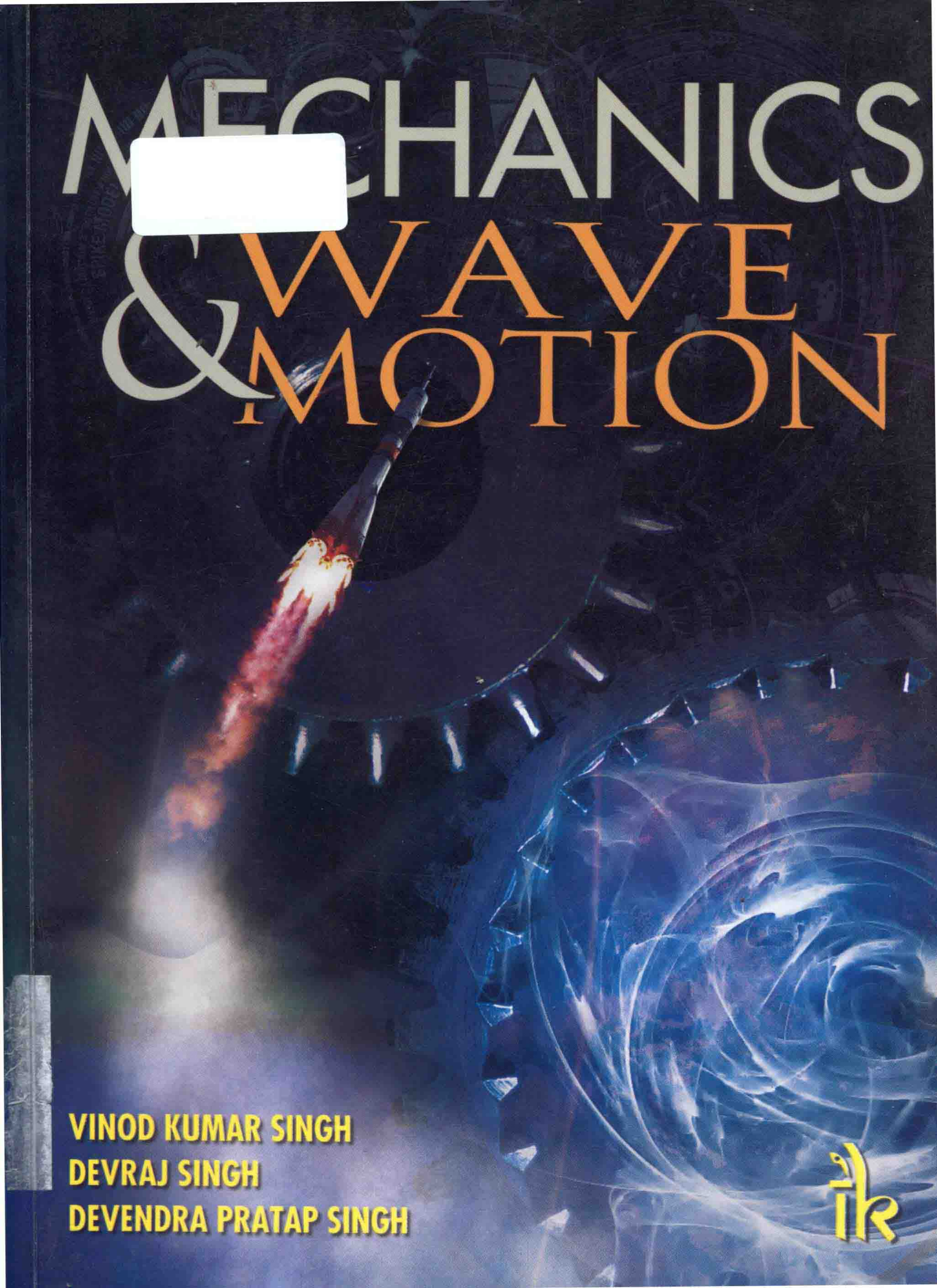
MECHANICS & WAVE MOTION



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Preface

Recently, most of the U.P. state universities have adopted uniform syllabus for three-year degree courses proposed by the University Grants Commission, New Delhi.

This book is intended as a text for a course on **'Mechanics and Wave Motion'** for the first year B.Sc. Students of U.P. state universities and covers the entire revised course as prescribed.

In the book, the subject matter has been divided into four units and efforts have been made to present the topics in a systematic way starting from basic concepts keeping in mind the actual difficulties of students. The presentation of topics is lucid, fluent and simple.

According to the latest trend of question papers of various universities, short and long answer type questions, numerical problems and multiple choice questions have been included at the end of each chapter. A large number of solved examples are provided for better grasp of the subject.

We feel highly obliged to numerous authors and publishers of various books which we have often consulted.

We don't find adequate words to express our gratitude to our family members, friends and well-wishers for their love, cooperation and constant encouragement without which this work would have never been accomplished.

Above all, we must thank the almighty without whose grace nothing is possible for us.

Despite utmost care, some misprints and omissions might have crept in. We shall be highly thankful to the readers who point out any errors and communicate for further improvement of the book. Please feel free to write to us.

Authors

Contents

Unit I

CHAPTER 1

Laws of Motion and Inertial Frame

1.1 INTRODUCTION

Mechanics is the study of the motion of objects. When we are concerned with the motion of the objects, we are dealing with that part of mechanics which is called kinematics. When we relate motion to the forces associated with it and to the properties of the moving objects, we are dealing with dynamics. The laws of motion and the force laws taken together constitute the law of mechanics.

Classical mechanics deals with great accuracy the problems associated with space-time invariance. But for the particle of linear size less than 10^{-6} m and large velocities comparable to speed of light, classical mechanics fails. Here, the mass of the particle m_0 is not invariant rather it changes with its speed v as

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where c is the speed of light. Thus, classical mechanics is true only when the speed of the object $v << c$.

The formulation of mechanics is rightly accorded to Isaac Newton. Newton translated various physical observations on the motion of bodies into a compact mathematical theory. The Newtonian theory of mechanics is condensed in three laws of motion.

1.2 NEWTON'S LAWS OF MOTION

Newton's laws of motion are given as follows:

First law (Law of Inertia)

Unless and until an external force is applied on a body, the body must maintain its state of rest or of uniform rectilinear motion.

Second law

The rate of change of momentum of a body is directly proportional to the force applied on it and the change takes place in the direction of force.

If an external force $\vec{F}$ is applied on a body of mass m whose instantaneous velocity is $\vec{v}$ at time t, then

$$\vec{F} = \frac{d\vec{P}}{dt} \qquad ...(1)$$

where momentum $$\vec{P} = m\vec{v} \qquad ...(2)$$

Thus, $$\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt}$$ [since m is constant in Newtonian mechanics.]

or, $$\vec{F} = m\vec{a} \qquad ...(3)$$

Therefore, the second law states also that by application of a force on a body, acceleration is produced in the body.

If force $$\vec{F} = 0, \vec{a} = 0$$

or, $$\frac{d\vec{v}}{dt} = 0$$

or, $$\vec{v} = \text{constant}$$

i.e., in the absence of external force, the body remains in its initial state of rest or of uniform motion which is Newton's first law. Thus, we can say that Newton's first law is a special case of the second law.

Third law

To every action there is an equal and opposite reaction. Action and reaction act on different bodies.

If a body '1' exerts a force $\vec{F}_{12}$ on the body '2' then the reaction force $\vec{F}_{21}$ will be exerted on the body '1' by body '2', i.e.,

$$\vec{F}_{12} = -\vec{F}_{21}$$

Limitations

1. First and second laws are valid only when the frame of reference is not accelerated.
2. If m is variable then the second law ($\vec{F} = m\vec{a}$) would not hold good, e.g., in the case of a rocket, a falling rain drop, etc.
3. Actions and reactions are equal and opposite provided both are measured at the same time. Since forces have a finite propagation velocity therefore, action

and reaction do not communicate simultaneously to each other rather take finite interval of time. So, at any instant $\vec{F}_{12}$ is not exactly equal to $\vec{F}_{21}$. This effect is significant for particles of atomic dimensions. This is the limitation of the third law.

Example 1 A soldier fires 50 gm bullets from a rifle, each at a speed of 1.0 km/sec. If he fires 10 bullets in 2 seconds what average force does he exert against the rifle, during this period ?

Solution According to Newton's second law of motion:

Average force exerted against the rifle = The rate of change of momentum of the rifle

$$= (\text{momentum of each bullet}) \times (\text{number of bullets fired in 1 second})$$

$$= (0.05 \text{ kg} \times 1000 \text{ m/sec.}) \times \left(\frac{10}{2}/\text{sec}\right)$$

$$= 250 \text{ newton}$$

Example 2 If two mutually perpendicular forces of magnitude 3 newtons and 4 newtons are acted on a body of mass 2.5 kg, find the magnitude and direction of the acceleration produced.

Solution Let the resultant force $\vec{F}$ make an angle θ with the X-axis.

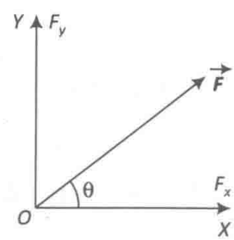

Fig. 1.1

Component of the force along X-axis, $F_x = F\cos\theta$ ...(1)

and component of the force along Y-axis, $F_y = F\sin\theta$ ...(2)

Squaring and adding Eqs. (1) and (2), we get

$$F = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9 + 16} = \sqrt{25} = 5 \text{ newtons}$$

Thus, acceleration

$$a = \frac{F}{m} = \frac{5 \text{ newtons}}{2.5 \text{ kg}} = 2 \text{ m/sec}^2$$

on dividing Eq. (2) by Eq. (1), we get

$$\tan\theta = \frac{F_y}{F_x} = \frac{4}{3}$$

or, $$\theta = \tan^{-1}(4/3)$$

Thus, acceleration makes an angle of $\tan^{-1}(4/3)$ with X-axis.

1.3 FRAME OF REFERENCE

A coordinate system attached to a rigid body with respect to which the position of a particle is described, is known as frame of reference.

For describing the motion of a particle, we may choose Cartesian coordinate system as a frame of reference. [Fig. 1.2]

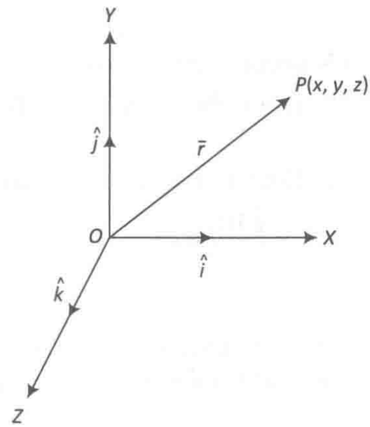

Fig. 1.2

The position vector of the particle P,

$$\vec{r} = \overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$$

The velocity of the particle,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\,\hat{i} + \frac{dy}{dt}\,\hat{j} + \frac{dz}{dt}\,\hat{k}$$

and the acceleration,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\,\hat{i} + \frac{d^2y}{dt^2}\,\hat{j} + \frac{d^2z}{dt^2}\,\hat{k}$$

For the determination of the position of an event, we need the specification of the time of occurrence of the event also. Thus, four coordinates (three position coordinates x, y, z and a time coordinate t) are required. This four-dimensional coordinate system is known as space-time frame of reference.

Inertial Frame of Reference

In a frame, if external force on a body is zero i.e., its acceleration $\vec{a} = \frac{d^2\vec{r}}{dt^2} = 0$, the frame is called an inertial frame of reference. In such frames, Newton's first and second laws hold good.