



# SPACE-TIME CODING THEORY AND PRACTICE

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# **Preface**

The use of multiple antennas in most future wireless communication systems seems to be inevitable. Today, the main question is how to include multiple antennas and what are the appropriate methods for specific applications. The academic interest in space-time coding and multiple-input multiple-output (MIMO) systems has been growing for the last few years. Recently, the industry has shown a lot of interest as well. It is amazing how fast the topic has emerged from a theoretical curiosity to the practice of every engineer in the field. It was just a few years ago, when I started working at AT&T Labs – Research, that many would ask "who would use more than one antenna in a real system?" Today, such skepticism is gone.

The fast growth of the interest and activities on space-time coding has resulted in a spectrum of people who actively follow the field. The range spans from mathematicians who are only curious about the interesting mathematics behind space-time coding to engineers who want to build it. There is a need for covering both the theory and practice of space-time coding in depth. This book hopes to fulfill this need. The book has been written as a textbook for first-year graduate students and as a reference for the engineers who want to learn the subject from scratch. An early version of the book has been used as a textbook to teach a course in spacetime coding at the University of California, Irvine. The goal of such a course is the introduction of space-time coding to anyone with some basic knowledge of digital communications. In most cases, we start with common ideas for single-input single-output (SISO) channels and extend them to MIMO channels. Therefore, students or engineers with no knowledge of MIMO systems should be able to learn all the concepts. While graduate students might be interested in all the details and the proofs of theorems and lemmas, engineers may skip the proofs and concentrate on the results without sacrificing the continuity of the presentation.

A typical course on space-time coding may start with some background material on wireless communications and capacity of MIMO channels as covered in Chapters 1 and 2. A review of design criteria for space-time codes is covered in Chapter 3.

Chapters 4 and 5 provide a detailed treatment of the theory behind space-time block codes. A practitioner who is only interested in the structure of the codes may bypass all the proofs in these chapters and concentrate only on the examples. Chapters 6 and 7 discuss space-time trellis codes in depth. Each chapter includes discussions on the performance analysis of the codes and simulation results. For those who are more interested in the practical aspects of the topic, simulation results are sufficient and the sections on performance analysis may be skipped. The practitioners may continue with Chapter 11 and its discussion on MIMO-OFDM and Chapter 9 on receiver design. On the other hand, those who are more interested in the theory of space-time codes can follow with Chapter 8 and its treatment of differential space-time modulation. Finally, for the sake of completeness, we discuss BLAST and some other space-time processing methods in Chapters 9 and 10. Homework problems have been included at the end of each chapter.

The book includes the contribution of many researchers. I am grateful to all of them for generating excitement in the field of space-time coding. My special thanks goes to my very good friend and former colleague Professor Vahid Tarokh who introduced space-time coding to me. Also, I should thank my other colleagues at AT&T Labs – Research who initiated most of the basic concepts and ideas in space-time coding. Without the support of my department head at AT&T Labs – Research, Dr. Behzad Shahraray, I would not be able to contribute to the topic and I am thankful to him for providing the opportunity. Also, Professor Rob Calderbank has been a big supporter of the effort.

The early versions of this book have been read and reviewed by my students and others. Their comments and suggestions have improved the quality of the presentation. Especially, comments from Professor John Proakis, Professor Syed Jafar, Dr. Masoud Olfat, and Hooman Honary have resulted in many improvements. My Ph.D. students, Li Liu, Javad Kazemitabar, and Yun Zhu, have helped with the proofreading of a few chapters. Many of the presented simulation results have been double checked by Yun Zhu. I also thank the National Science Foundation (NSF) for giving me an NSF Career Award to support my research and educational goals related to this book.

Last but not least, I would like to thank my wife, Paniz, for her support and love.

# Standard notation

```
||\cdot||
         Euclidean norm
        Frobenius norm
||\cdot||_{\mathrm{F}}
        tensor product
 \otimes
        conjugate
  +
         Moore-Penrose generalized inverse (pseudo-inverse)
         determinant of a matrix
 det
         expectation
  E
         pdf of X
f_X(x)
  Η
         Hermetian
         imaginary part
  3
  I
         imaginary part
         N \times N identity matrix
  I_N
         \sqrt{-1}
  j
         covariance of vector X
 K_X
  \Re
         real part
  R
         real part
  T
         transpose
         trace of a matrix
  Tr
 Var
         variance
```

L number of orthogonal (data) blocks in a SOSTTC

L size of IFFT and FFT blocks in OFDM

L-PSK a PSK constellation with  $L = 2^b$  symbols

*M* number of receive antennas

N number of transmit antennas

 $\mathcal{N}$   $T \times M$  noise matrix

 $N_0$  noise samples have a variance of  $N_0/2$  per complex dimension

P number of trellis transitions (two trellis paths differ in P transitions)

Pout outage probability

Q the memory of a trellis

r transmission rate in bits/(s Hz)

r received signal

 $\mathbf{r}$   $T \times M$  received matrix

R rate of a STC

*IR* set of real numbers

s transmitted signal

 $S_t$  the state of the encoder at time t

x indeterminant variable

**Z** set of integers

# Abbreviations

ADC Analog to Digital Converter AGC Automatic Gain Control

**AWGN** Additive White Gaussian Noise

**BER** Bit Error Rate

**BLAST** Bell Labs Layered Space-Time **BPSK** Binary Phase Shift Keying BSC Binary Symmetric Channel

**CCDF** Complementary Cumulative Distribution Function

**CDF** Cumulative Distribution Function **CDMA** Code Division Multiple Access **CSI** Channel State Information

CTCordless Telephone

DAST Diagonal Algebraic Space-Time

DASTBC Diagonal Algebraic Space-Time Block Code

**D-BLAST** Diagonal BLAST

**DECT** Digital Cordless Telephone **DFE Decision Feedback Equalization DPSK** Differential Phase Shift Keying

**EDGE** Enhanced Data for Global Evolution

**FER** Frame Error Rate

FFT Fast Fourier Transform FIR Finite Impulse Response **GSM** Global System for Mobile **IFFT** Inverse Fast Fourier Transform iid independent identically distributed **IMT** International Mobile Telephone

ISI Intersymbol Interference

LAN Local Area Network

LDSTBC Linear Dispersion Space-Time Block Code

LOS Line of Sight

MGF Moment Generating Function
MIMO Multiple-Input Multiple-Output
MISO Multiple-Input Single-Output

MMAC Multimedia Mobile Access Communication

MMSE Minimum Mean Squared Error MRC Maximum Ratio Combining ML Maximum-Likelihood

MTCM Multiple Trellis Coded Modulation

OFDM Orthogonal Frequency Division Multiplexing

OSTBC Orthogonal Space-Time Block Code

PAM Pulse Amplitude Modulation PAN Personal Area Network

PAPR Peak-to-Average Power Ratio
PDA Personal Digital Assistant
PDC Personal Digital Cellular
pdf probability density function
PEP Pairwise Error Probability
PHS Personal Handyphone System

PSK Phase Shift Keying

QAM Quadrature Amplitude Modulation

QOSTBC Quasi-Orthogonal Space-Time Block Code

QPSK Quadrature Phase Shift Keying

RF Radio Frequency

RLST Random Layered Space-Time

RV Random Variable SER Symbol Error Rate

SISO Single-Input Single-Output
SIMO Single-Input Multiple-Output

SM Spatial Multiplexing SNR Signal to Noise Ratio

SOSTTC Super-Orthogonal Space-Time Trellis Code

SQOSTTC Super-Quasi-Orthogonal Space-Time Trellis Code

STBC Space-Time Block Code STTC Space-Time Trellis Code

TAST Threaded Algebraic Space-Time

TASTBC Threaded Algebraic Space-Time Block Code

TCM Trellis Coded Modulation
TDD Time Division Duplexing
TDMA Time Division Multiple Access

V-BLAST Vertical BLAST ZF Zero Forcing

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## Introduction

### 1.1 Introduction to the book

Recent advances in wireless communication systems have increased the throughput over wireless channels and networks. At the same time, the reliability of wireless communication has been increased. As a result, customers use wireless systems more often. The main driving force behind wireless communication is the promise of portability, mobility, and accessibility. Although wired communication brings more stability, better performance, and higher reliability, it comes with the necessity of being restricted to a certain location or a bounded environment. Logically, people choose freedom versus confinement. Therefore, there is a natural tendency towards getting rid of wires if possible. The users are even ready to pay a reasonable price for such a trade-off. Such a price could be a lower quality, a higher risk of disconnection, or a lower throughput, as long as the overall performance is higher than some tolerable threshold. The main issue for wireless communication systems is to make the conversion from wired systems to wireless systems more reliable and if possible transparent. While freedom is the main driving force for users, the incredible number of challenges to achieve this goal is the main motivation for research in the field. In this chapter, we present some of these challenges. We study different wireless communication applications and the behavior of wireless channels in these applications. We provide different mathematical models to characterize the behavior of wireless channels. We also investigate the challenges that a wireless communication system faces.

Throughout the book, we provide different solutions to some of the challenges in wireless communication by using multiple antennas. The main topic of this book is how to overturn the difficulties in wireless communication by employing multiple antennas. We start with a study of the capacity increase due to the use of multiple antennas. Then, we show how to design a space-time architecture for multiple transmit antennas to improve the performance of a wireless system

while keeping the transmission power intact. Most of the book discusses different space-time coding methods in detail. The detailed discussion of each method includes design, properties, encoding, decoding, performance analysis, and simulation results. We pay close attention to the complexity of encoding and decoding for each method and to different trade-offs in terms of throughput, complexity, and performance. Not only do we provide the theoretical details of each method, but also we present the details of the algorithm implementation. Our overall goal is to keep a balance between the theory and the practice of space-time coding.

## 1.2 Wireless applications

There are many systems in which wireless communication is applicable. Radio broadcasting is perhaps one of the earliest successful common applications. Television broadcasting and satellite communications are other important examples. However, the recent interest in wireless communication is perhaps inspired mostly by the establishment of the first generation cellular phones in the early 1980s. The first generation of mobile systems used analog transmission. The second generation of cellular communication systems, using digital transmission, were introduced in the 1990s. Both of these two systems were primarily designed to transmit speech. The success of cellular mobile systems and their appeal to the public resulted in a growing attention to wireless communications in industry and academia. Many researchers concentrated on improving the performance of wireless communication systems and expanding it to other sources of information like images, video, and data.

Also, the industry has been actively involved in establishing new standards. As a result, many new applications have been born and the performance of the old applications has been enhanced. Personal digital cellular (PDC), global system for mobile (GSM) communications, IS-54, IS-95, and IS-136 are some of the early examples of these standards. While they support data services up to 9.6 kbits/s, they are basically designed for speech. More advanced services for up to 100 kbits/s data transmission has been evolved from these standards and are called 2.5 generation. Recently, third generation mobile systems are being considered for high bit-rate services. With multimedia transmission in mind, the third generation systems are aiming towards the transmission of 144–384 kbits/s for fast moving users and up to 2.048 Mbits/s for slow moving users.

The main body of the third generation standards is known as international mobile telephone (IMT-2000). It includes the enhanced data for global evolution (EDGE) standard, which is a time division multiple access (TDMA) system and an enhancement of GSM. It also includes two standards based on wideband code division

multiple access (CDMA). One is a synchronous system called CDMA2000 and the other one is an asynchronous system named WCDMA. In addition to applications demanding higher bit rates, one can use multiple services in the third-generation standards simultaneously. This means the need for improved spectral efficiency and increased flexibility to deploy new services. There are many challenges and opportunities in achieving these goals.

Of course the demand for higher bit rates does not stop with the deployment of the third-generation wireless systems. Another important application that drives the demand for high bit rates and spectral efficiency is wireless local area networks (LANs). It is widely recognized that wireless connection to the network is an inevitable part of future communication networks and systems in the emerging mobile wireless Internet. Needless to say, the design of systems with such a high spectral efficiency is a very challenging task. Perhaps the most successful standard in this area is the IEEE 802.11 class of standards. IEEE 802.11a is based on orthogonal frequency division multiplexing (OFDM) to transmit up to 54 Mbits/s of data. It transmits over the 5 GHz unlicensed frequency band. IEEE 802.11b provides up to 11 Mbits/s over the 2.45 GHz unlicensed frequency band. IEEE 802.11g uses OFDM over the 2.45 GHz unlicensed frequency band to provide a data rate of up to 54 Mbits/s. Other examples of wireless LAN standards include high performance LAN (HiperLAN) and multimedia mobile access communication (MMAC). Both HiperLAN and MMAC use OFDM. The main purpose of a wireless LAN is to provide high-speed network connections to stationary users in buildings. This is an important application of wireless communications as it provides freedom from being physically connected, portability, and flexibility to network users.

There are many other applications of wireless communications. Cordless telephone systems and wireless local loops are two important examples. Cordless telephone standards include the personal handyphone system (PHS), digital cordless telephone (DECT), and cordless telephone (CT2). Wireless personal area network (PAN) systems are utilized in applications with short distance range. IEEE 802.15 works on developing such standards. Bluetooth is a good example of how to build an ad hoc wireless network among devices that are in the vicinity of each other. The Bluetooth standard is based on frequency hop CDMA and transmits over the 2.45 GHz unlicensed frequency band. The goal of wireless PANs is to connect different portable and mobile devices such as cell phones, cordless phones, personal computers, personal digital assistants (PDAs), pagers, peripherals, and so on. The wireless PANs let these devices communicate and operate cohesively. Also, wireless PANs can replace the wire connection between different consumer electronic appliances, for example among keyboard, mouse, and computers or between television sets and cable receivers.

## 1.2.1 Wireless challenges

While various applications have different specifications and use different wireless technologies, most of them face similar challenges. The priority of the different challenges in wireless communications may not be the same for different applications; however, the list applies to almost all applications. Some of the challenges in wireless communications are:

- · a need for high data rates;
- · quality of service;
- · mobility;
- · portability;
- · connectivity in wireless networks;
- · interference from other users;
- · privacy/security.

Many of the demands, for example the need for high data rates and the quality of service, are not unique to wireless communications. But, some of the challenges are specific to wireless communication systems. For example, the portability requirement results in the use of batteries and the limitation in the battery life creates a challenge for finding algorithms with low power consumptions. This requires special attention in the design of transmitters and receivers. Since the base station does not operate on batteries and does not have the same power limitations, it may be especially desirable to have asymmetric complexities in different ends.

Another example of challenges in wireless communications is the connectivity in wireless networks. The power of the received signal depends on the distance between the transmitter and the receiver. Therefore, it is important to make sure that if, because of the mobility of the nodes, their distance increases, the nodes remain connected. Also, due to the rapidly changing nature of the wireless channel, mobility brings many new challenges into the picture. Another important challenge in a wireless channel is the interference from other users or other sources of electromagnetic waves. In a wired system, the communication environment is more under control and the interference is less damaging.

While the demand for data rates and the performance of the signal processors increase exponentially, the spectrum and bandwidth are limited. The limited bandwidth of the wireless channels adds increases impairment. Increases in battery power grows slowly and there is a growing demand for smaller size terminals and handset devices. On the other hand, the users want the quality of wire-line communication and the wire-line data rates grow rapidly. Researchers face many challenges to satisfy such high expectations through the narrow pipeline of wireless channels.

The first step to solve these problems is to understand the behavior of the wireless channel. This is the main topic of the next section. We provide a brief introduction