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BERNARD
BOLZANO

THEORY OF SCIENCE

VOLUME ONE

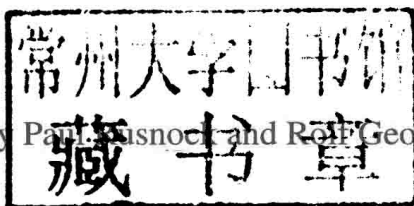
Translated by
PAUL RUSNOCK AND ROLF GEORGE

Bernard Bolzano

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Bernard Bolzano

To my parents (PR) and to the memory of Angus Kerr-Lawson

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GENERAL INTRODUCTION

Bernard Placidus Johann Nepomuk Bolzano was born in Prague on 5 October, 1781. His early education, at home and at the Piarist *Gymnasium*, was conducted in the spirit of the Josephinian Enlightenment (named after the second Habsburg Emperor of that name), which emphasised utility, practical morality, and a somewhat pedantic concern with the common good. Bolzano later wrote of his father that he was a man with “a genuine reverence for God, purified of every trace of superstition, courage that even the hardest blows of fate could not bend, a lively compassion, a patriotism that was not feigned, but rather grew from a well-organised love of mankind, and diligence that would not allow him to spend even a single hour without useful occupation.”¹ He could have used these words to describe himself.

Beginning in 1796, Bolzano attended the Charles University, where he studied philosophy, mathematics, and theology. He was especially drawn to the “purely speculative” part of mathematics, “that part of it which is at the same time philosophy.” By this he meant proofs for opinions which everyone already holds, “seeking out the grounds on which our judgements rest.”² His interest, in short, lay with the foundations of mathematics, and his first published work, completed towards the end of his doctoral studies, accordingly dealt with some aspects of elementary geometry. In philosophy, his preferred subject was logic. As was then the vogue, he also spent a good deal of time studying Kant but, although he acknowledged his debt on several occasions, he remained critical of the Kantian system and opposed to many of its tenets. It seems that none of his teachers had a profound or lasting influence upon his opinions nor, according to his own testimony, was there any philosophical system which he took to be the only true one, or for which he harboured much admiration.³

In 1804, soon after he had finished his studies, Bolzano applied for two jobs at the Charles University, one the chair in mathematics recently vacated by his teacher Stanislas Vydra, the other a newly instituted chair in the science of religion (he also applied for a position at a secondary school in Prague). A strong candidate for both university positions, Bolzano expressed his preference for the latter, citing his interest in working for the betterment of society.

¹*Lebensbeschreibung des Dr. B. Bolzano*. ed. M. Fesl, (Sulzbach: Seidel, 1836), p. 5.

²*Ibid.*, p. 19.

³From a brief manuscript entitled “Zur Lebensbeschreibung,” *Bernard Bolzano-Gesamtausgabe* (Stuttgart-Bad Cannstatt: Frommann-Holzboog, 1969–), hereafter **BBGA**, Series 2A, Vol. 12/1, p. 67.

Soon after, he was appointed to the new professorship in the science of religion, the chair in mathematics going to his friend Ladislav Jandera.

Bolzano saw in the new post exactly what he had longed for: an opportunity to present an enlightened view of religion to the educated classes and thereby to contribute to the common good by spreading better conceptions. This was far from what the authorities had had in mind when instituting the new chairs, however. For, by this time, Joseph II was long dead (†1790), and the current Emperor, Francis, rather than championing the cause of enlightenment as Joseph had, was keen to stamp it out, believing that it inevitably led to unrest and revolution. The new professors of religion were expected to make obedient subjects of their students rather than to encourage independent thought among them, and Bolzano was accordingly instructed to give interpretations of religious dogma in his lectures based upon a book by the Emperor's confessor, Jakob Friedrich Frint (he was also expected to read homilies on Sundays and holidays, hear confession, etc.). It must be understood that Bolzano was a devout man, and that his opposition to irreligion was as unfaltering as anyone could hope. At the same time, the contrast between the simplistic Catholicism which was officially enforced and Bolzano's rational faith was bound sooner or later to lead to conflict.

The foundation of Bolzano's ethics was a version of the principle of utility, and his understanding of religion was a natural outgrowth of his ethical convictions. Religion, he wrote at one point, is "the sum of such doctrines or opinions which have either a detrimental or a beneficial influence upon the virtue and happiness of a man",¹ and a proposition was declared to be religious if its consideration "not only moves us in our heart to declare either for or against it, but if through the acceptance or rejection of this proposition our virtue or happiness is altered."² By virtue, Bolzano means "the persistent striving to make the sum of pain in this world as small as possible, and to enlarge the sum of well-being as much as possible."³ Revealed doctrines, as he explained elsewhere, must also pass the test of the principle of utility. For since God only wills what is good, anything that He has revealed must, if believed by human beings, promote their virtue and happiness.⁴ Revelation

¹*Lebensbeschreibung*, p. 199.

²Bolzano, *Lehrbuch der Religionswissenschaft*, Sulzbach 1834, Vol. I, p. 60.

³Homily on the first Sunday of Advent, 1810, in *Erbauungsreden*, Vol. IV, Prague-Vienna 1852, p. 19.

⁴"Über das Recht der Geistlichkeit, ihren Lebensunterhalt von Personen zu beziehen, welche nicht ihres Glaubens sind. [On the Right of the Clergy to obtain their Livelihood from Persons not of their Faith] Eine Abhandlung nach B. Bolzanos Ansichten von einem seiner Schüler bearbeitet." [actually written by Bolzano] *Freimüthige Blätter* (Stuttgart, 1838)

is thus not only compatible with reason on his view, but must be confirmed by it. This position had political consequences which Bolzano did not shy away from. One cannot, for example, appeal to revealed religion in order to support institutions that demonstrably harm the common good, for once we have established that they are harmful, we prove by the same token that God could not have spoken in favour of them. And if the harm of an institution is great enough, he explained in another sermon, one may even be justified in disobeying lawful authorities.¹ This was most decidedly not the sort of preaching the Emperor and his circle expected.

Bolzano was professor of the science of religion from 1805 to the end of 1819. His weekly sermons became immensely popular, frequently drawing large numbers of listeners, and resulting in a kind of movement, sometimes called the “Bohemian Enlightenment”, which combined a rationally clarified Catholic faith with an ambitious programme for social and political reform. It was partly this popularity, partly the general ferment of the Napoleonic wars, which kept him in this position for such a long time.

His dismissal was part of a purge of unreliable elements, freethinkers, nationalists, and progressives, which took place in Germany and Austria after the assassination of the conservative playwright and diplomat August von Kotzebue. Charges of heterodoxy and political unreliability had been placed against Bolzano much earlier, and personal grievances also seem to have played a role. As early as 1806, Frint had complained that his book did not sell well in Prague, and later Bolzano was expressly asked to justify himself for lecturing from his own notes rather than from Frint’s treatise. Eventually, presentations were made to the Emperor, and objectionable passages were excerpted from his writings. The most offensive of these came from a volume of sermons of 1813:

Each century furnishes us . . . with new proofs of how harmful war is; of the abuses which certain social institutions inevitably lead to; under which constitutions the people are better off. And should it be impossible for our God to make us all wiser through this, to finally open our eyes, so that we will recognise with wonder, how easily we might have had things better all along? O! he

Vol. 11, pp. 291–331 and Vol. 12, pp. 5–47, §8 (Vol. 11, p. 313). English translation in B. Bolzano, *Selected Writings on Ethics and Politics* (Amsterdam: Rodopi, 2007), p. 153–154). Cf. “Von den Mißbräuchen der Religion,” [On abuses of religion] *Erbauungsreden*, Vol. 3 (Prague and Vienna, 1851), no. 13.

¹“On Duties towards Unjust Authorities,” *Selected Writings on Ethics and Politics*, pp. 85–95. See also “On the right of clergy . . .,” *ibid.*, pp. 153–167.

can certainly do that, our God; he will certainly make it happen. There will come a time—I say this with complete confidence—there will come a time when war—that absurd attempt to prove one's right by force—will be looked upon with the same disgust that duelling is now! There will come a time when all the thousandfold divisions and distinctions of rank between people, which bring about so much evil, will be put back within their proper bounds, so that each will deal with his neighbours as a brother with his brother! There will come a time when constitutions will be introduced which are not open to the horrible abuses which our present one is; a time . . . when no one will think himself deserving of honour and respect because he, a single person, has taken for himself as much as would be sufficient to satisfy the needs of a thousand!¹

Saurau, then Chancellor, maintained that Bolzano's "innovations" could not be justified. He pointed out that in German universities, where professors must live on students' fees, new doctrines are a necessity; but in Austria, professors are paid by the state, "so that they teach propositions that are approved by the Church and the civil administration. It is a dangerous error for a professor to think that he can instruct the youth entrusted to his care according to the drift of his individual convictions or according to his own views."²

An imperial decree dismissing Bolzano was issued on 24 December, 1819; it forbade him to teach or preach in public; ecclesiastic charges against him were ordered to be laid at once. The resulting proceedings did not come to a conclusion until 1825, when Bolzano dictated a final response to the charges, withdrawing none of his earlier utterances, but expressing regret about any evil consequences that might have resulted from their being misunderstood.

From 1823 until 1841, he spent much of his time on the estate of his friends Anna and Joseph Hoffmann, in Těchobuz, a village near Tábor in southern Bohemia. During these years, freed from his academic duties and removed from public life, he was a prolific author, writing a book on immortality to console Anna Hoffmann after her daughter Karoline had died, an autobiography, and numerous works on mathematics, philosophy and religion.³

¹Bernard Bolzano, *Erbauungsreden 1813*, BBGA, Series 1, Vol. 2 (ed. J. Loužil, Stuttgart-Bad Cannstatt, 1985), p. 83–84.

²Quoted after E. Winter, *Der Bolzanoprozeß*, Brno 1944, p. 35 f.

³*Athanasia, oder Gründe für die Unsterblichkeit der Seele* (Sulzbach, 1827; 2nd ed.

A significant part of the twenties was spent in carrying out a plan conceived some years earlier, of writing a logic. He had concerned himself with the subject from the beginning of his academic career, and his mathematical studies, which continued throughout the years of his professorship, only deepened his interest. In one of his earliest publications,¹ he wrote that “a discussion of mathematical method is basically nothing but logic, and hence does not belong to mathematics.” He must already have had in mind logic as a methodology of the sciences, a *Wissenschaftslehre*. A few years later, he recorded in a notebook his intention “to publish a logic under the title *Essay Concerning a New Logic, Which Would Necessitate a Restructuring of All Science; Offered for Examination to All Friends*. [. . .] The first chapter should be: There are truths (§1 concept of truth); the sense ordinary people attach to it; not what philosophers quite unnaturally call *subjective*, but what they (confusingly) call objective truth. Chapter two: We know several truths. Chapter three: Sometimes we commit errors. Chapter four: Making certain (purpose of ordinary, not of scientific method). Chapter five: There is an objective connection between truths. [. . .] Chapter six: It is sometimes possible to indicate this objective connection. Chapter seven: Scientific method. Chapter eight: Different kinds of truths or judgements. Chapter nine: various ways truths are connected, etc.”² He appended the remark that probability judgements must be discussed, a proof that there are synthetic judgements must be included, and plenty of examples are to be provided.

Thus many of the central thoughts of the *Theory of Science* were already present; especially noticeable, aside from his recognition of truths in themselves and an objective order among them, is his resolute opposition to the Kantian maxim that logic must under no circumstance be envisaged as an organon. From the very beginning, Bolzano made no distinction between logic and scientific method; his aim was to provide a method that would aid inquiry and lead to better grounded presentations of all sciences.

Work on the *Theory of Science* occupied most of the decade of the 1820s,

Sulzbach, 1838); *Lebensbeschreibung des Dr. B. Bolzano* (Sulzbach, 1836); *Von dem besten Staate* (around 1830; first printed Prague, 1931; new ed. in **BBGA**, Series 2A, Vol. 14; English translation in *Selected Writings on Ethics and Politics*); *Dr. Bolzano und seine Gegner* (Sulzbach, 1839); *Ueber die Perfektibilität des Katholizismus*, an exchange of letters with A. Stoppani (2 Vols. Leipzig, 1845; new edition in **BBGA**, Series 1, Vols. 19/1-19/2), among many others.

¹ *Beyträge zu einer begründeteren Darstellung der Mathematik* (Prague, 1810), II, §1; hereafter *Contributions*. An English translation is available in *The Mathematical Works of Bernard Bolzano*, ed. and tr. S. B. Russ (Oxford: Oxford University Press, 2004).

² **BBGA**, Series 2B, Vol. 16/1, p. 34–35.

and by 1830 the manuscript was complete. During the next several years only a few alterations were made, and the work was published in 1837.¹

After the completion of the *Theory of Science*, Bolzano returned to his mathematical work. He intended to write a comprehensive treatise (to be entitled *Größenlehre*, or *Theory of Quantities*) providing foundations for all of contemporary mathematics. Due to his declining health in the 1840s, he was unable to finish this work, though hundreds of pages of his manuscripts survive.² His theory of the infinite, which he discussed at length with his student and friend František (Franz) Příhonský, was published by the latter in 1851.³ Although Bolzano never had much opportunity to discuss mathematical problems, nor the benefit of professional contacts with students or mathematicians of rank, his contributions are impressive.⁴ The most outstanding are perhaps the following: he defined convergence for infinite series and indicated convergence criteria several years before Cauchy. He formulated the Bolzano-Weierstraß theorem (every infinite set of points contained in a closed, bounded interval has an accumulation point in the interval) and used it to prove central theorems of analysis. He constructed a continuous, nowhere differentiable function. Finally, he realised that any infinite set contains a subset that can be mapped one to one onto it; more importantly, he saw that this was not a contradiction. Bolzano's work is outstanding for its conceptual precision. Here as everywhere else, he is remarkable because he refused to accept what he had not carefully proved, and because of his close critical examination of received theories.

During the late 1830s and the 1840s efforts were made to publicise Bol-

¹Bolzano continued to draft revisions to his work after the manuscript left his hands. Many of these are preserved in manuscripts that have been published in the Bolzano *Gesamtausgabe*: “Zusätze oder Verbesserungen zur Logik” and “Verbesserungen und Zusätze zur Logik,” **BBGA**, Series 2A, Vol. 12/2.

²These are being published in the *Gesamtausgabe*: **BBGA**, Series 2A, Vols. 7–10. English translations of some of these writings may be found in S. B. Russ ed. and tr., *The Mathematical Works of Bernard Bolzano* (Oxford: Oxford University Press, 2004).

³B. Bolzano, *Paradoxien des Unendlichen*, Leipzig, 1851. New editions by Alois Höfler (Leipzig: Meiner, 1921, Hamburg, 1955) and Christian Tapp (Hamburg, 2012). English translations by D. A. Steele (*Paradoxes of the Infinite* [London: Routledge and Kegan Paul, 1950]) and S. B. Russ (*The Mathematical Works of Bernard Bolzano*). Příhonský also wrote a critical assessment of Kant based on Bolzano's principles: *Neuer Anti-Kant* (Bautzen, 1850; new ed. St. Augustin: Academia, 2003).

⁴For discussions of Bolzano's mathematical works, see A. Behboud, *Bolzanos Beiträge zur Mathematik und ihrer Philosophie* (Bern: Bern Studies in the History and Philosophy of Science, 2000); P. Rusnock, *Bolzano's Philosophy and the Emergence of Modern Mathematics* (Amsterdam: Rodopi, 2000); J. Sebestik, *Logique et mathématique chez Bernard Bolzano* (Paris: Vrin, 1992).