ELECTROMAGNETISM

Principles and Applications

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PREFACE

Like *Electromagnetic Fields and Waves* by the same authors, this book aims to give the reader a working knowledge of electromagnetism. Those who will use it should refer to a collection of essays by Alfred North Whitehead entitled *The Aims of Education,*† particularly the first essay, which carries the same title. At the outset, Whitehead explains his point: "The whole book is a protest against dead knowledge, that is to say, against inert ideas . . . ideas that are merely received into the mind without being utilized, or tested, or thrown into fresh combinations." It is in this spirit that we have given more than 90 examples and set 332 problems, 27 of which are solved in detail.

The book is intended for courses at the freshman or sophomore level, either as an introduction to the subject for physicists and engineers, or as a last course in electromagnetism for students in other disciplines.

It is assumed that the reader has had a one-term course on differential and integral calculus. No previous knowledge of vectors, multiple integrals, differential equations, or complex numbers is assumed.

The features of our previous book that have been so appreciated will be found here as well: the examples mentioned above, problems drawn from the current literature, a summary at the end of each chapter, and three-dimensional figures, this time drawn in true perspective.

The book is divided into twenty short chapters suitable for self-paced instruction. Following an introductory chapter on vectors, the discussion of electromagnetism ranges from Coulomb's law through plane waves in dielectrics. Electric circuits are discussed at some length in three chapters: 5, 17, and 18. Chapter 16 is entirely devoted to complex numbers and phasors. There is no way of dealing properly with alternating currents without using complex numbers, and, contrary to what is often assumed, complex numbers are not much of

†Free Press, New York, 1976.

an obstacle at this level. Our discussion of relativity occupies only part of one chapter (Chapter 10)—just enough to explain briefly the Lorentz transformation and the transformation of electric and magnetic fields. This naturally leaves a host of questions unanswered, but should be sufficient in the present context.

THE PROBLEMS

The longer problems concern devices and methods of measurement described in the physics and electrical engineering literature of the past few years. They are "programmed" in the sense that they progress by small steps and that the intermediate results are usually given.

These longer problems are meant to give the reader the opportunity to learn how to make approximations and to build models amenable to quantitative analysis. It is of course hoped that they can teach the heuristic process involved in solving problems in the field of electromagnetism. In fact, after working through many such problems, students end up surprised to see that they can deal with real situations.

These problems contain a large amount of peripheral information and should provide some interesting reading. They should also incite the reader to apply his newly acquired knowledge to other fields and should stimulate creativity. Many of them can serve for open-ended experiments.

The easier problems are marked *E*, from Chapter 2 on. They can be solved in just a few lines, but they nonetheless often require quite a bit of thought. Some problems are marked *D*. Those are quite difficult.

A number of problems require curve plotting. This is because curves are so much more meaningful than formulas. It is assumed that the student has access to a programmable calculator, or possibly to a computer. Otherwise the calculations would be tedious.

On the average, about two problems are solved in detail at the end of each chapter.

UNITS AND SYMBOLS

The units and symbols used are those of the *Système International d'Unités,* designated SI in all languages.† The system originated with the proposal made

† See *ASTM/IEEE Standard Metric Practice,* published by the Institute of Electrical and Electronics Engineers, Inc., 345 East 47th Street, New York, N.Y. 10017.

by the Italian engineer Giovanni Giorgi in 1901 that electrical units be based on the meter-kilogram-second system. The Giorgi system grew with the years and came to be called, first the MKS system, and later the MKSA system (A for ampere). Its development was fostered mostly by the International Bureau of Weights and Measures, but also by several other international bodies such as the International Council of Scientific Unions, the International Electrotechnical Commission, and the International Union of Pure and Applied Physics.

Appendix D provides a conversion table for passing from cgs to SI units, and inversely. "Further use of the cgs units of electricity and magnetism is deprecated."†

SUPPLEMENTARY READING

The following books are recommended for supplementary reading:

Electromagnetic Fields and Waves: Second Edition, by the same authors and the same publisher, published in 1970. There are several references to this book in the footnotes.

Standard Handbook for Electrical Engineers, McGraw-Hill, New York, 1968.

Electronics Engineers' Handbook, McGraw-Hill, New York, 1975.

Reference Data for Radio Engineers, Howard Sams and Company, Inc., Indianapolis, 1970.

Electrostatics and Its Applications, A. D. Moore, Editor, John Wiley, New York, 1973.

These books can be found in most physics and engineering libraries. The reader will find the second and third references to be inexhaustible sources of information on practical applications.

ACKNOWLEDGMENTS

I am indebted first of all to my students, who have taught me so much in the course of innumerable discussions. I am also indebted to the persons who assisted me in writing this book: to François Lorrain, who took part in the preparation of the manuscript and who prepared sketches of the three-dimensional objects that appear in the figures in true perspective; to Ronald Liboiron, who also worked on the preparation of the figures; to Jean-Guy Desmarais and

†*Ibid.*, p. 11.

CONTENTS

CHAPTER 1

VECTORS

Electric and magnetic phenomena are described in terms of the *fields* of electric charges and currents. For example, one expresses the force between two electric charges as the product of the magnitude of one of the charges and the field of the other.

The object of this first chapter is to describe the mathematical methods used to deal with fields. All the material in this chapter is essential for a proper understanding of what follows.

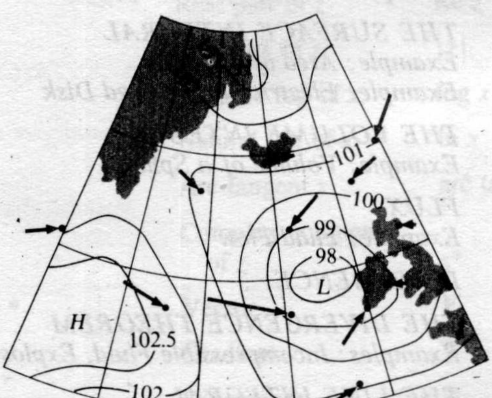

Figure 1-1 Pressure and wind-velocity fields over the North Atlantic on November 1, 1967, at 6 hours, Greenwich Mean Time. The curved lines are *isobars*, or lines of equal pressure. Pressures are given in kilopascals. High-pressure areas are denoted by *H*, and low-pressure areas by *L*. In this case an arrow indicates the direction and velocity of the wind at its tip; arrow lengths are proportional to wind velocities (the longest arrow in this figure represents a wind velocity of 25 meters per second). Wind vectors are given only at a few points, where actual measurements were made.

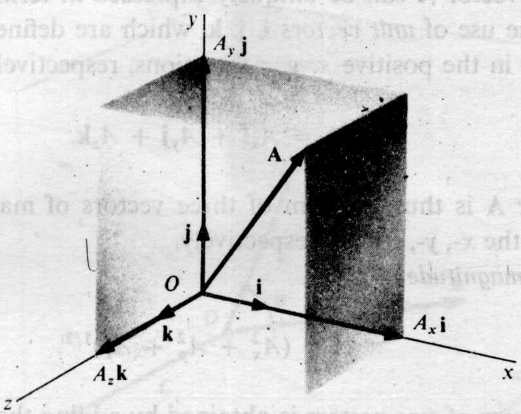

Figure 1-2 A vector **A** and the three vectors A_x**i**, A_y**j**, A_z**k**, which, when placed end-to-end, are equivalent to **A**.

Mathematically, a field is a function that describes a physical quantity at all points in space. In *scalar fields*, this quantity is specified by a single number for each point. Pressure, temperature, and electric potential are examples of scalar quantities that can vary from one point to another in space. For *vector fields*, both a number and a direction are required. Wind velocity, gravitational force, and electric field intensity are examples of such vector quantities. Both types of field are illustrated in Fig. 1-1.

Vector quantities will be indicated by **boldface** type; *italic* type will indicate either a scalar quantity or the magnitude of a vector quantity.†

We shall follow the usual custom of using *right-hand Cartesian coordinate systems*, as in Fig. 1-2; the positive z direction is the direction of advance of a right-hand screw rotated in the sense that turns the positive x-axis into the positive y-axis through the 90° angle.

1.1 VECTORS

A vector can be specified by its *components* along any three mutually perpendicular axes. In the Cartesian coordinate system of Fig. 1-2, for example, the components of the vector **A** are A_x, A_y, A_z.

† In a handwritten text, it is convenient to identify a vector by means of an arrow over the symbol, for example, $\vec{A}$.

The vector **A** can be uniquely expressed in terms of its components through the use of *unit vectors* **i**, **j**, **k**, which are defined as vectors of unit magnitude in the positive x, y, z directions, respectively:

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}. \qquad (1\text{-}1)$$

The vector **A** is thus the sum of three vectors of magnitude A_x, A_y, A_z, parallel to the x-, y-, z-axes, respectively.

The *magnitude of* **A** is

$$A = (A_x^2 + A_y^2 + A_z^2)^{1/2}. \qquad (1\text{-}2)$$

The *sum of two vectors* is obtained by adding their components:

$$\mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}. \qquad (1\text{-}3)$$

Subtraction is simply addition with one of the vectors changed in sign:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k}. \qquad (1\text{-}4)$$

1.2 SCALAR PRODUCT

The *scalar*, or *dot product*, is the scalar quantity obtained on multiplying the magnitude of the first vector by the magnitude of the second and by the cosine of the angle between the two vectors. In Fig. 1-3, for example,

$$\mathbf{A} \cdot \mathbf{B} = AB\cos(\varphi - \theta). \qquad (1\text{-}5)$$

It follows from this definition that the usual commutative and distributive rules of ordinary arithmetic multiplication apply to the scalar product:

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}, \qquad (1\text{-}6)$$

and, for any three vectors,

$$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}. \qquad (1\text{-}7)$$

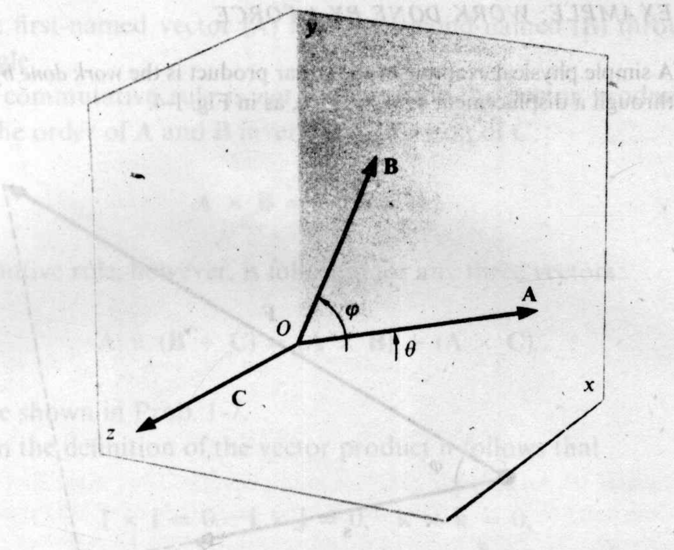

Figure 1-3 Two vectors **A** and **B** in the xy-plane. Their scalar product is $AB \cos(\varphi - \theta)$. The vector **C** is their vector product **A** × **B**.

The latter property will be verified graphically in Prob. 1-3. It also follows that

$$\mathbf{i} \cdot \mathbf{i} = 1, \ \mathbf{j} \cdot \mathbf{j} = 1, \ \mathbf{k} \cdot \mathbf{k} = 1, \tag{1-8}$$

$$\mathbf{j} \cdot \mathbf{k} = 0, \ \mathbf{k} \cdot \mathbf{i} = 0, \ \ \mathbf{i} \cdot \mathbf{j} = 0. \tag{1-9}$$

Then,

$$\mathbf{A} \cdot \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}), \tag{1-10}$$

$$= A_xB_x + A_yB_y + A_zB_z. \tag{1-11}$$

It is easy to check that this result is correct for two vectors in a plane, as in Fig. 1-3:

$$\mathbf{A} \cdot \mathbf{B} = AB\cos(\varphi - \theta) = AB\cos\varphi\cos\theta + AB\sin\varphi\sin\theta, \tag{1-12}$$

$$= A_xB_x + A_yB_y. \tag{1-13}$$

1.2.1 EXAMPLE: WORK DONE BY A FORCE

A simple physical example of the scalar product is the *work done by a force* **F** acting through a displacement **s**: $W = \mathbf{F} \cdot \mathbf{s}$, as in Fig. 1-4.

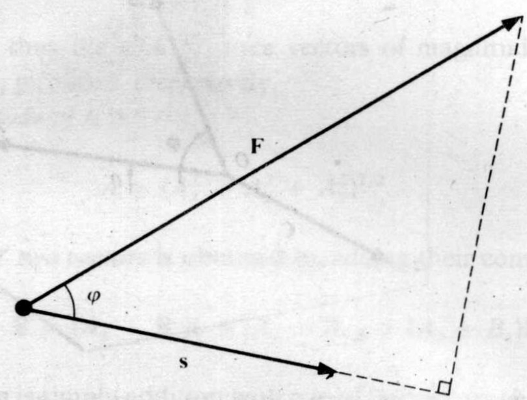

Figure 1-4 The work done by a force **F** whose point of application moves by a distance **s** is $(F \cos \varphi)s$, or $\mathbf{F} \cdot \mathbf{s}$.

1.3 VECTOR PRODUCT

The *vector*, or *cross product*, of two vectors is a vector whose direction is perpendicular to the plane containing the two initial vectors and whose magnitude is the product of the magnitudes of those vectors and the sine of the angle between them. We indicate the vector product thus:

$$\mathbf{A} \times \mathbf{B} = \mathbf{C}. \tag{1-14}$$

The magnitude of **C** is

$$C = |AB \sin (\varphi - \theta)|, \tag{1-15}$$

with φ and θ defined as in Fig. 1-3. The direction of **C** is given by the right-hand screw rule: it is the direction of advance of a right-hand screw whose axis, held perpendicular to the plane of **A** and **B**, is rotated in the sense that rotates the first-named vector (**A**) into the second-named (**B**) through the smaller angle.

The commutative rule is *not* followed for the vector product, since inverting the order of **A** and **B** inverts the direction of **C**:

$$\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A}). \tag{1-16}$$

The distributive rule, however, is followed for any three vectors:

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{C}). \tag{1-17}$$

This will be shown in Prob. 1-7.

From the definition of the vector product it follows that

$$\mathbf{i} \times \mathbf{i} = 0, \quad \mathbf{j} \times \mathbf{j} = 0, \quad \mathbf{k} \times \mathbf{k} = 0, \tag{1-18}$$

and, for the usual right-handed coordinate systems, such as that of Fig. 1-2,

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}, \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k}, \text{ and so on.} \tag{1-19}$$

Writing out the vector product of **A** and **B** in terms of the components,

$$\mathbf{A} \times \mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}), \tag{1-20}$$

$$= (A_yB_z - A_zB_y)\mathbf{i} + (A_zB_x - A_xB_z)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}, \tag{1-21}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}. \tag{1-22}$$

We can check this result for the two vectors of Fig. 1-3 by expanding $\sin(\varphi - \theta)$ and noting that the vector product is in the positive z direction.

1.3.1 *EXAMPLES: TORQUE, AREA OF A PARALLELOGRAM*

A good physical example of the vector product is the *torque* **T** produced by a force **F** acting with a moment arm **r** about a point O, as in Fig. 1-5, where $\mathbf{T} = \mathbf{r} \times \mathbf{F}$.

A second example is the *area of a parallelogram*, as in Fig. 1-6, where the area $\mathbf{S} = \mathbf{A} \times \mathbf{B}$. The area is thus represented by a vector perpendicular to the surface.

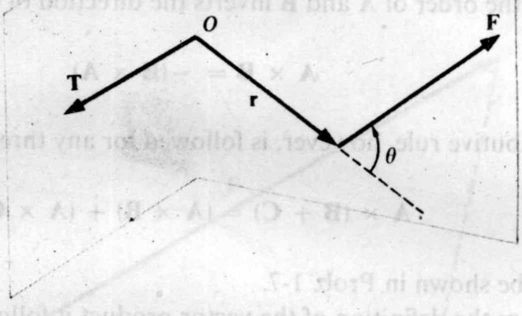

Figure 1-5 An example of vector multiplication. The torque **T** of the force **F** about the point O is $\mathbf{r} \times \mathbf{F}$. This vector has a magnitude of $rF \sin \theta$ and is oriented as shown.

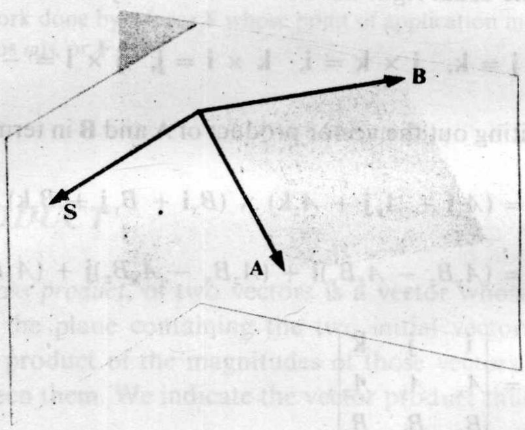

Figure 1-6 The area of the parallelogram is $\mathbf{A} \times \mathbf{B} = \mathbf{S}$. The vector **S** is normal to the parallelogram.

1.4 THE TIME DERIVATIVE

We shall often be concerned with the rates of change of scalar and vector quantities with both time and space coordinates, and thus with the time and space derivatives.