GRADUATE STUDENT SERIES IN PHYSICS

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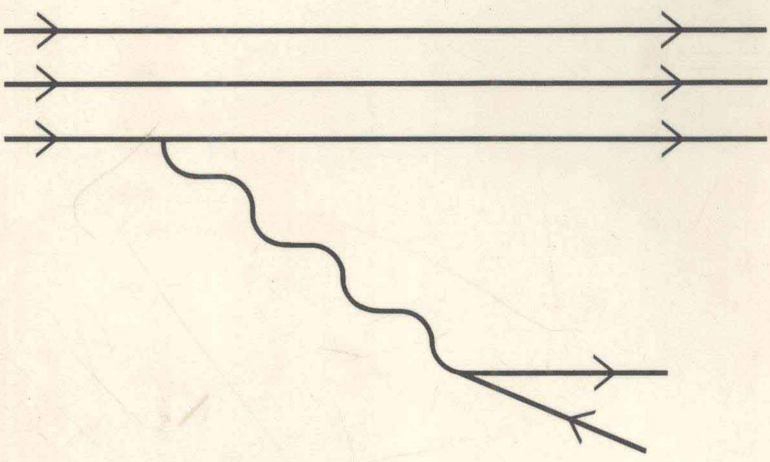
Weak Interactions



Second Edition

DAVID BAILIN

GRADUATE STUDENT SERIES IN PHYSICS

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WEAK INTERACTIONS

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Second edition

ADAM HILGER LTD, BRISTOL
Published in association with
the University of Sussex Press

British Library Cataloguing in Publication Data

Bailin, David
Weak interactions.—2nd ed. (Graduate student series in physics, ISSN 0261-7242)
1. Weak interactions (Nuclear physics)
I. Title II. Series
539.7'54 QC794.8.W4

ISBN 0-85274-539-7

First edition 1977
Second edition 1982

Published by Adam Hilger Ltd, Techno House, Redcliffe Way, Bristol BS1 6NX, in association with the University of Sussex Press

The Adam Hilger book-publishing imprint is owned by The Institute of Physics

Printed in Great Britain by J W Arrowsmith Ltd, Bristol

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PREFACE

rnerstone of the modern theory of weak interactions was laid by iman and Gell-Mann[1] in 1958. Their theory, "the universal V-A int-current theory", was itself a development from the pioneering of Fermi[2] twenty years earlier. For more than fifteen years the it -current theory was subjected to extensive experimental probing vived virtually unscathed. With the passage of time, developments areas of particle physics led to refinement of the theory; it was ted in terms of the strong interaction symmetry SU(3), and its was enhanced by the theoretical development and application of algebra. The discovery of CP-violation in 1964 was not predicted, magnitude of the observed violation is so small as to be negligible st areas of application of the theory. Thus today the theory is seen providing an economical description of the vast majority of low energy weak phenomena.

For this reason the first five chapters of this book are devoted to an exposition of this theory and its application to experiment. Several excellent and full books on Weak Interactions have appeared in recent years[3], so I have not attempted to be exhaustive. Rather, I have sought to provide an introductory text which illustrates how one does actual calculations for processes accessible to experimentalists. The emphasis is on theory and calculation rather than experimental measurement, but naturally the theoretical predictions are compared with the current experimental data whenever possible. In doing this I have aspired to emulate Okun upon whose admirable little book[4] was reared a generation of students, including myself.

It has been known since its inception that the Feynman Gell-Mann theory could not possibly be correct at high energies, but, with only relatively low energies accessible, this problem was left for theorists to worry about when they had nothing else to do. However, in 1973 (after this t started) an entirely new type of weak process was discovered whic be explained by the "old" theory: the new processes involve "neut currents", as distinct from the "charged currents" in the Feynmar Mann theory. The strength of the new interactions is comparable that of the well-established phenomena, so a unified explanation is called for.

The most natural framework in which to achieve this unificatior provided by quantum field theory. In any case, it is my own pr that elementary particles are described by operator fields, so I written this book using the formalism of quantum field theory. it I have particularly had in mind first and second year graduate of particle physics. Thus it seemed appropriate to include som on those aspects of elementary relativistic free field theory wh especially important in weak interactions. This is presented as a preliminary to the definition of the "leptonic current". S tions precluded any coverage of Feynman diagrams or the LSZ r formulae. I anticipate that most readers will be following in par course on these topics, if they do not already possess the minimal co tence required here. In any case, all of these topics are developed extensively in other texts, as is the free field theory which I have include In particular I have repeatedly referred to those by Bjorken and Drell[5]. The reader who is already familiar with this material can safely omit the first four sections of Chapter 2. Similar remarks apply to the first section of Chapter 4. This includes a modicum of SU(3) group theory prior to the statement of the Cabibbo hypothesis. This topic too is well-covered elsewhere[6].

The recent experimental developments, to which I have alluded above, have been matched by corresponding theoretical advances, both before and after the event. Naturally these theories are not (yet ?) grounded in experiment in the way that the Feynman Gell-Mann theory is. However their manifest power and elegance are, I hope, characteristic of the sort of theory we might eventually expect to have, and I feel that their pedagogic value outweighs their speculative nature. This material is contained in hapter 6.

The observation of CP-violation in weak processes has led to the tradi- of this topic being discussed alongside known weak phenomena, even ugh the interaction responsible is not yet known to be weak. This book forms with this tradition and the subject is dealt with in Chapter 7.

My view of this subject has of course been developed by continuous osure to the literature and by personal contact with my colleagues in international community of physicists. It is impossible to acknowledge individually all those who have helped me in this way. But I must at least apologise to those whose work is uncited either because of my own ignorance of it or because space limitations have prevented me from treating the subject as fully as it merits. I would like to thank Mrs. Jean Hafner for converting my almost illegible manuscript into a beautifully produced typescript. In the course of writing this book, as well as an earlier review, I have benefited from inumerable discussions with my colleagues at the niversity of Sussex. It is a pleasure to thank Professors J. P. Elliott, R.J. Tayler and Drs. G. Barton, J. Byrne, N. Dombey, W.D. Hamilton, D.R.T. Jones and A. Love for their continued advice. Dr. H.F. Jones of Imperial College read the final typescript and made a number of very helpful suggestions, for which I am most grateful. I also acknowledge my debt to Professor R.J. Blin-Stoyle. He urged me to write this book, encouraged me when I was wrestling with it and undertook a critical reading when it was finished. His helpful advice is much appreciated. My family, too, have participated, in a very real sense, in the production of this book. My wife's endurance of an absent and absorbed husband is repaid only in part by the dedication of this book: to Anjali.

David Bailin

Note added for Second Edition

It is now more than seven years since most of this book was written. The intervening period has been especially fruitful for experimentalists and theorists alike. The experimental harvest has largely consisted of the verification of the detailed predictions made by the theories which were described in Chapter 6 of the first edition. The power and elegance of the unified theory of weak and electromagnetic interactions, now called the standard electroweak theory, and of the non-abelian gauge theory of strong interactions, now called quantum chromodynamics, have always been evident. The broad experimental support which now exists for these theories thus makes the caution which was appropriate in 1974 seem misplaced in 1982. These theories are now the leading contenders for being regarded as true descriptions of all fundamental interactions (except gravity) in the same way that quantum electrodynamics is regarded as the fundamental description of all electromagnetic interactions.

For these reasons I felt that the reprinting of the book afforded an excellent opportunity to update the material contained in Chapter 6. There have, of course, been many developments which are relevant to the other chapters, but they have not seriously affected the treatment of these topics, and, in order to save time and minimize the amendments, all of these chapters remain unaltered. Chapter 6 has been essentially rewritten in order to exhibit fully the spectacular success that the theory has enjoyed.

It is a pleasure to thank Professor D. F. Brewer and Adam Hilger Ltd. for their decision to re-issue this book.

D. B.

CONTENTS

Chapter 1

PRELIMINARY SURVEY

The weak interactions of elementary particles are characterised by the slowness of reactions in which they participate. Typically, the weak decay of a π^+ meson has a mean lifetime of 10^{-8} s, while the electromagnetic decay of a π^o has a lifetime of 10^{-16} s. However, within this class of slow processes there is a tremendous range of observed lifetimes; the neutron decays in 10^3 s, while the hyperons have lifetimes of the order of 10^{-10} s. Clearly, one would like to ascribe this variation to kinematical differences between the various decay modes rather than to some intrinsic difference in the dynamical interaction responsible for the decay.

Such a hope is motivated by the fantastic success of the theory of quantum electrodynamics (QED), which apparently can predict the observable properties of the electron (and muon) to arbitrary accuracy. This theory is formulated within the framework of relativistic quantum field theory, in which the electron and photon are represented by an operator field which is defined at each space-time point; the field is an "operator" in the sense that the electron's field, for example, "operates" on the space of physical states and can either destroy an electron or create a positron. The time development of any (Schrödinger picture) state is then determined by the specification of the Lagrangian or Hamiltonian for the system in terms of the field operators, just as in Classical Field Theory. The whole theory of QED is based on the assumption of a specific simple form for the Hamiltonian, the only input being the mass and charge of the electron.

In view of the success of field theory in QED, it is natural to hope that a similar approach to the weak interactions will yield an equally successful theory of all weak phenomena. In this book we shall see how far this approach has progressed. The theory is certainly not complete, as we shall see, though as at present formulated it has had a measure of success in explaining quite disparate weak phenomena. However, we shall see also that in its present form the theory is certainly wrong, in as much as it makes predictions which cannot be correct. It is, therefore, believed that the present version of the theory is an approximation to the 'correct' theory; the success of the present formulation is thought to be due to the fact that it has until quite recently been applied only to relatively low energy phenomena; more accurately, it has in fact been derived from low energy decay processes. In this connection, it is interesting that at the present time it is becoming feasible to perform weak scattering experiments using high energy neutrino beams. It may well be that these high energy processes will yield the clues needed to get at the next version of the theory. It is certainly true that those experiments already performed have yielded fascinating results and, perhaps, the seeds of a new theory.

We have already noted that the weak interactions are in general significantly slower than electromagnetic interactions. It is, therefore, to be expected that we may calculate weak matrix elements by using relativistic perturbation theory, just as one does in QED. In this latter case, the amplitude for any electromagnetic process is expanded as a power series in e, the electron charge; in the language of quantum field theory we may regard e as the electromagnetic 'coupling constant', which determines the strength with which the electron and photon fields are coupled in the Hamiltonian. The quantity e, as we have already noted, is input and in 'natural' units ($\hbar = c = 1$) satisfies $\alpha \equiv e^2/4\pi \approx 1/137$. Since the fine structure constant α is small compared with unity, it is believed that the perturbation series converges in QED, and consequently that any matrix element can be calculated to arbitrary accuracy. The slowness of weak interactions compared with electromagnetic processes leads one to hope that the weak coupling of the fields in the Hamiltonian has a coupling constant which is even smaller than the fine structure constant α. However, as we shall see, it turns out that the weak coupling constant G is not dimensionless. Its magnitude is about 10^{-49} erg cm^3. In order to compare with α we must scale G with some dimensional quantity. In natural units, if we scale with the square of the nucleon mass, then $G m_n^2$ is about 10^{-5}. Thus, if the nucleon mass is a suitable unit with which to scale G, then one could hope that the perturbation approach is applicable to weak processes. Further, if this is so, then it is an excellent approximation to retain only the first term in the perturbation series, since hardly any weak processes are measured accurately to one part in 10^5. This is what is usually done, and it is the approach we shall adopt for most of this book.

However, it is important to appreciate from the outset that one does not have to scale G with the nucleon mass. It may well appear to be an appropriate unit for the relatively low energy decay phenomena from which the present form of the theory has been obtained by induction. At higher energies though, which are becoming available in the weak scattering experiments for example, it is not so clear what the scaling unit should be, although it would evidently have to be fairly massive, if it were to lead us to retain even the second term in the perturbation series. At very high energies, where it is anticipated that all masses are negligible, the only mass unit available is the centre-of-mass energy W. If W is sufficiently large, it is clear that eventually GW^2 is of the order of unity. This occurs when W is around 350 GeV, and at this enormous energy all terms in the perturbation series have the same order of magnitude. Thus at this energy, and beyond, the perturbation approach is useless, and we must solve the problem by 'other means', which have yet to be devised.

The study of weak interactions has been considerably simplified by the experimental observation that some of the elementary particles which occur in nature do not participate in strong interactions. This is of considerable importance, because, while it is possible to treat the weak interactions to lowest order in the perturbation series, as we have argued, it is clear that this is not possible for the strong interactions, which are characterized by the pion-nucleon coupling constant g satisfying $g^2/4\pi = 13.6$. The particles which do not have strong interactions are the photon γ, which has only electromagnetic interactions, the neutrinos and antineutrinos, ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$, which only have weak interactions, and the electron and muon and their antiparticles, e^-, μ^-, e^+, μ^+, which have both electromagnetic and weak interactions. Excluding the photon, these particles are referred to collectively as the leptons. The other known particles, which do participate in strong interactions are defined as hadrons.

If, as a first approximation, we neglect the electromagnetic interactions of the charged leptons, then we may infer the precise way in which the leptonic field operators enter the weak Hamiltonian; an experimental study of the weak processes in which the leptons participate in principle enables one to determine the matrix element of the process being studied. The form of this matrix element precisely mirrors the way in which the lepton field operators enter the weak Hamiltonian, since there are no renormalization effects for the leptons, so long as we ignore electromagnetic and higher order weak effects which we assume to be small. As a result of such experimental studies, it is now believed that the leptons' field operators enter the weak Hamiltonian in a combination known as the "leptonic current". We shall define this notion precisely in the following chapters. For the moment we confine ourselves to some more general observations. The word "current" is used because, like the electromagnetic current in QED, it is a bilinear combination of the field operators which transforms as a vector under restricted Lorentz transformations. However, the analogy ends there; the use of the word "current" should not be taken as implying (necessarily) that the leptonic current is generated by a gauge transformation of some underlying field (i. e. a Noether current). And in certain respects the leptonic current is quite different from the electromagnetic current. We shall see that under orthochronous Lorentz transformations (i. e. including the discrete operation of space inversion or parity reversal P) the leptonic current is found to be the sum of vector and axial vector pieces. (The space components of a vector change sign under parity reversal, like the electric field $\underline{E}$, while those of an axial vector are unaltered, like the magnetic field $\underline{B}$). In contrast the electromagnetic current transforms just as a vector.

What can be said about the way in which the hadrons enter the weak Hamiltonian ? Let us restrict ourselves for the moment to that part of the Hamiltonian which is responsible for semileptonic processes, i. e. weak processes involving both hadrons and leptons. Since angular momentum is found to be conserved in all known processes, it follows that the weak Hamiltonian, or more precisely the Hamiltonian density $\mathcal{H}_w$, must be invariant under restricted Lorentz transformations. Thus, since we know that the leptons' fields enter $\mathcal{H}_w$ in the form of the leptonic current, it follows that the hadrons' fields enter $\mathcal{H}_w$ in some form which also transforms as a vector under restricted Lorentz transformations; only a vector can be coupled to another vector via their scalar product to give a scalar or invariant product. This vector combination of the hadronic fields is called the "hadronic current", the word "current" being used in the same sense as before. Like the leptonic current, the hadronic current is the sum of vector and axial vector pieces. Under orthochronous Lorentz transformations the resultant Hamiltonian is the sum of scalar and pseudoscalar pieces; the scalar part is invariant under parity reversal, while the pseudoscalar part changes sign. Thus $\mathcal{H}_w$ is not invariant under parity reversal. This implies that the transition probability for a semileptonic process contains observable pseudoscalars, and is therefore not invariant under parity reversal. The existence of parity violation in nuclear beta decays was established in 1957. As we shall see, the observations are most elegantly explained by a theory in which parity invariance is maximally violated. Since experiments have put very stringent limits on possible parity violation in strong and electromagnetic interactions, it has come to be believed that the only interactions which violate parity invariance are the weak interactions. Thus to some extent the existence of parity violation has now become one of the defining features of the weak processes.

One might hope that it would be possible to infer the precise form of the hadronic current in terms of the hadronic field operators, in just the same way that the form of the leptonic current has been derived. The semileptonic processes certainly determine, in principle, the matrix elements of the hadronic current between hadronic states. Unfortunately, however, *this does* not enable us to determine the form of the hadronic current itself, because there is no reliable way of calculating the effects of the strong interactions. The best that can be done is to characterise the hadronic current in terms of the quantities which are believed to be conserved by the strong interactions. For example, the observation of the beta decay of a neutron, $n \to p\, e^- \bar{\nu}_e$, indicates that the hadronic current contains a piece which has a non-vanishing matrix element between the initial neutron state and the final proton state. Thus this piece of the hadronic current must have zero hypercharge Y, since the neutron and proton have the same hypercharge and Y is conserved by the strong interactions. Likewise, we may infer that this piece of the hadronic current has third component of isospin I_3 equal to unity. In the same way, we may classify the pieces of the hadronic current according to their total isospin I, since isospin is believed to be an exact symmetry of the strong interactions. It is known, however, that the electromagnetic interactions do not conserve isospin. Thus, in order to test any hypothesis we may make about the isospin properties of the hadronic current, it is important that we are able to calculate the electromagnetic effects in any process in which the experimental accuracy is of the same order as the fine structure constant α.

In addition to the semileptonic decays of the type already discussed, there is another type, in which the hypercharge of the initial and final hadron states differ by one unit; for example, the beta decay of a lambda hyperon, $\Lambda \to p e^- \bar{\nu}_e$, must proceed by a piece of the hadronic current which has $Y = 1$, $I_3 = \frac{1}{2}$. No semileptonic decays have ever been observed in which the hypercharges of the initial and final hadron states differ by two or more units; for example, the decay $\Xi^0 \to p e^- \bar{\nu}_e$ has not been observed. Thus until very recently it was believed that all semileptonic processes were accounted for by a Hamiltonian involving only the leptonic current and the two pieces of the hadronic current which we have mentioned. However, it is now clear that a different type of semileptonic process exists, which cannot be described by such a Hamiltonian. In the "common" semileptonic processes the leptons invariably carry away one unit of charge. This is apparent from the