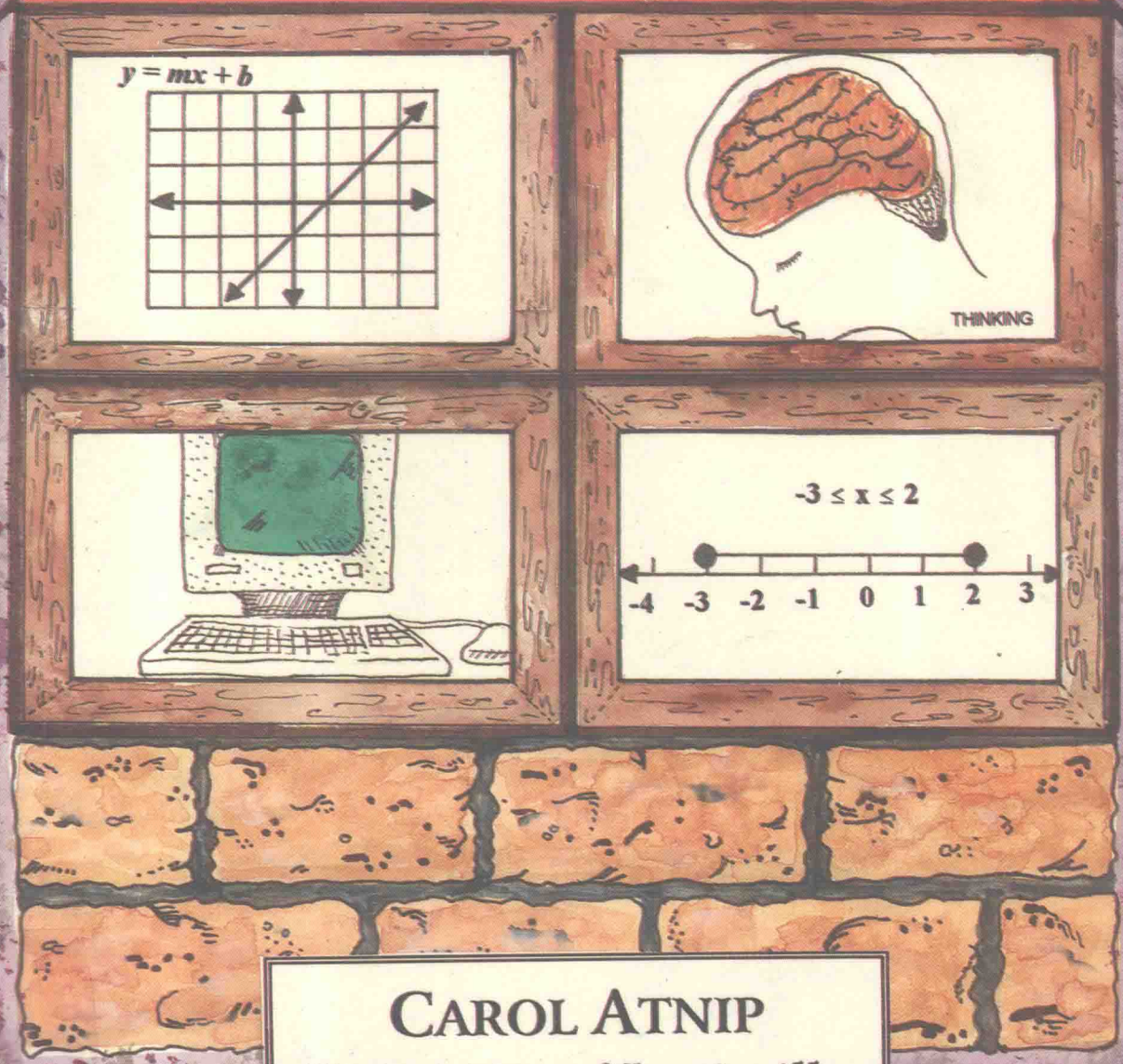
FOUNDATIONS OF ALGEBRA



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Foundations of Algebra

First Edition
1998

by

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I would like to thank the hard work and dedication of my teammate and friend, **Richard Benner**, who was co-author on the first draft of this text.

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Also contributing, Allison Atnip, who at 14 years old is a better typist than her mom.

To the Instructor:

In the past several years mathematics education in the United States has been criticized in major reports like the National Commission on Excellence in Education's document, "A Nation at Risk: The Imperative for Education Reform" , The Secretary's Commission on Achieving Necessary Skills (SCANS) report, "What Work Requires of School: A SCANS Report for America 2000" and the National Research Council's work, *Everybody Counts*. Mathematics educators are in the process of responding to this criticism by reforming the way mathematics is taught from grade school through calculus. Major mathematics organizations such as the National Council of Teachers of Mathematics and the American Mathematical Association for Two-Year Colleges have written standards for teaching and learning mathematics.

This text book was created to include the changes recommended by the documents mentioned above as well as research on the teaching and learning of mathematics. In addition to changes in course content, the text is designed to include the processes required to improve student's learning. These key processes are discovery learning, problem solving, critical thinking, cooperative learning, communicating, and self-assessing.

Students in this class will be expected to: *think critically, work in cooperative groups, experience discovery, self-assess, use resources provided, learn faster, and have fun!* You, as instructor/facilitator, will promote the process by asking critical questions, facilitating cooperative learning, and getting students to be actively involved with their own learning by using the theme of the course: **"TRY IT!"**

Each of the topics in this text are presented in a learning activity format with learning objectives and performance criteria followed by models and discussion as needed. One key component of each activity is a set of Critical Thinking Questions which are designed to get the student to think about and articulate particular aspects of the models. The Critical Thinking Questions are followed by skills exercises which are typical problems found in most newer texts. The book is divided into two parts Chapters 1-5, which have 31 math activities designed to be about 45-50 minutes in length, and Chapters 6-8, which have 12 activities on how to learn mathematics more efficiently. The first five chapters are: Algebraic Expressions and Solving Equations, Linear Equations and Graphing, Polynomial Expressions, Rational Expressions, and Radical Expressions and Quadratic Equations. The last three chapters include, a chapter on managing course resources such as the course syllabus, the Learning Assessment Journal, working through an activity. The next chapter introduces the student to the ideas of cooperative learning and the processes of learning, and the last chapter has two activities on managing anxiety. The last activity is on preventing common errors and should be done before the review for the final exam. The first 5 chapters are designed to be sequential, but part two can and should be interspersed as needed.

For more information about Process Education visit our web site: *http://www.pcrest.com/process* or join the Internet group ProcessEd at *processes@miser.suffolk.edu.*

To the student:

Foundations of Algebra presents a way of learning math that is based on the experience and research of many mathematics educators across the country. You will be learning to use key processes that will enable you to increase your mathematical knowledge as well as your ability to learn mathematics. As you begin this course, think about how you would answer the following questions.

How successful have you been in learning mathematics?
How good do you feel about the process that you used to learn math?
Are you willing to try a new process for learning math?

In this class you will be expected to work in teams with assigned roles, think critically, experience discovery, learn to self-assess, learn how to learn faster, and **have fun**! You will be using a powerful computer program called *MathSkills* that will help you gain the necessary speed and accuracy in working practice problems to excel in this and your next math courses. One major part of learning how to assess your own progress in class is to write about it in your *Learning Assessment Journal*. Your instructor will facilitate your learning by asking directed and critical thinking questions, not by lecturing on material you can discover on your own or with your team. This will be an active learning environment where you are the center of the learning process, not your instructor. Your instructor may answer your questions with the one phrase that captures the spirit of the class: **"Try it!"**

Table of Contents

Chapter 1

Algebraic Expressions and Solving Equations

1.1 Order of Operations

Why Learn This

When simplifying and solving arithmetic expressions and equations, it is sometimes unclear which operation to perform first. If we didn't agree on a plan, we could all have different answers to a problem. The order of operations is a universally agreed upon plan for working problems. The *key to success* in algebra is a complete understanding of order of operations and the ability to apply the process in appropriate situations.

Learning Objectives

1. To investigate the methodology (or list of steps) for order of operations.
2. To evaluate expressions using the correct order of operations.

Performance Criteria

1. Be able to apply the methodology for order of operations by correctly evaluating any given expression.

Language

Evaluate Methodology Absolute Value Integer

Resource

Appendix 1: Review of Integer Operations

Methodology

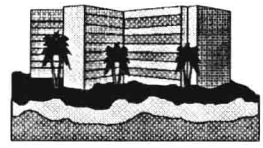
Methodology for Simplifying Expressions Using Order of Operations

Step 1	Simplify within parenthesis.
Step 2	Perform exponentiation.
Step 3	Perform multiplication and division *as they appear from left to right.*
Step 4	Perform addition and subtraction *as they appear from left to right.*

Discussion of the Methodology

- Expressions within absolute value symbols ($|a|$) are simplified in the same order as parenthesis, that is, as if there are parenthesis around the expression within the absolute value symbols.
- Expressions with fractions are simplified as if the numerator is in parenthesis and the denominator is in parenthesis.
- Order of operations is used when simplifying equations.
- When validating a solution or evaluating a formula for specific values, follow the order of operations.

Models



A. Motel Rooms:

34 rooms	@ $\$64$	Single
150 rooms	@ $\$78$	Double-double
45 rooms	@ $\$98$	Suite

Assuming that all rooms are rented on a given day, how much revenue will the motel collect?

$$(34 \cdot 64) + (150 \cdot 78) + (45 \cdot 98) = \text{Revenue}$$
$$2176 + 11700 + 4410 = \text{Revenue}$$
$$\$18286 = \text{Revenue}$$

B. Evaluate: $24 - 3(4 \cdot 2 + 3)$

Step 1 Simplify within parenthesis.

$$24 - 3(8 + 3)$$
$$24 - 3(11)$$

What changed?

Step 2 No exponents; skip this step.

Step 3 Perform multiplication.

$$24 - 33$$

Step 4 Perform Subtraction.

$$-9$$

C. Evaluate: $3-(4\cdot 2-5)^2+4-2^3$

$3-(4\cdot 2-5)^2+4-2^3$

If you are doing work on your own paper a good first step is to copy the problem exactly as it appears.

Step 1 Parenthesis

$3-(8-5)^2+4-2^3$

$3-3^2+4-2^3$

Step 2 Exponents

$3-9+4-8$

Step 3 No Multiplication or division, skip this step.

Step 4 Addition and subtraction from left to right.

$-6+4-8$

$-2-8$

-10

D. Evaluate: $2\cdot 5\cdot(-3)\div 6+8\div 4(-5)\div 2$

Step 1 No parenthesis to simplify; skip this step.
Step 2 No exponents; skip this step.
Step 3 Multiplication and division ***as they appear from left to right***.

$10\cdot(-3)\div 6+2(-5)\div 2$

$-30\div 6-10\div 2$

$-5-5$

-10

Notice that multiplication was done twice in the same step. Why can that work?

E. Evaluate: $|-2-3|\div 5+4^2$

$|-5|\div 5+4^2$

$5\div 5+4^2$

$5\div 5+16$

$1+16$

17

For absolute value symbols, see appendix 1.

Here is a way to remember but it is NOT a rule:

Parenthesis
Exponents
Multiply & Divide
Add & Subtract.

P.E.M.D.A.S.

Critical Thinking Questions

1. What two mathematical operations are used to calculate the revenue in model A?

2. In what order are these operations performed? Why does it make sense to use that order?

3. Why is it incorrect to combine the $24 - 3$ part of the expression in model B before simplifying within the parenthesis?

4. What would happen if the parenthesis were left out in model C?

5. Using exponents is a simpler way of writing repeated multiplication. With that in mind, explain why exponentiation is a higher order operation than multiplication or division.

6. What is a good reason to explain why multiplication and division are performed before addition and subtraction in the order of operations?

7. When can steps in the methodology be skipped or rearranged?

Skill Exercises *(For help with integer operations see appendix 1)*

1. Complete the computer assignment for this topic.
2. Simplify the following:

a. $15 - 10 \div 5 \cdot 2$

b. $25 \div 5 \cdot 5 - 10 + 15$

c. $-3(5-9) - 5(3-6)$

d. $|-20| - 6 + 1$

e. $25 - 2(3 \cdot 2 - 4) + 10$

f. $|7 - 15| \div 2 \cdot 4(2-5)$

g. $6(6-1)-9(2-4)$

h. $18 \div 9 \cdot 2 - 6 + 2$

i. $\dfrac{4(2-7)-2(5-2)}{-10-2-1}$

j. $\dfrac{26-7\cdot(1-5)}{18}$

k. $16 - 2^3 \div 4 \cdot 2$

l. $6(6-7)^2 - 9(3-6)^2$

m. $-3(4-7)^2 - 5(3-8)^2$

n. $|-26| - 3^3 + 1$

o. $18 - 2(3 \cdot 3 - 4) + 6$

p. $(2^4 - 25)^2$

q. $\dfrac{-3(4-7)-5(7-2)}{-5-2-1}$

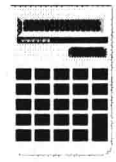
r. $\dfrac{26-14(1-3)^2}{30}$

s. $|5-7\cdot 2|-4^2\div 8$

t. $7-2|1-8(-1)|\div 3$

Problem Solving

1. Test your calculator: Using your calculator work problems 2a and 2b as they are written. What did you get? Justify your calculator answer with your previous work. Does your calculator follow the approved order of operations?



2. Sometimes it is convenient to use a different way to type an expression. Some programming languages still use an "*" to indicate multiplication and a "^" to indicate exponentiation and "/" to indicate division.

   a. Write the following expression using math notation.

   3^2 – 2[5 + (–3)^2/3]*

   b. Write the following expression using computer notation.

   $5^2-(3+2)^3-6\div 3$

Self Assessment

List three things you learned from this activity.

-
-
-

List three things that were the hardest about this activity.

-
-
-

List three ways you will know when you can use the order of operations methodology with complete confidence. (How will you *know* when you know?)

-
-
-

1.2 The Language of Algebra

Why Learn This

Every discipline and occupation has its own vocabulary and expressions. It is difficult to master new concepts without understanding the language needed to develop the new material. Learning to recognize the importance of discipline specific words and phrases will improve your performance in this and all classes you take as well as any occupation you choose.

Learning Objectives

1. To learn the basic vocabulary of algebraic expressions.
2. To learn how each algebraic component interacts with the other components.
3. To learn how to identify and name the components of an algebraic expression.

Performance Criteria

1. Be able to identify each of the components of any given algebraic expression.
2. Be able to identify the like terms of any given list of expressions.

Language

Algebraic Expression: a collection of variables, constants, grouping symbols and signs of operation.

Terms: combinations of constants, variables, products or quotients that are separated by plus (addition) or minus (subtraction) signs.

Variables: symbols (usually letters) that represent unspecified numbers (values).

Constants: symbols (usually numbers) that do not change their values.

Like Terms: terms whose variable factors have the same exponents.

Coefficients (Numerical Coefficients): constants that precede variables.

Constant term: usually the last term, but any term that does not have a variable factor.