Eberhard Zeidler

Nonlinear Functional Analysis and its Applications II/B

Nonlinear Monotone Operators

非线性泛函分析及其应用

第2B卷

Springer

世界图书出版公司
www.wpcbj.com.cn

Eberhard Zeidler

Nonlinear Functional Analysis and its Applications

II/B: Nonlinear Monotone Operators

Translated by the Author and by Leo F. Boron†

With 74 Illustrations

Eberhard Zeidler
Sektion Mathematik
Karl-Marx-Platz
7010 Leipzig
German Democratic Republic

Leo F. Boron†
Department of Mathematics
University of Idaho
Moscow, ID 83843
U.S.A.

Mathematics Subject Classification (1980): 46xx

Library of Congress Cataloging in Publication Data
(Revised for vol. 2 pts. A-B)
Zeidler, Eberhard.
Vol. 2, pts. A-B: Translated by the author and
Leo L. Boron.
Vol. 3: Translated by Leo L. Boron.
Vol. 4: Translated by Juergen Quandt.
Includes bibliographies and indexes.
Contents: 1. Fixed point theorems — 2. pt A. Linear
monotone operators. Pt. B Nonlinear operators — [etc.]
— 4. Applications to mathematical physics.
1. Nonlinear functional analysis. I. Title.
QA321.5.Z4513 1985 515.7 83-20455

Previous edition, *Vorlesungen über nichtlineare Funktionalanalysis*, Vols. I–III, published by B. G. Teubner Verlagsgesellschaft, 7010 Leipzig, Sternwartenstrasse 8, Deutsche Demokrati Republik.

© 1990 by Springer-Verlag New York Inc.
All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer-Verlag, 175 Fifth Avenue, New York, New York 10010, U.S.A.), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden.

This reprint has been authorized by Springer-Verlag (Berlin/Heidelberg/New York) for sale in the People's Republic of China only and not for export therefrom.

ISBN 0-387-97167-X Springer-Verlag New York Berlin Heidelberg
ISBN 3-540-97167-X Springer-Verlag New York Berlin Heidelberg

图书在版编目（CIP）数据

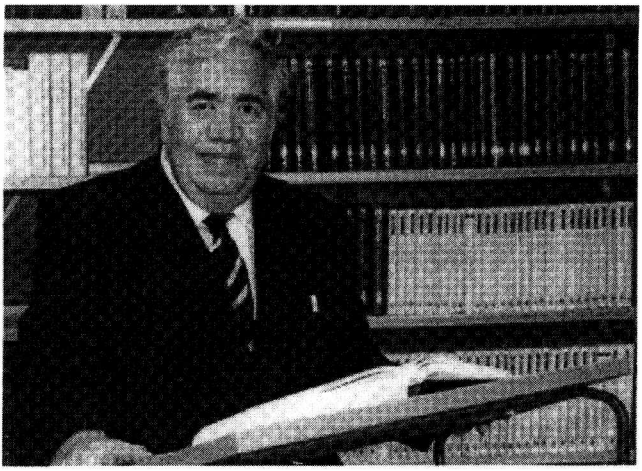
非线性泛函分析及其应用．第2B卷，非线性单调算子
= Nonlinear Functional Analysis & It's Applications Vol. 2/B
Nonlinear Monotone Operators：英文/（德）宰德勒著．
—北京：世界图书出版公司北京公司，2009.8
ISBN 978-7-5100-0521-3

Ⅰ．非… Ⅱ．宰… Ⅲ．①非线性—泛函分析—英文
②非线性算子：单调算子—英文 Ⅳ．0177.91

中国版本图书馆CIP数据核字（2009）第126347号

书　　名：	Nonlinear Functional Analysis & Its Applications Vol. 2/B
作　　者：	E. Zeidler
中 译 名：	非线性泛函分析及其应用 第2B卷
责任编辑：	高蓉
出 版 者：	世界图书出版公司北京公司
印 刷 者：	三河国英印务有限公司
发　　行：	世界图书出版公司北京公司（北京朝内大街137号　100010）
联系电话：	010-64021602，010-64015659
电子信箱：	kjb@wpcbj.com.cn
开　　本：	24开
印　　张：	31.5
版　　次：	2009年08月
版权登记：	图字：01-2009-1090
书　　号：	978-7-5100-0521-3/O·737　　**定　　价：** 89.00元

世界图书出版公司北京公司已获得Springer授权在中国大陆独家重印发行



Eberhard Zeidler
德国 Max Planck 研究所数学领头教授，Leipzig 大学数学系荣誉教授。

影印版前言

自 1932 年，波兰数学家 Banach 发表第一部泛函分析专著“Théorie des operations linéaires”以来，这一学科取得了巨大的发展，它在其他领域的应用也是相当成功。如今，数学的很多领域没有了泛函分析恐怕寸步难行，不仅仅在数学方面，在理论物理方面的作用也具有同样的意义，M. Reed 和 B. Simon 的“Methods of Modern Mathematical Physics ”在前言中指出：“自 1926 年以来，物理学的前沿已与日俱增集中于量子力学，以及奠定于量子理论的分支：原子物理、核物理固体物理、基本粒子物理等，而这些分支的中心数学框架就是泛函分析。”所以，讲述泛函分析的文献已浩如烟海。而每个时代，都有这个领域的代表性作品。例如上世纪 50 年代，F. Riesz 和 Sz.-Nagy 的《泛函分析讲义》(中译版，科学出版社，1985)，就是那个时代的一部具有代表性的著作；而 60 年代，N. Dunford 和 J. Schwartz 的三大卷“Linear Operators”则是泛函分析学发展到那个时代的主要成果和应用的一个较全面的总结。泛函分析一经产生，它的发展就受到量子力学的强有力的推动，上世纪 70 年代，M. Reed 和 B. Simon 的 4 卷“Methods of Modern Mathematical Physics”是泛函分析对于量子力学应用的一个很好的总结。

呈现在我们眼前这部 5 大卷鸿篇巨制——E. Zeidler 的“Nonlinear Functional Analysis and its Applications”是非线性泛函分析到了上世纪 80 年代的主要成果和最典型应用的一个全面的论述，是一部百科全书。该书写作思想是：

(1) 讲述什么样的概念是基本的具有支配地位的概念，它们之间的关系是什么?

(2) 上述思想与经典分析以及线性泛函分析已有结果的关系是什么?

此为试读，需要完整PDF请访问：www.ertongbook.com

(3) 最典型的应用是什么?

一般的泛函分析书往往注重抽象理论的阐述，写应用常常不够详尽。而 Zeidler 这部书大为不同，其最大的特点是，书中讲了大量的各方面的应用，而且讲得非常清楚深入。

首先，这部书讲清楚了泛函分析理论对数学其他领域的应用。例如，第 2A 卷讲述线性单调算子。他从椭圆型方程的边值问题出发，讲问题的古典解，由于具体物理背景的需要，问题须作进一步推广，而需要讨论问题的广义解。这种方法背后的分析原理是什么？其实就是完备化思想的一个应用！将古典问题所依赖的连续函数空间，完备化成为 Sobolev 空间，则可讨论问题的广义解。在这种讨论中间，我们可以看到 Hilbert 空间的作用。书中不仅有这种理论讨论，而且还讲了怎样计算问题的近似解（Ritz 方法）。

其次，这部书讲清楚了分析理论在诸多领域（如物理学、化学、生物学、工程技术和经济学等等）的广泛应用。例如，第 3 卷讲解变分方法和优化，它从函数极值问题开始，讲到变分问题及其对于 Euler 微分方程和 Hammerstein 积分方程的应用；讲到优化理论及其对于控制问题（如庞特里亚金极大值原理）、统计优化、博弈论、参数识别、逼近论的应用；讲了凸优化理论及应用；讲了极值的各种近似计算方法。比如第 4 卷，讲物理应用，写作原理是：由物理事实到数学模型；由数学模型到数学结果；再由数学结果到数学结果的物理解释；最后再回到物理事实。

再次，该书由浅入深地讲透了基本理论的发展历程及走向，它既讲清楚了所涉及学科的具体问题，也讲清楚了其背后的数学原理及其作用。数学理论讲得也非常深入，例如，不动点理论，就从 Banach 不动点定理讲到 Schauder 不动点定理，以及 Bourbaki-Kneser 不动点定理等等。

这套书的写作起点很低，具备本科数学水平就可以读；应用都是从最简单情形入手，应用领域的读者也可以读；全书材料自足，各部分又尽可能保持独立；书后附有极其丰富的参考文献及一些文献评述；该书文字优美，引用了许多大师的格言，读之你会深受启发。这套书的优点不胜枚举，每个与数理学科相关的人，搞理论的，搞应用的，搞研究的，搞教学的，都可读该书，哪怕只是翻一翻，都不会空手而返！

全书共有 4 卷（5 本）：

第 1 卷 不动点定理　　第 2 卷 A. 线性单调算子，B. 非线性单调算子

第 3 卷 变分方法与优化　第 4 卷 数学物理的应用

Zeidler 教授著述很多，他后来于 90 年代又写了两本“Applied Functional Analysis”（Springer-Verlag, Applied Mathematical Sciences, 108,109），篇幅虽然比眼前这套书小了很多，但特点没有变。近期，他又在写 6 大卷“Quantum Field Theory”，第 1 卷“Basics in Mathematics and Physics, a Bridge between Mathematicians and Physicists”，已经由 Springer-Verlag 出版社出版。

非常感谢刘景麟对本文建议。

南京理工大学 黄振友

To the memory of my parents

Preface to Part II/B

The present book is part of a comprehensive exposition of the main principles of nonlinear functional analysis and its numerous applications to the natural sciences and mathematical economics. The presentation is self-contained and accessible to a broader audience of mathematicians, natural scientists, and engineers. The material is organized as follows:

Part I: Fixed-point theorems.
Part II: Monotone operators.
Part III: Variational methods and optimization.
Parts IV/V: Applications to mathematical physics.

Here, Part II is divided into two subvolumes:

Part II/A: Linear monotone operators.
Part II/B: Nonlinear monotone operators.

These two subvolumes form a *unit* equipped with a uniform pagination. The contents of Parts II/A and II/B and the basic strategies of our presentation have been discussed in detail in the Preface to Part II/A. The present volume contains the complete index material for Parts II/A and II/B.

For valuable hints I would like to thank Ina Letzel, Frank Benkert, Werner Berndt, Günther Berger, Hans-Peter Gittel, Matthias Günther, Jürgen Herrler, and Rainer Schumann. I would also like to thank Professor Stefan Hildebrandt for his generous hospitality at the SFB in Bonn during several visits in the last few years. In conclusion, I would like to thank Springer-Verlag for a harmonious collaboration.

Leipzig
Summer 1989

Eberhard Zeidler

Contents (Part II/B)

Preface to Part II/B vii

GENERALIZATION TO NONLINEAR STATIONARY PROBLEMS 469

Basic Ideas of the Theory of Monotone Operators 471

CHAPTER 25
Lipschitz Continuous, Strongly Monotone Operators, the Projection–Iteration Method, and Monotone Potential Operators 495

§25.1. Sequences of k-Contractive Operators 497
§25.2. The Projection–Iteration Method for k-Contractive Operators 499
§25.3. Monotone Operators 500
§25.4. The Main Theorem on Strongly Monotone Operators, and the Projection–Iteration Method 503
§25.5. Monotone and Pseudomonotone Operators, and the Calculus of Variations 506
§25.6. The Main Theorem on Monotone Potential Operators 516
§25.7. The Main Theorem on Pseudomonotone Potential Operators 518
§25.8. Application to the Main Theorem on Quadratic Variational Inequalities 519
§25.9. Application to Nonlinear Stationary Conservation Laws 521
§25.10. Projection–Iteration Method for Conservation Laws 527
§25.11. The Main Theorem on Nonlinear Stationary Conservation Laws 535
§25.12. Duality Theory for Conservation Laws and Two-sided *a posteriori* Error Estimates for the Ritz Method 537
§25.13. The Kačanov Method for Stationary Conservation Laws 542
§25.14. The Abstract Kačanov Method for Variational Inequalities 545

CHAPTER 26
Monotone Operators and Quasi-Linear Elliptic Differential Equations 553
§26.1. Hemicontinuity and Demicontinuity 554
§26.2. The Main Theorem on Monotone Operators 556
§26.3. The Nemyckii Operator 561
§26.4. Generalized Gradient Method for the Solution of the Galerkin Equations 564
§26.5. Application to Quasi-Linear Elliptic Differential Equations of Order $2m$ 567
§26.6. Proper Monotone Operators and Proper Quasi-Linear Elliptic Differential Operators 576

CHAPTER 27
Pseudomonotone Operators and Quasi-Linear Elliptic Differential Equations 580
§27.1. The Conditions (M) and (S), and the Convergence of the Galerkin Method 583
§27.2. Pseudomonotone Operators 585
§27.3. The Main Theorem on Pseudomonotone Operators 589
§27.4. Application to Quasi-Linear Elliptic Differential Equations 590
§27.5. Relations Between Important Properties of Nonlinear Operators 595
§27.6. Dual Pairs of B-Spaces 598
§27.7. The Main Theorem on Locally Coercive Operators 598
§27.8. Application to Strongly Nonlinear Differential Equations 604

CHAPTER 28
Monotone Operators and Hammerstein Integral Equations 615
§28.1. A Factorization Theorem for Angle-Bounded Operators 619
§28.2. Abstract Hammerstein Equations with Angle-Bounded Kernel Operators 620
§28.3. Abstract Hammerstein Equations with Compact Kernel Operators 625
§28.4. Application to Hammerstein Integral Equations 627
§28.5. Application to Semilinear Elliptic Differential Equations 632

CHAPTER 29
Noncoercive Equations, Nonlinear Fredholm Alternatives, Locally Monotone Operators, Stability, and Bifurcation 639
§29.1. Pseudoresolvent, Equivalent Coincidence Problems, and the Coincidence Degree 643
§29.2. Fredholm Alternatives for Asymptotically Linear, Compact Perturbations of the Identity 650
§29.3. Application to Nonlinear Systems of Real Equations 652
§29.4. Application to Integral Equations 653
§29.5. Application to Differential Equations 653
§29.6. The Generalized Antipodal Theorem 654
§29.7. Fredholm Alternatives for Asymptotically Linear (S)-Operators 657
§29.8. Weak Asymptotes and Fredholm Alternatives 657

§29.9. Application to Semilinear Elliptic Differential Equations of the Landesman–Lazer Type 661
§29.10. The Main Theorem on Nonlinear Proper Fredholm Operators 665
§29.11. Locally Strictly Monotone Operators 677
§29.12. Locally Regularly Monotone Operators, Minima, and Stability 679
§29.13. Application to the Buckling of Beams 697
§29.14. Stationary Points of Functionals 706
§29.15. Application to the Principle of Stationary Action 708
§29.16. Abstract Statical Stability Theory 709
§29.17. The Continuation Method 712
§29.18. The Main Theorem of Bifurcation Theory for Fredholm Operators of Variational Type 712
§29.19. Application to the Calculus of Variations 722
§29.20. A General Bifurcation Theorem for the Euler Equations and Stability 730
§29.21. A Local Multiplicity Theorem 733
§29.22. A Global Multiplicity Theorem 735

GENERALIZATION TO NONLINEAR NONSTATIONARY PROBLEMS 765

CHAPTER 30
First-Order Evolution Equations and the Galerkin Method 767
§30.1. Equivalent Formulations of First-Order Evolution Equations 767
§30.2. The Main Theorem on Monotone First-Order Evolution Equations 770
§30.3. Proof of the Main Theorem 771
§30.4. Application to Quasi-Linear Parabolic Differential Equations of Order $2m$ 779
§30.5. The Main Theorem on Semibounded Nonlinear Evolution Equations 783
§30.6. Application to the Generalized Korteweg–de Vries Equation 790

CHAPTER 31
Maximal Accretive Operators, Nonlinear Nonexpansive Semigroups, and First-Order Evolution Equations 817
§31.1. The Main Theorem 819
§31.2. Maximal Accretive Operators 820
§31.3. Proof of the Main Theorem 822
§31.4. Application to Monotone Coercive Operators on B-Spaces 827
§31.5. Application to Quasi-Linear Parabolic Differential Equations 829
§31.6. A Look at Quasi-Linear Evolution Equations 830
§31.7. A Look at Quasi-Linear Parabolic Systems Regarded as Dynamical Systems 832

CHAPTER 32
Maximal Monotone Mappings 840
§32.1. Basic Ideas 843
§32.2. Definition of Maximal Monotone Mappings 850

此为试读,需要完整PDF请访问: www.ertongbook.com

§32.3. Typical Examples for Maximal Monotone Mappings 854
§32.4. The Main Theorem on Pseudomonotone Perturbations of Maximal Monotone Mappings 866
§32.5. Application to Abstract Hammerstein Equations 873
§32.6. Application to Hammerstein Integral Equations 874
§32.7. Application to Elliptic Variational Inequalities 874
§32.8. Application to First-Order Evolution Equations 876
§32.9. Application to Time-Periodic Solutions for Quasi-Linear Parabolic Differential Equations 877
§32.10. Application to Second-Order Evolution Equations 879
§32.11. Regularization of Maximal Monotone Operators 881
§32.12. Regularization of Pseudomonotone Operators 883
§32.13. Local Boundedness of Monotone Mappings 884
§32.14. Characterization of the Surjectivity of Maximal Monotone Mappings 886
§32.15. The Sum Theorem 888
§32.16. Application to Elliptic Variational Inequalities 892
§32.17. Application to Evolution Variational Inequalities 893
§32.18. The Regularization Method for Nonuniquely Solvable Operator Equations 894
§32.19. Characterization of Linear Maximal Monotone Operators 897
§32.20. Extension of Monotone Mappings 899
§32.21. 3-Monotone Mappings and Their Generalizations 901
§32.22. The Range of Sum Operators 906
§32.23. Application to Hammerstein Equations 908
§32.24. The Characterization of Nonexpansive Semigroups in H-Spaces 909

CHAPTER 33
Second-Order Evolution Equations and the Galerkin Method 919
§33.1. The Original Problem 921
§33.2. Equivalent Formulations of the Original Problem 921
§33.3. The Existence Theorem 923
§33.4. Proof of the Existence Theorem 924
§33.5. Application to Quasi-Linear Hyperbolic Differential Equations 928
§33.6. Strong Monotonicity, Systems of Conservation Laws, and Quasi-Linear Symmetric Hyperbolic Systems 930
§33.7. Three Important General Phenomena 934
§33.8. The Formation of Shocks 935
§33.9. Blowing-Up Effects 937
§33.10. Blow-Up of Solutions for Semilinear Wave Equations 944
§33.11. A Look at Generalized Viscosity Solutions of Hamilton–Jacobi Equations 947

GENERAL THEORY OF DISCRETIZATION METHODS 959

CHAPTER 34
Inner Approximation Schemes, A-Proper Operators, and the Galerkin Method 963

§34.1. Inner Approximation Schemes 963
§34.2. The Main Theorem on Stable Discretization Methods with Inner Approximation Schemes 965
§34.3. Proof of the Main Theorem 968
§34.4. Inner Approximation Schemes in H-Spaces and the Main Theorem on Strongly Stable Operators 969
§34.5. Inner Approximation Schemes in B-Spaces 972
§34.6. Application to the Numerical Range of Nonlinear Operators 974

CHAPTER 35
External Approximation Schemes, A-Proper Operators, and the Difference Method 978
§35.1. External Approximation Schemes 980
§35.2. Main Theorem on Stable Discretization Methods with External Approximation Schemes 982
§35.3. Proof of the Main Theorem 984
§35.4. Discrete Sobolev Spaces 985
§35.5. Application to Difference Methods 988
§35.6. Proof of Convergence 990

CHAPTER 36
Mapping Degree for A-Proper Operators 997
§36.1. Definition of the Mapping Degree 998
§36.2. Properties of the Mapping Degree 1000
§36.3. The Antipodal Theorem for A-Proper Operators 1000
§36.4. A General Existence Principle 1001

Appendix 1009

References 1119

List of Symbols 1163

List of Theorems 1174

List of the Most Important Definitions 1179

List of Schematic Overviews 1182

List of Important Principles 1183

Index 1189

GENERALIZATION TO NONLINEAR STATIONARY PROBLEMS

When the answers to a mathematical problem cannot be found, then the reason is frequently the fact that we have not recognized the general idea, from which the given problem appears only as a single link in a chain of related problems.
David Hilbert (1900)

In the preceding chapters we studied linear monotone problems. In the following chapters we want to generalize these results to nonlinear monotone problems.

(i) In Chapters 25 through 29 we investigate stationary problems, i.e., we study operator equations of the form

$$Au = b, \qquad u \in X,$$

together with applications to quasi-linear elliptic differential equations and to Hammerstein integral equations.

In this connection we consider the following cases:

Lipschitz continuous, strongly monotone operators on H-spaces (Chapter 25);
monotone coercive operators on B-spaces (Chapter 26);
pseudomonotone operators (Chapter 27);
maximal monotone operators (Chapter 32).

For example, strongly continuous perturbations of monotone continuous operators are pseudomonotone. In Part I we considered the two fundamental fixed-point principles of Banach and Schauder. Figure 25.1 shows that these two principles also play a crucial role in the theory of monotone operators.

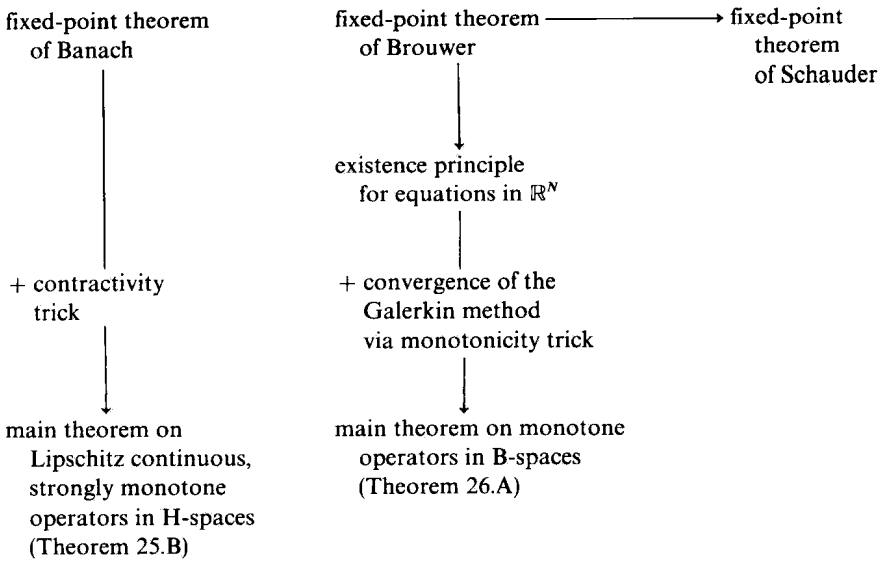


Figure 25.1

The special case of variational methods will be considered in Chapter 25. In this connection, the solutions of the convex minimum problem

$$f(u) = \min!, \qquad u \in X,$$

are solutions of the monotone operator equation

$$f'(u) = b, \qquad u \in X,$$

which generalizes the classical Euler equation.

(ii) In Chapters 30 through 33 we consider nonstationary problems, i.e., we study evolution equations of first and second order

$$u^{(n)} + Au = b, \qquad n = 1, 2,$$

together with applications to quasi-linear parabolic and hyperbolic equations.

In (i) and (ii), the operator A is monotone or, more generally, pseudo-monotone. Here, the notion of *maximal monotone operators* plays a fundamental role.

(iii) In Chapters 34 through 36 we investigate a general theory of discretization methods together with applications to Galerkin methods (inner approximation schemes) and difference methods (external approximation schemes).

In this connection, the notion of *A-proper operators* is crucial.

Basic Ideas of the Theory of Monotone Operators

> Riemann has shown us that proofs are better achieved through ideas than through long calculations.
>
> David Hilbert (1897)

> One can understand a mathematical statement, if one
>
> (i) can use it,
> (ii) has completely understood the proof, or
> (iii) one can independently find the proof again at any time.
>
> Only when one reaches the third step can one speak of understanding in a real sense.
>
> Aleksander Ostrowski (1951)

> Male lovers sometimes lack experience before their fortieth year, and later on, there is often a lack of opportunity. In the same way, younger mathematicians often lack the knowledge, and older ones lack the ideas.
>
> Wilhelm Blaschke (1942)

The theory of nonlinear monotone operators is based on only a few tricks. For the convenience of the reader, we summarize these tricks here. This way, we want to make the proofs of the main results as transparent as possible. In the following Chapters 26 through 36, the items (1), (2), etc. below, will be quoted as (25.1), (25.2), etc.

The theory of nonlinear monotone operators generalizes the following elementary result. We consider the real equation

$$\text{(E)} \qquad F(u) = b, \qquad u \in \mathbb{R},$$

and assume that:

(i) The function $F: \mathbb{R} \to \mathbb{R}$ is monotone.
(ii) F is continuous.
(iii) $F(u) \to \pm\infty$ as $u \to \pm\infty$.

Then, for each $b \in \mathbb{R}$, equation (E) has a solution. If F is strictly monotone, then the solution is unique (cf. Fig. 25.2).

This classical existence theorem follows from the intermediate value theorem of Bolzano, whereas the uniqueness statement is obvious.

In particular, if $f: \mathbb{R} \to \mathbb{R}$ is convex and C^1, then $F = f'$ is monotone, and (E) with $b = 0$ is the Euler equation to the minimum problem

$$\text{(M)} \qquad f(u) = \min!, \qquad u \in \mathbb{R}.$$

This observation is the key to the application of variational methods in the theory of monotone operators. However, note that the general theory of monotone operators concerns operator equations which are not necessarily the Euler equations of extremal problems.

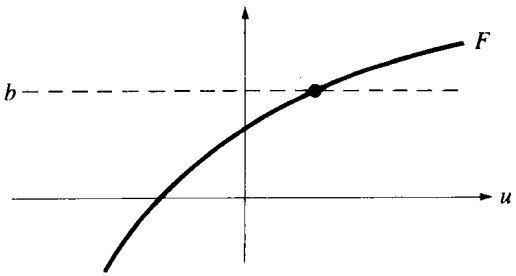


Figure 25.2

We now want to generalize the result above to monotone operator equations of the form

(E*) $$Au = b, \qquad u \in X.$$

Suppose that:

(i*) The operator $A: X \to X^*$ is monotone on the real reflexive B-space X, i.e.,

$$\langle Au - Av, u - v \rangle \geq 0 \qquad \text{for all} \quad u, v \in X.$$

(ii*) A is hemicontinuous, i.e., the map

$$t \mapsto \langle A(u + tv), w \rangle$$

is continuous on $[0, 1]$ for all $u, v, w \in X$.

(iii*) A is coercive, i.e.,

$$\lim_{\|u\| \to \infty} \frac{\langle Au, u \rangle}{\|u\|} = +\infty.$$

Then the main theorem on monotone operators (Theorem 26.A) tells us the following: For each $b \in X^*$, equation (E*) has a solution.

By Theorem 32.H, this fundamental result remains true if we replace (iii*) with the weaker condition that A is weakly coercive, i.e.,

$$\lim_{\|u\| \to \infty} \|Au\| = \infty.$$

If, in addition, A is strictly monotone, i.e.,

$$\langle Au - Av, u - v \rangle > 0 \qquad \text{for all} \quad u, v \in X \quad \text{with } u \neq v,$$

then the solution of (E*) is unique.

The result for equation (E) above is a special case of this theorem. In this connection, set $X = \mathbb{R}$ and

$$F(u) = Au.$$

Note that $X^* = \mathbb{R}$ and

$$\langle Au - Av, u - v \rangle = (F(u) - F(v))(u - v)$$

此为试读,需要完整PDF请访问: www.ertongbook.com