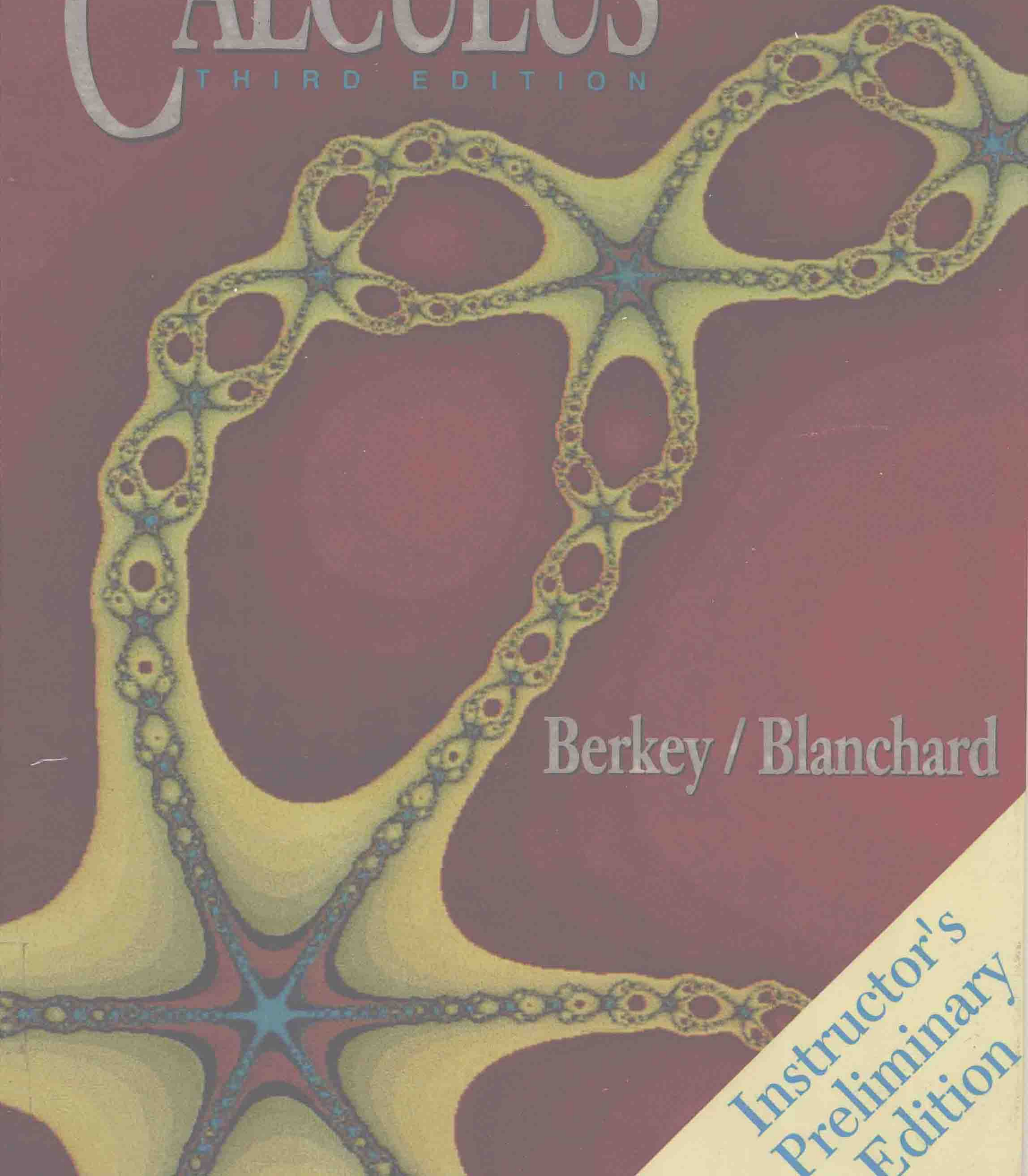
CALCULUS

THIRD EDITION



Berkey / Blanchard

Instructor's Preliminary Edition

Calculus

THIRD EDITION

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Please note that this Instructor's Review Copy has been prepared for review and examination purposes only. This preliminary edition does not contain the answer section so that we can subject it to another rigorous error check to ensure the accuracy of the Student Edition. Final bound books of the Student Edition will be available May 1, 1992. To order the Student Edition through your bookstore, use the following ISBN: 0-03-046927-9. For further information o[redacted]tive or the Saunders sales office: 1-800-2[redacted]7-2665.

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About the cover: The cover is a small-scale study of the convergence of Newton's Method applied to the solution of a cubic polynomial in the complex plane. For this particular polynomial $z - 1 = 0$, the initial points that do not lead to a root form the Julia set of this iterative algorithm. The Julia set is the fractal highlighted in this picture. Details and related computer experiments are given in Appendix IV.

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Preface

This text is intended for use in a traditional three-semester or four-quarter sequence of courses on the calculus, populated principally by mathematics, science and engineering students. It reflects our philosophy that calculus should be taught so as to produce skilled practitioners who understand the mathematical issues underlying the techniques they have acquired. It is written in order to provide students of widely varying interests and abilities with a readable exposition of the principal results, including ample motivation, numerous well-articulated examples, a rich discussion of applications, and a serious description and use of numerical techniques. In particular, we explain how calculators and computers can be used to illustrate the theory, to provide approximate solutions to problems on which more elegant techniques fail, and to relieve the tedium of extensive computations.

Changes for the Third Edition

Content

The Table of Contents of this edition has not changed substantially from that of the Second Edition. The text, however, has been extensively revised and edited. We have expanded our treatment of numerical methods, and we have entirely rewritten the discussion of several key topics. In particular, Chapters 3 and 4 on the derivative and its applications have been significantly revised to place greater emphasis on the derivative as a rate of change and to emphasize its role in extremal problems. Our introduction to the definite integral in Chapter 6 has been entirely rewritten to emphasize the definition as a limit of Riemann sums and to streamline the treatment of the Fundamental Theorem of Calculus. In Chapters 16 and 17 we have reorganized the material on vectors, curves, and surfaces to highlight the geometric issues involved. Finally, we added a brief section on Jacobians at the end of Chapter 19 on multiple integration in response to user requests. For more information about these changes and the order in which various topics can be treated using this text, see our brief guide, *Notes to the Instructor,* immediately following this preface.

Exercises

Over 1000 new exercises designed to challenge the student's understanding have been added to this edition. Many of these are a result of our consideration of the ''lean and lively'' calculus reform movement and a desire on our part for the student to have geometric and numerical as well as algebraic familiarity with the ideas of calculus.

Artwork

Over 20% of the figures in this text are new to this edition. All of the artwork has been redrawn by computer in a four-color format with careful attention to the use of color as a pedagogical tool. (A guide to the pedagogical use of color is given on page xix.) Much of the art has been rendered using the computer-based mathematics system *Mathematica* to provide greater accuracy and more realistic three-dimensional images.

Increased Utilization of Technology

Graphing Calculator Exercises: Over 300 graphing calculator exercises have been added to the third edition. These problems (marked with the logo) can be used with any graphing calculator currently available and have been carefully selected to explore essential concepts of calculus in more depth, not merely to teach students how to push buttons. In fact, many of these problems are actually mini-projects, rather than exercises. Of course, these exercises can also be done with an appropriately equipped personal computer.

For those who need more information about getting started, Appendix V, Calculus and the Graphing Calculator, discusses various graphing calculators and their utility for learning calculus. This appendix includes instructions, explanations, and programs for the Casio, TI-81, HP-28S and HP-48S graphing calculators.

Appendix I on Calculus and the Computer: Appendix I has been significantly revised for this edition. It now contains Pascal versions of the BASIC computer programs previously offered as well as a section that discusses how to use *Mathematica* to illustrate many of the fundamental concepts of calculus.

Appendix IV: We have added a short appendix on Newton's Method in the complex plane. It discusses interesting aspects of the method that are usually ignored in the standard treatment of Newton's Method in calculus courses. More importantly, this appendix gives the interested student a glimpse of an area of current mathematical research.

Features

Even though this revision has been substantial, the distinctive approach, flexibility, and pedagogy of the previous editions have been retained.

Approach

We present the traditional calculus curriculum using informal discussion and geometric arguments whenever possible. Chapter 2 begins with a flexible introduction to the limit, providing both informal and formal discussions of limits. We develop the derivative in Chapter 3 with an emphasis on rates and linear approximation.

Our short Chapter 5 on antidifferentiation precedes the introduction of the definite integral and includes material on differential equations. This chapter is positioned so that antidifferentiation can be treated either as the final application of the derivative or as the first topic in a unit on integration. This ordering reflects our concern that the definite integral is often presented almost simultaneously with the notion of the antiderivative and the Fundamental Theorem of Calculus. As a result, students may leave their first calculus course thinking of a definite integral as a difference between two values of an antiderivative that, coincidentally, might also be identified with the area of a planar region. By introducing antiderivatives first, together with their applications to solving simple differential equations, the distinction between antiderivatives and limits of approximating Riemann sums can be clearly established before the important connection provided by the Fundamental Theorem is introduced. This approach helps students grasp the notion of approximate integration and makes the development of the various applications of the definite integral more accessible.

We define the definite integral as a limit of Riemann sums (Chapter 6), and the natural logarithm as an integral (Chapter 8). We also discuss fully the principal theorems of the calculus, including the theorems of Green and Stokes, as well as the Divergence Theorem (all in Chapter 20).

Flexibility

The organization of this text continues to provide as much flexibility as possible for the individual instructor. We have resisted the current trend towards fewer (and longer) chapters so that instructors can conveniently treat topics in different orders if they so desire. In the Notes to the Instructor, we elaborate on these options.

Examples

Many of the over 800 examples, especially in the early parts of the text, use the two-column Strategy-Solution format. The ''Strategy'' column gives the student, in abbreviated form, a description of the principal steps involved in the more complete solution. This format helps students identify the more general aspects of the particular solution and develop problem-solving strategies of their own.

Chapter Summaries

Each chapter ends with a brief list of the key concepts of the chapter. Page references are included so that students can easily refer back to the material as they review.

Review Exercises

A comprehensive set of exercises reviewing all of the topics of the chapter follows the chapter summary. Students who work these review exercises can feel comfortable that they have mastered the chapter.

Historical Part Openers

Each of the seven parts of the text begins with an essay describing the key mathematicians whose discoveries underpin the material of that part. These essays illustrate how mathematics is a human endeavor carried out over centuries and continents.

Accuracy and the Saunders Solution

Because accuracy in mathematics texts is critical, this edition has been subjected to an exhaustive series of accuracy checks. Initially, corrections and suggestions from users of the second edition were incorporated in the reprint of that edition which subsequently became the basis for the third edition. The accuracy of the worked examples and the answers to the odd-numbered exercises were checked during manuscript, galleys and page proof stages by numerous professors as well as by us. All exercises were solved separately by the three professors who wrote the Instructor's Manual and Student Solutions Manual. Finally, a completely independent check of all the exercises was completed by four graduate students at Boston University.

Supplements

The extensive package in support of *Calculus,* Third Edition, reflects Saunders College Publishing's continuing commitment to provide the most helpful, thoughtful, and up-to-date ancillaries.

The **Instructor's Manual,** by Judy Coomes (William Paterson College), Dennis Kletzing (Stetson University), and Michael Motto (Ball State University), is available free to adopters. It contains complete detailed solutions to all the exercises in the text and is arranged in two volumes for convenience. To ensure accuracy, solutions figures are computer-generated, and selected solutions have been checked again using *Mathematica.*

The **Student Solutions Manual,** also by Judy Coomes, Dennis Kletzing, and Michael Motto, is available in two convenient volumes for students who need help with homework. They contain detailed solutions to the odd-numbered exercises; answers to these exercises are also given at the back of the text.

The **Test Bank** by Ken Kramer of CUNY, Queens College, offers over 1300 open-ended test questions corresponding in level and difficulty with the examples and exercises in the text. For each chapter there are four test forms of approximately 15 questions each. The three final exams are divided into two parts (for Chapters 1–11 and 12–21). Answers for each test are also provided and have been checked independently to ensure accuracy.

For adopters with IBM or Macintosh computers, this Test Bank is also available free in computerized form. The ***ExaMaster*™ Computerized Test Bank** software offers two easy, flexible ways to create and edit tests. Each version of the *ExaMaster* software comes with *ExamRecord*™, a gradebook program for each recording, curving, and graphing of grades.

A kit of **transparencies and transparency masters** of the most important figures from the text is available. The 25 four-color overhead transparencies illustrate essential figures that are difficult to draw accurately and effectively. In addition, there are 100 transparency masters of other important and useful figures chosen from throughout the text.

For more material on graphing calculators, we offer two manuals on using the HP-28 and HP-48S graphing calculators to enrich the teaching and learning of calculus. **Calculator Enhancement for Single Variable Calculus** by James H. Nicholson and J.W. Kenelly, consultant, and **Calculator Enhancement for Multivariable Calculus** by J.A. Reneke and D.R. LaTorre, consultant, provide procedures, calculator programs, examples and exercises designed to remove the computational burden normally associated with these courses. Each supplement helps students appreciate the geometrical and graphical aspects of calculus, master its theory and methods, and explore new topics and applications.

Calculus and *Mathematica* by John Emert and Roger Nelson of Ball State University is a computer laboratory manual of 30 projects to supplement the traditional calculus sequence. These experiments use the computer to lead a process of discovery, conjecture, and verification. The experiments cover a wide variety of topics, from the expected Newton's Method and Riemann Sum explorations to adventuresome investigations of symmetric derivatives, chaotic sequences, and global/local behavior.

Calculus and *Derive* by David Olwell and Pat Driscoll of the United States Military Academy contains almost 40 modules (exercises and projects) designed specifically to use the computer to illustrate symbolically and graphically the essential concepts of calculus and to eliminate the drudgery of computation. The average student will take 45 minutes to complete each exercise and four hours to explore each project.

CalcAide by Elizabeth Chang of Hood College provides 17 graphing and numerical utility software modules and is intended for use in the classroom, in a mathematics laboratory, or on students' own computers. CalcAide can draw the graph of most functions and provides numeric and graphic illustrations of many concepts of calculus. The flexible menu system allows you to graph a function on any portion of the coordinate plane, change some features of the graph or function while leaving others unchanged, examine value, slope or other features at a specific point on the graph, and offers many other options. (CalcAide requires an IBM-PC with CGA capabilities.)

Electronic Bulletin Board

The Department of Mathematics at Boston University is committed to excellence in teaching, and as part of this commitment, it has established on its computer system (math.bu.edu) a library of public domain teaching materials available by anonymous login over the Internet. Those who have access to the Internet can use anonymous ftp to retrieve these materials during off-peak hours. Up-to-date materials related to this text will be posted in this library. Those who have access to electronic mail but who do not have access to the Internet can obtain information related to these materials by sending electronic mail to berkey-blanchard@math.bu.edu.

Acknowledgments

Many individuals played instrumental roles in the development of this text. First, we would like to recognize all those who participated in the development of the first two editions. This edition relies heavily on the work of those individuals.

When we began to plan this revision, we already had many helpful suggestions from users of the second edition who kept diaries containing their impressions and comments. These individuals were:

Jake Beard, Tennessee Tech University
Patricia Burgess, Monroe Community College
Peter Collinge, Monroe Community College
M. Hilary Davies, University of Alaska—Anchorage
Edmond D. Dixon, Tennessee Tech University
John Dyer-Bennet, Carleton College
Carol Edwards, St. Louis Community College—Florissant Valley
George Feissner, SUNY–Cortland
Thomas Gruszka, Grand Valley State University
Douglas Kelly, University of North Carolina—Chapel Hill

Calvin Jongsma, Dordt College
Peter M. Knopf, Pace University
Michael Schneider, Belleville Area College
Lowell Stultz, Kalamazoo Valley Community College

These diaries identified topics that needed attention, and they helped us to refine every section in the text.

Ideas for major revisions also came from a focus group that was hosted by Saunders College Publishing in the summer of 1989. At that session, Scott Farrand (California State University–Sacramento), Michael Schneider (Belleville Area College), Bob McFadden (Northern Illinois University), and Ken Kramer (CUNY—Queens College) provided frank evaluations of our plans for this revision as well as a number of excellent suggestions of their own.

We benefited greatly from the considered opinions of the following mathematicians who read parts or all of the initial draft of the revision:

Shih-Chuan Cheng, Creighton University
H. Jay Davis, Rancho Santiago College
Scott Farrand, California State University—Sacramento
Francis G. Florey, University of Wisconsin—Superior
Wayne Dell Gibson, Rancho Santiago College
Fredric T. Howard, Wake Forest University
Elgin Johnston, Iowa State University
Dan Kemp, South Dakota State University
Peter M. Knopf, Pace University
John Kroll, Old Dominion University
Hendrik J. Kuiper, Arizona State University
Jim McKinney, California State Polytechnic University—Pomona
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Gloria Langer, University of Colorado
Giles Maloof, Boise State University
Len Miller, Mississippi State University
Allan Mundsack, Los Angeles Mission College
Hugo Sun, California State University—Fresno

All exercises were solved by the three professors who wrote the Instructor's Manual and Student Solutions Manual: Judy Coomes (William Paterson College), Dennis Kletzing (Stetson University), and Michael Motto (Ball State University). Answers to the exercises were checked independently by four graduate students at Boston University: Duff Campbell, Thomas LoFaro, Michèle Nichols, and Mario Casella.

The graphing calculator exercises and appendix were designed and written by James Angelos of Central Michigan University. His knowledge and experience with these remarkable devices is much superior to ours, and we are fortunate to be able to include his materials in this text. David Olwell of the United States Military Academy reviewed every graphing calculator exercise, and his comments helped us to refine several of them. The graphing calculator appendix was reviewed for accuracy by Thomas LoFaro and Duff Campbell of Boston University and Don LaTorre of Clemson University. The Test Bank was checked by Gloria Langer of the University of Colorado.

All of these individuals worked meticulously to ensure the accuracy of the text. Whatever errors might remain are, of course, our sole responsibility. We have worked hard to produce the most accurate text possible, but we welcome your comments on improving or correcting the text. Those who prefer to use electronic mail can reach us at berkey-blanchard@math.bu.edu.

The historical notes, which provide an important human contrast, were written by Duane Deal of Ball State University. They were reviewed by our colleague, Thomas Hawkins of Boston University.

We used *Mathematica* 2.0 to generate many of the three-dimensional figures in this edition, and we thank Wolfram Research, Inc. for providing a beta test copy of version 2.0 so that we could generate these pictures and test the code in the appendix using the current version of *Mathematica*.

The cover of this text is an image obtained in a computer study of Newton's Method, and it was computed at Boston University using *Citool,* a computer program written by Scott Sutherland, Gert Vegter, and Paul Blanchard. Mario Casella helped produce this image, and the final slides were shot by Laura Giannitrapani of the Graphics Lab at Boston University. The images in the appendix on Newton's Method were also produced using *Citool*.

We are greatly appreciative of the support that Saunders College Publishing has shown throughout this lengthy revision. They have enthusiastically provided the resources necessary to complete such a large project in a timely fashion. In particular, we are pleased to recognize the encouragement of Senior Mathematics Editor Robert Stern and Editor-in-Chief Liz Widdicombe. Our schedules made the production of this text especially complicated, and we wish to thank Senior Project Manager Sally Kusch, Manager of Art and Design Carol Bleistine, and Director of Editorial, Design, Production, and Manufacturing Tim Frelick for their extra efforts during the prolonged production of this project. Finally, we must make special note of the work done by Developmental Editor Alexa Barnes, who ably helped us through every step of this revision over a three-year period.

At Boston University, we were assisted by numerous colleagues in addition to those already mentioned. Lisa Doherty, Barbara Leonard, Diurka Rodriguez, and Angelique Thayer all helped manage the flow of materials between Boston and Philadelphia. Thomas LoFaro and Michèle Taylor helped revise many of the exercise sets, and Reza Behnam produced preliminary versions of many of the three-dimensional figures. Daniel Alexander helped with the production of preliminary *Mathematica* notebooks, which were incorporated into the final version of Appendix I. Thomas LoFaro and Farzan Nadim helped compile the accuracy reports and the answer manuscript.

On a more personal level, the writing of this text was supported by three very special groups of people. First, our students at Boston University, who have helped sharpen our thinking about teaching and calculus for more than a decade. Second, our colleagues in the faculty and administration of Boston University, who believe deeply in the importance of effective teaching. And, most importantly, our families—

Cristin, Aaron, Jessica, Eric, and especially Catherine and Dottie, who understood our need to write this book and who shared fully and willingly in the sacrifices that were required. To all we are truly grateful.

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January 1992

Notes to the Instructor

These notes provide a brief overview of the organization of this text. They also discuss options regarding possible rearrangement of topics.

Calculator Exercises: Before discussing the individual chapters, it is important to emphasize one issue regarding the calculator and, especially, the graphing calculator exercises. When assigning these exercises, the instructor must remember that these exercises usually take the student more time to complete than standard exercises. In particular, many of the graphing calculator exercises are closer to projects than to exercises. Consequently, we strongly recommend that instructors complete any graphing calculator exercise on their own before assigning it so that they are aware of exactly what is involved. The Instructor's Manual provides annotated solutions to these exercises.

We also note that these exercises are appropriate for personal computers, and in Appendix I we cite the graphing calculator exercises that make effective use of the routines provided there.

Finally, we also note that some of the exercises marked with a graphing calculator icon can also be solved with a programmable calculator.

Overall Organization: The general order of topics in this text is consistent with most popular calculus texts. We begin with a discussion of limits (Chapter 2), develop the derivative and its applications (Chapters 3 and 4), briefly discuss antidifferentiation (Chapter 5), develop the definite integral (Chapter 6), and discuss applications of the integral (Chapter 7). These chapters are covered at Boston University in our standard first-semester calculus course, and we usually must omit a few sections in a 13-week semester.

Next we discuss certain important transcendental functions: logarithms and exponentials (Chapter 8) and the inverse trigonometric functions (Chapter 9). We also develop the standard techniques of integration (Chapters 9 and 10). Then we provide a brief discussion of l'Hôpital's Rule and improper integrals (Chapter 11) in anticipation of their use in the analysis of sequences and series (Chapter 12). Taylor polynomials, Taylor's Theorem, and power series follow immediately (Chapter 13). At Boston University, we finish the second semester with the (optional) topic of conic sections (Chapter 14) and polar coordinates and parametric equations in the plane (Chapter 15). Again, we usually omit a few sections in order to complete these chapters.

Multivariate calculus can either start with a review of the material in Chapter 15 or with the discussion of vectors (Chapter 16). We continue the study of geometry by discussing vector-valued functions, curves, and surfaces (Chapter 17). At that point, we treat the standard topics of multivariate calculus: partial derivatives (Chapter 18), multiple integration (Chapter 19), and vector analysis (Chapter 20). These topics easily fill a complete semester.

Our text ends with a chapter on differential equations (Chapter 21), which is a brief treatment of many of the ideas that are developed in more detail in courses on differential equations.

Here are more specific details to aid with the writing of a syllabus:

Chapter 1: This chapter is a review of precalculus concepts. The sections that should be covered in detail depend very much on the class. In this edition, we have added a Readiness Test, which the students can use either as a self-study guide to help with their own review or as an indicator that the student is ready to study the calculus after a review of Chapter 1.

Chapter 2: Our treatment of the limit is designed to accommodate those instructors who desire a rigorous development as well as those instructors who are comfortable with a more intuitive approach. The ϵ-δ definition is discussed in Section 2.3, and the limit theorems are proved rigorously at the end of Sections 2.4, 2.5, and 2.6. Instructors who do not desire an ϵ-δ approach can simply skip Section 2.3 as well as the material at the ends of Sections 2.4 through 2.6.

Chapter 3: The derivative is developed in the standard manner in Sections 3.1 through 3.5. Then Section 3.6 discusses related rates immediately after the Chain Rule (Section 3.5). We prefer this early exposure to related rate problems for two reasons: it reinforces the fact that the Chain Rule involves rates of change, and it introduces the difficulties associated with setting up and solving word problems as early as possible. We realize that many instructors may prefer to treat this topic after discussing the remainder of Chapter 3. In fact, this topic along with linear approximation in Section 3.7 and Newton's Method in Section 3.10 is optional.

Chapter 4: We have completely reorganized this chapter so that applied extremum problems are discussed early in the chapter. The theory of critical points and extrema, both absolute and relative, is introduced in Section 4.1, and this theory is related to applied extremal problems in Section 4.2. Then Sections 4.3 through 4.5 discuss the Mean Value Theorem, monotonicity, and concavity. Section 4.6 treats limits at infinity and infinite limits. The material in Sections 4.1 through 4.6 is central to our presentation of calculus. Section 4.7 is a traditional section on curve sketching, and Section 4.8 uses the first derivative version of Taylor's Theorem to analyze the error involved in linear approximation. Either or both of these sections are certainly optional.

Chapter 5: This is a brief chapter on antidifferentiation and differential equations. The first two sections on antidifferentiation and substitution are crucial to the treatment in Chapter 6, but Section 5.3 (in which differential equations are introduced) can be omitted if desired.

Chapter 6: This chapter introduces the definite integral with a focus on area and Riemann sums and then presents the Fundamental Theorem of Calculus. In order to emphasize that the definite integral is a limit of Riemann sums, we briefly discuss approximation of integrals by the Midpoint Rule in Section 6.2. A more comprehensive discussion of numerical approximation procedures is the topic of the final section of this chapter (Section 6.6). The error analysis in that section is somewhat more geometric than the typical presentation.

Chapter 7: In this chapter, we present eight different applications of the definite integral, and here the instructor has much flexibility about what topics to include or omit.

Chapter 8: The natural logarithm and exponential are defined and developed using the definite integral and the Fundamental Theorem of Calculus. Section 8.1 begins with a brief review of a precalculus definition of the logarithm and moves on to a discussion of inverse functions. The amount of time that should be allotted to this section depends on the background of the class. Section 8.5 presents the application of this theory to exponential growth and decay problems.

Chapter 9: This chapter is a combination of a number of topics that involve the calculus of trigonometric functions. Sections 9.1 and 9.2 describe integration techniques for the trigonometric functions and their powers. Therefore, the instructor has some flexibility about how to treat these two sections based on the manner in which they plan to cover Chapter 10 (techniques of integration). Sections 9.3 and 9.4 present the inverse trigonometric functions and their calculus, so complete coverage of these two sections is important. Section 9.5 discusses the hyperbolic functions, and it can be safely skipped if the instructor so desires.

Chapter 10: We have kept our treatment of techniques of integration to a minimum, but those who are interested in a leaner presentation of the calculus can omit much of this material. However, we believe that integration by parts (Section 10.1) still merits careful treatment.

Chapter 11: This brief chapter discusses l'Hôpital's Rule and improper integrals, and both of these topics are necessary in what follows.

Chapter 12: This chapter on sequences and series is a standard presentation of these two related concepts. Most calculus sequences would contain a complete treatment of this chapter.

Chapter 13: The three concepts of Taylor polynomials, power series, and Taylor series are presented with a complete treatment of Taylor's Theorem. If Section 4.8 was covered earlier, the students benefit from being reminded that the ideas presented here are simply generalizations of the idea of linear approximation. This is another core chapter.

Chapter 14: In this chapter, we review briefly the properties of conics with an emphasis on their analytic description. In what manner and when an instructor covers this material should be determined by the backgrounds of the students. Much of this material can be treated earlier if the instructor so desires.

Chapter 15: There are two basic geometric topics in this chapter: polar coordinates and parametric equations in the plane. Again this is another case where the instructor has a great deal of flexibility as to when and how this material is treated. However, a solid understanding of these topics is crucial for students to succeed in multivariate calculus.

Chapter 16: This chapter has been reorganized to highlight geometric problems involving lines and planes. The entire chapter is crucial to the development of multivariate calculus.

Chapter 17: In this chapter, we introduce vector-valued functions, associated curves in space, and surfaces in space. The section on curvature (Section 17.5) can be safely omitted if the instructor desires. There is also merit to the idea of waiting until triple integrals are discussed in Chapter 19 to introduce cylindrical and spherical coordinates. However, knowledge of these coordinates provides natural examples of the multivariable Chain Rule, which is discussed in Section 18.6.

Chapter 18: This is a standard presentation of partial differentiation. Sections 18.1 through 18.7 are fundamental. Section 18.8 discusses constrained extrema and provides a nice application of the gradient. However, this section can safely be omitted if desired. Section 18.9 discusses the problem of reconstructing a function of two variables from its gradient. This technique is used in the study of vector fields (Section 20.1) and in the study of exact differential equations (Section 21.2). If desired, a discussion of Section 18.9 can be delayed until either of these two topics is presented.

Chapter 19: This is another standard chapter in multivariate calculus. In this edition, we have added an optional section (Section 19.8) on change of variables and Jacobians, and that section has been written so that it has two distinct parts. If an instructor wants to discuss Jacobians for functions of two variables immediately after discussing double integrals in polar coordinates (Section 19.3), the first part of Section 19.8 can be covered at that time. Then the remainder of this section can be completed after triple integrals in cylindrical and spherical coordinates are discussed (Section 19.7).

Chapter 20: This is a standard chapter on vector analysis which should essentially be covered in the order it is written. However, if an instructor has time to cover only Stokes' Theorem or the Divergence Theorem, either topic can be covered independent of the other.

Chapter 21: Our final chapter is a brief discussion of differential equations. Instructors who want to study differential equations earlier can cover many of these sections at appropriate times in the second semester course. For example, first-order linear equations (Section 21.1) can be discussed immediately following Chapter 8. The same is true for Sections 21.3, 21.4, and 21.6. Section 21.5 is an application of the theory of power series in Chapter 13. However, the topic of exact equations (Section 21.2) needs the material presented in Section 18.9 on reconstructing a function from its gradient.

Appendix I: This appendix illustrates ways in which the computer can be used as a tool in the study of the calculus. The first part of the appendix presents short programs written in BASIC and Pascal that are referenced throughout the exercise sets. The *Mathematica* routines in the second part of the appendix illustrate a number of ways the power of a computer-based mathematics system can be used in the teaching of calculus. Relevant graphing calculator exercises from the main part of the text are cited.

Appendix II: Here we complete a number of proofs as promised in the main exposition.

Appendix III: This is a brief summary of the properties of complex numbers.

Appendix IV: As mentioned in the preface, this appendix presents a brief introduction to Newton's Method in the complex plane as well as related computer graphics. Relevant computer projects are also presented.

Appendix V: Many of the graphing calculator exercises are more accessible if certain short programs are available. This appendix by James Angelos of Central Michigan University contains relevant programs and gives a brief introduction to the use of the graphing calculator in calculus. However, it is not meant to replace the more complete manuals on the use of the graphing calculator in calculus (see the descriptions of the Clemson Calculator Enhancement manuals for calculus mentioned in the *Preface*).

Appendixes VI, VII, and VIII: For easy reference, the remaining three appendixes provide tables of transcendental functions, a complete table of integrals, and useful geometry formulas, respectively.

Guide to the Pedagogical Use of Color

The new four-color art program has been computer-generated for increased clarity and accuracy. Color is used pedagogically and consistently to help students grasp key concepts. A guide to the system of colors with references to sample figures is given below.

Color	Purposes	Sample
Blue	primary curve	Chapter 6, p. 324, Figure 2.5
	Riemann rectangle above x-axis	
	positive area