quantum mechanics. However, because some of the ideas touched on in this book illustrate fundamental issues of quantum physics, we would find it appropriate to turn "back to the roots of quantum mechanics" in a brief and certainly incomplete survey. Moreover, not all readers may have mastered quantum statistics – the formalism of density matrices – and so it is worthwhile to explain this theory here, with apologies to those who know it already. Let us first sketch, in a couple of lines, *one* possible way of motivating the principal ideas of quantum theory.

1.2.1 Axioms

"At the heart of quantum mechanics lies the *superposition principle*" – to quote from the first chapter of Dirac's classic treatise (Dirac, 1984);

[1] This book evolved from my monograph *Measuring the Quantum State of Light* (Leonhardt, 1997a). Hopefully it turned into a fully fledged textbook of Quantum Optics.

"... any two or more states may be superposed to give a new state" (Schleich et al., 1991). We denote the state of a perfectly prepared quantum object by $|\psi\rangle$. Then, according to this principle, the complex superposition $c_1|\psi_1\rangle + c_2|\psi_2\rangle$ of two states $|\psi_1\rangle$ and $|\psi_2\rangle$ is a possible state as well. In other words, perfectly prepared states, called *pure states*, are vectors in a complex space. The superposition principle alone does not make physical predictions, it only prepares the ground for quantum mechanics. Nevertheless, the principle is highly nontrivial and can hardly be derived or taken for granted. In the history of quantum mechanics the superposition principle was motivated by the wavelike interference of material particles. Note, however, that this simple principle experienced a dramatic generalization such that we cannot consider its historical origin as a physical motivation any more.

Let us now turn to more physical assumptions. When we observe a physical quantity of an ensemble of equally prepared states, we obtain certain measurement values a (real numbers) with probabilities p_a. Given a result a, we assume that we would obtain the same result if we repeated the experiment immediately after the first measurement (provided, of course, that the physical object has not been destroyed). This assumption is certainly plausible. As a consequence, the object must have jumped into a state $|a\rangle$, called an *eigenstate*, which gives the measurement result with certainty, an event called the *collapse of the state vector*. Or, if we prefer to assign states only to ensembles of objects, a measurement produces a statistical ensemble of states $|a\rangle$ with probability p_a. According to the superposition principle we can expand the state vector $|\psi\rangle$ before the measurement in terms of the eigenstates $|a\rangle$, written as $|\psi\rangle = \sum_a \langle a|\psi\rangle |a\rangle$, with some complex numbers denoted by the symbol $\langle a|\psi\rangle$. What is the probability for the transition from $|\psi\rangle$ to a particular $|a\rangle$? Clearly, the larger the $\langle a|\psi\rangle$ component is (compared to all other components) the larger should be p_a. However, this component is a complex number in general. So the simplest possible expression for the transition probability is the ratio

$$p_a = \frac{|\langle a|\psi\rangle|^2}{\langle\psi|\psi\rangle} . \tag{1.1}$$

Here $\langle\psi|\psi\rangle$ abbreviates simply the sum of all $|\langle a|\psi\rangle|^2$ values. It is a special case of the more general symbol

$$\langle\psi'|\psi\rangle = \sum_a \langle\psi'|a\rangle\langle a|\psi\rangle \tag{1.2}$$

with the convention

$$\langle\psi|a\rangle = \langle a|\psi\rangle^* . \tag{1.3}$$

The mathematical construction (1.2) of the symbol $\langle\psi'|\psi\rangle$ fulfils all requirements of a scalar product in a vector space. However, at this stage the scalar product depends critically on a particular set of eigenstates $|a\rangle$ or, in other words, on a particular experiment. Let us assume that all possible sets of physical eigenstates form the same scalar product so that no experimental setting is favoured or discriminated against in principle. This assumption seems to be natural yet it is nontrivial. If we accept this, then the symbol $\langle\psi'|\psi\rangle$ describes *the* scalar product in the linear state space. We can employ Dirac's convenient bracket formalism, and in particular we can understand the $\langle a|\psi\rangle$ components as orthogonal projections of the $|\psi\rangle$ vector onto the eigenstates $|a\rangle$.

Formula (1.1) is the key axiom of quantum mechanics. It makes a quantitative prediction about an event in physical reality (the occurrence of the measurement result a), and it contains implicitly the superposition principle for describing quantum states. The historical origin of this fundamental principle is Born's probability interpretation of the modulus square of the Schrödinger wave function.

Now we are in a position to reproduce the basic formalism of quantum mechanics. Because the probability p_a does not depend on the normalization of the state vector $|\psi\rangle$, we may simplify formula (1.1) by considering only normalized states, that is we set

$$\langle\psi|\psi\rangle = 1\,. \tag{1.4}$$

Because the eigenstates produce the measurement result a with certainty, they must be *orthonormal*

$$\langle a|a'\rangle = \delta_{a'a}\,. \tag{1.5}$$

Furthermore, the system of eigenvectors must be *complete*

$$\sum_a |a\rangle\langle a| = 1 \tag{1.6}$$

if we assume that any observation gives at least one of the values a so that $\sum_a p_a = \langle\psi|\sum_a |a\rangle\langle a|\,|\psi\rangle$ equals unity for all states $|\psi\rangle$. The average $\langle A\rangle$ of the measurement values a is given by

$$\langle A\rangle = \sum_a a\,p_a = \langle\psi|\hat{A}|\psi\rangle \tag{1.7}$$

where we have introduced the *operator*

$$\hat{A} \equiv \sum_a a\,|a\rangle\langle a| \tag{1.8}$$

with *eigenvalues* a and *eigenvectors* $|a\rangle$. (The structure (1.8) explains the term *eigenvectors* for the measurement produced states $|a\rangle$.) The operator $\hat{A}$ is *Hermitian*,

$$\hat{A}^\dagger = \hat{A}\,, \tag{1.9}$$

because the measurement results a are real. Observable quantities thus correspond to Hermitian operators; their eigenstates are the states assumed after measurement and the eigenvalues are the measurement results.

Suppose that after the measurement of the observable quantity $\hat{A}$ the object is in the eigenstate $|a\rangle$. What happens when we subsequently perform another measurement of the observable $\hat{B}$? Clearly, the quantum object will now assume one of the eigenstates of $\hat{B}$. But the two sets of eigenstates, $\{|a\rangle\}$ and $\{|b\rangle\}$, may differ. As the overlap $|\langle a|b\rangle|^2$ gives the probability of observing b after a, the measurement result b is statistically uncertain if the overlap is not perfect, $|\langle a|b\rangle|^2 < 1$. In this case, the two operators $\hat{A}$ and $\hat{B}$ do not commute,

$$\hat{A}\hat{B} \neq \hat{B}\hat{A}\,, \tag{1.10}$$

because otherwise they would share the same system of eigenstates. The degree of commutation is characterized by the *commutator*

$$\left[\hat{A}, \hat{B}\right] \equiv \hat{A}\hat{B} - \hat{B}\hat{A}\,, \tag{1.11}$$

an anti-Hermitian operator $\hat{C}$ for Hermitian $\hat{A}$ and $\hat{B}$, with $\hat{C}^\dagger = -\hat{C}$. Incompatible observables correspond to non-commuting operators. They cause mutual statistical uncertainty of measurement results, an uncertainty that can be quantified in *uncertainty relations*, for example in relation (5.61) that we use later on.

We must mention another fundamental axiom of quantum mechanics concerning the composition of physical objects. If one system consists of, say, two subsystems, then the theory should allow us to experiment on each of the subsystems independently. We would obtain two real measurement values (a_1, a_2) and if we had repeated the same experiment immediately after the first measurement we would read the same values (a_1, a_2). Furthermore, we would also obtain a_1 if we had performed the repeated measurement only on the first subsystem, irrespective of what happens on the other (irrespective of which measurement is performed there) and, of course, vice versa. So it is natural to assume that independent measurements correspond to factorized eigenstates

$$|a_1, a_2\rangle = |a_1\rangle \otimes |a_2\rangle\,. \tag{1.12}$$

As usual, the symbol $\otimes$ denotes the tensor product. Note, however, that the innocent looking statement (1.12) is capable of peculiar physical effects when it is combined with the superposition principle. The state space of the total system is the tensor product of the subspaces. However, the superposition of two different states $|a_1\rangle \otimes |a_2\rangle$ and $|a_1'\rangle \otimes |a_2'\rangle$ will not factorize in general, producing an *entangled state*. The total