天元基金 影印系列丛书

Alexandre J. Chorin 著

旋度与湍流

Vorticity and Turbulence

清华大学出版社

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Some practical information about approximation procedures is provided in the book, as well as tools for assessing the plausibility of approximation schemes. The emphasis, however, is on the understanding of turbulence---its origin, mechanics, spectra, organized structures, energy budget, and renormalization. The physical space methodology is natural, and makes the reasoning particularly straightforward. Open questions are indicated as such throughout the book.

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Preface

This book provides an introduction to the theory of turbulence in fluids based on the representation of the flow by means of its vorticity field. It has long been understood that, at least in the case of incompressible flow, the vorticity representation is natural and physically transparent, yet the development of a theory of turbulence in this representation has been slow. The pioneering work of Onsager and of Joyce and Montgomery on the statistical mechanics of two-dimensional vortex systems has only recently been put on a firm mathematical footing, and the three-dimensional theory remains in parts speculative and even controversial.

The first three chapters of the book contain a reasonably standard introduction to homogeneous turbulence (the simplest case); a quick review of fluid mechanics is followed by a summary of the appropriate Fourier theory (more detailed than is customary in fluid mechanics) and by a summary of Kolmogorov's theory of the inertial range, slanted so as to dovetail with later vortex-based arguments. The possibility that the inertial spectrum is an equilibrium spectrum is raised.

The remainder of the book presents the vortex dynamics of turbulence, with as little mathematical and physical baggage as is compatible with clarity. In Chapter 4, the Onsager and Joyce-Montgomery discoveries in the two-dimensional case are presented from a contemporary point of view, and more rigorous recent treatments are briefly surveyed. This is where the peculiarities of vortex statistics, in particular negative (trans-infinite) temperatures, first appear. Chapter 5 summarizes the fractal geometry of vortex stretching, and Chapter 6 provides a brief but self-contained introduction to the tools needed for further analysis, in particular polymer statistics, percolation, and real-space renormalization. In Chapter 7, these tools are used to analyze a simple model of three-dimensional vortex statistics. The Kolmogorov theory is revisited; a rationale is provided for the effectiveness of some large-eddy approximations; and an instructive contrast is drawn between classical and superfluid turbulence.

Some practical information about approximation procedures is provided in the book, as well as tools for assessing the plausibility of approximation schemes. The emphasis, however, is on the understanding of turbulence—its origin, mechanics, spectra, organized structures, energy budget, and renormalization. The physical space methodology is natural, and makes the reasoning particularly straightforward. Open questions are indicated as such throughout the book.

February 1993

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Introduction

This book provides an introduction to turbulence in vortex systems, and to turbulence theory for incompressible flow described in terms of the vorticity field. I hope that by the end of the book the reader will believe that these subjects are identical, and constitute a special case of fairly standard statistical mechanics, with both equilibrium and non-equilibrium aspects. The special properties of fluid turbulence are due to the unusual constraints imposed by the Euler and Navier-Stokes equations, which include topological constraints in the three-dimensional case. The main consequence of these constraints is that turbulent flows typically have negative temperatures. Despite this peculiar feature turbulence fits well within the standard framework of statistical mechanics; in particular, the Kolmogorov exponent appears as a fairly standard critical exponent and large-eddy simulation appears as a renormalization. In the course of the discussion a comparison with certain properties of vortices in superfluids arises in a natural way, and it points out similarities as well as differences between quantum and classical vortices.

The book is rather concise, but I have tried to make it self-sufficient. I assume that the reader is familiar with the bread-and-butter techniques of mathematical physics—Green's functions, Fourier analysis, distributions—and with basic incompressible hydrodynamics. I have provided introductions to those aspects of probability theory—stochastic processes, statistical mechanics, percolation and polymer physics—that are needed in the analysis. I have tried to give enough mathematics to make the physical ideas understandable, without going overboard. Given a choice between a clear heuristic derivation and a much more difficult mathematical one, I have usually chosen the former; for example, I have stressed the original heuristic derivation of the Joyce-Montgomery equation in preference to the more rigorous and more demanding recent versions. The material in this book has been taught in a graduate course at Berkeley with students drawn from mathematics, physics, and engineering, and I hope it is accessible to a first-year graduate student in any one of these fields.

My main goal is to relate turbulence to statistical mechanics, and many interesting issues that do not contribute to this goal have been omitted. For example, there is no discussion of correlations in time nor of the recent developments in turbulent diffusion. There is no mention of turbulent boundary layers. There is no extensive discussion of mathematical issues relating to the Euler and Navier-Stokes equations beyond their conservation and invariance properties. Implied criticism of some recent theories expressed in spectral variables remains for the most part implied.

The book is organized as follows: The first three chapters constitute a fairly standard introduction to turbulence in incompressible flow. Chapter 1 is a quick survey of incompressible hydrodynamics; Chapter 2 uses probability and random fields to define the Fourier transform and the energy, dissipation, and vorticity spectra of homogeneous flow. Chapter 3 contains an account of the Kolmogorov theory and of intermittency. This account departs in several respects from the usual accounts; I believe that the departures are necessary. The next four chapters present the statistical theory of vortex motion. In Chapter 4 the two-dimensional theory is discussed, following the work of Onsager, Joyce and Montgomery, J. Miller, Robert, and others. Negative temperatures make their appearance. Three-dimensional flow differs from two-dimensional flow mainly because it is dominated by vortex stretching. In Chapter 5 the basic facts about vortex stretching and folding are presented. In Chapter 6 the tools needed for a statistical description of the stretching (percolation and polymer statistics in particular) are introduced, and they are then applied in Chapter 7 to vortex statistics in three dimensions. It is reasonable to view the earlier parts of the book as the background material needed for understanding Chapter 7. The applications to the computation of turbulent flows are also discussed in Chapter 7. Open questions remain, and they are presented as such.

The basic premise that the large scales of turbulent flow in three space dimensions are problem-dependent and that the general theory is an analysis of the small scales is explained in Chapter 3. The observation that renormalization in the vortex setting leads to a form of dealiasing and provides a theoretical justification for large-eddy simulation is presented in Chapter 7.

I hesitated a lot before I put the Kolmogorov theory at the beginning of the analysis of turbulence, before the discussion of vortex statistics is even begun. The Kolmogorov theory, for all its brilliant intuition, is imprecise and, I believe, partly misguided. It does however provide a useful framework for later analysis. Its conclusions are revisited in Chapter 7. Much more can be said about the two-dimensional case than I said in Chapter 4; the interested reader is directed to the references. My main interest is in the three-dimensional case, and the most important aspect of Chapter 4 is that it allowed me to introduce negative temperatures in a context where their existence is beyond doubt. The Williams-Shenoy extension of Kosterlitz-Thouless renormalization to three space dimensions is notoriously controversial; its use in the context of classical fluid mechanics should not be, because in this context the primacy of vortex interactions is not in doubt.

My major conclusion is that turbulence can no longer be viewed as incomprehensible. There are many kinds of turbulence and a wide variety of phenomena; e.g., compressible turbulence differs from incompressible turbulence, quantum turbulence differs from classical turbulence, etc. Yet there is a reasonable and consistent general approach to the problem, one that is in harmony with the equations of motion and with results in other areas of statistical physics, and that provides correct information about practical approaches to problem solving. This is all one can expect to have in a problem with so many different aspects. There are many mathematical questions that remain open, and much work remains to be done both in the general theory and in specific problems, yet the overall picture is clear. Maybe the most startling contention in this book is that the inertial range of turbulence can be described in terms of equilibrium statistical mechanics; the cascade picture, so familiar and so nicely celebrated in verse, describes the formation of the inertial range but not situations where that range is already formed.

In many important respects, and certainly in point of view, this book is a second version of my *Lectures on Turbulence Theory* of 1975. The differences are due to the extensive progress made in the last 20 years in understanding turbulent flow.

A comment is needed about notation: I have used the same symbol (μ) for both chemical potential and for Flory exponents, and the same symbol (Z) for both enstrophy and partition function. Which one of the meanings of these symbols is meant should be apparent from the context. These symbols are commonly used in this way, and I have chosen (possibly mistakenly) to risk confusing the reader slightly here rather than have him or her be confused more when he or she turns to other books or papers. The references are collected at the end. In the text, I have mentioned in footnotes some of the references that are most relevant to whatever topic is at hand, giving enough information to enable the reader to find the full citation in the bibliography at the end of the book.

I have greatly benefitted from discussions with Profs. T. Buttke, P. Colella, G. Corcos, J. Goodman, O. Hald, A. Majda, J. Sethian, F. Sherman and A.K. Oppenheim, whom I warmly thank. The responsibility for all errors, however, is mine alone. I would like to thank the Institute for Advanced Study at Princeton for its hospitality in 1991–92, when a first draft of this book was written, and the Basic Energy Sciences program of the U.S. Department of Energy for its support over the past decade.

1
The Equations of Motion

In this chapter we present the Euler and Navier-Stokes equations for a fluid of constant density, in several forms that will be useful later. In particular, the vorticity and vortex magnetization are introduced, and a first discussion of spectral variables is given. Useful results whose proofs are available in elementary textbooks[1] will be merely stated.

1.1. The Euler and Navier-Stokes Equations

We consider a region $\mathcal{D}$, in either two-dimensional or three-dimensional space, that is filled with fluid. In this book, we shall only consider constant density fluids; the density can be taken equal to 1 without loss of generality. A point in $\mathcal{D}$ has coordinates $\mathbf{x} = (x_1, x_2, x_3)$. Vectors will always be denoted by boldface. Let $\mathbf{u}(\mathbf{x}, t) = (u_1, u_2, u_3)$ denote the velocity field of a fluid particle located at $\mathbf{x}$ at time t. In a fluid of constant density the total flow in and out of any subset of $\mathcal{D}$ must add up to zero; elementary vector calculus yields the equation of continuity

$$\operatorname{div} \mathbf{u} = 0 \ . \tag{1.1}$$

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[1] See, e.g., A. Chorin and J. Marsden, 1979, 1990, 1992.

If one takes a particular material point in the fluid, initially (i.e., when $t = 0$) located at $\mathbf{a}$, one will note that it will move in time. The equation of its trajectory is

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{u}(\mathbf{x}, t) \; ; \tag{1.2}$$

integrated with the initial condition $\mathbf{x}(0) = \mathbf{a}$, this equation will yield the trajectory of the fluid particle. The map from the initial locations of the fluid particle to their locations at time t is the flow map, ϕ_t (note that the subscript does not denote differentiation). Since the fluid has constant density, a collection of particles that occupies a volume V will move into sites that occupy an equal volume. Thus if $\boldsymbol{\phi}(\mathbf{x}, t)$ denotes the location of the particle that ϕ_t has moved from $\mathbf{a}$ to $\mathbf{x}$, the Jacobian $J = |\partial\phi_i/\partial x_j|$ must equal 1. The equation $J \equiv 1$ is equivalent to the equation of continuity.

Note, for later use, that the equation of continuity can be written in the symbolic form

$$\sum_i \frac{\partial}{\partial x_i} \frac{dx_i}{dt} = 0, \tag{1.3}$$

where $\mathbf{x}(t) = (x_1(t), x_2(t), x_3(t))$ and $d\mathbf{x}/dt$ is given by equation (1.2).

If $q(\mathbf{x}, t)$ is a function of position and time in the fluid, its derivative with respect to t is

$$\frac{dq}{dt} = \sum_j \frac{\partial q}{\partial x_j} \frac{dx_j}{dt} + \frac{\partial q}{\partial t} = \sum u_j \partial_j q + \frac{\partial q}{\partial t} = \left(\frac{\partial}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla}\right) q \; ,$$

where $\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \frac{\partial}{\partial x_3}\right)$ is the differentiation vector. We shall often write ∂_1 or ∂_{x_1} for $\frac{\partial}{\partial x_1}$. The operator $(\partial_t + \mathbf{u} \cdot \boldsymbol{\nabla})$ will be denoted by D/Dt (even though it is in fact identical to $\frac{d}{dt}$).

Newton's law, force = mass × acceleration, takes for a fluid the form

$$\frac{D\mathbf{u}}{Dt} = -\text{grad } p + \nu \Delta \mathbf{u} + \mathbf{f} \; .$$

The left-hand side is the acceleration, the right-hand side the force. The mass does not appear because the density is 1. The force is the sum of pressure forces, –grad p, viscous friction forces $\nu\Delta\mathbf{u}$ (Δ is the Laplace operator $\sum \partial_j^2$), where ν is the viscosity coefficient, and body forces $\mathbf{f}$ such as gravity. As usual, in each problem one picks a typical velocity U, a typical length L (and thus a time scale $T = L/U$), and refers the variables to them:

$$\mathbf{x}' = \frac{\mathbf{x}}{L}, \qquad \mathbf{u}' = \frac{\mathbf{u}}{U}, \qquad t' = \frac{t}{T}.$$

After dropping the primes, one obtains the dimensionless form of the equations:

$$\frac{D\mathbf{u}}{Dt} = -\text{ grad } p + \frac{1}{R}\Delta\mathbf{u} + \mathbf{f}, \tag{1.4}$$

where $R = \frac{UL}{\nu}$ is the Reynolds number. If $R^{-1} \neq 0$ these equations are known as the Navier-Stokes equations. If $R^{-1} = 0$ they are known as Euler's equations. Flow with $R^{-1} = 0$ will also be called inviscid. On the boundary $\partial\mathcal{D}$ of a bounded domain $\mathcal{D}$ the appropriate boundary conditions are:

$$\begin{aligned} \mathbf{u} &= 0 \quad \text{when } R^{-1} \neq 0, \\ \mathbf{u}\cdot\mathbf{n} &= 0 \quad \text{when } R^{-1} = 0 \end{aligned} \tag{1.5}$$

where $\mathbf{n}$ is the normal to $\partial\mathcal{D}$.

If $\mathcal{D}$ is a bounded domain, and $\mathbf{w}$ a sufficiently smooth vector, $\mathbf{w}$ can be written uniquely as a sum of the form

$$\mathbf{w} = \mathbf{u} + \text{ grad } \phi,$$

where div $\mathbf{u} = 0$, $\mathbf{u}\cdot\mathbf{n} = 0$ on $\partial\mathcal{D}$. The vectors $\mathbf{u}$ and grad ϕ are orthogonal in function space:

$$\int \mathbf{u}\cdot \text{ grad } \phi \, d\mathbf{x} = -\int (\text{ div } \mathbf{u})\phi \, d\mathbf{x} = 0.$$

$\mathbf{u}$ can then be viewed as the orthogonal projection of $\mathbf{w}$ on the space of divergence-free vectors tangential to the boundary; $\mathbf{u} = \mathbb{P}\mathbf{w}$. Clearly, $\mathbb{P}$ grad $\phi = 0$ for all ϕ; div $\mathbf{u} = 0$ implies div $\partial_t\mathbf{u} = 0$, thus equation (1.4) can be written

$$\partial_t\mathbf{u} = \mathbb{P}[-(\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} + R^{-1}\Delta\mathbf{u} + \mathbf{f}]. \tag{1.6}$$

(Note that $\mathbb{P}$ and Δ do not necessarily commute and thus $\mathbb{P}\Delta\mathbf{u} \neq 0$ in general.) This is the projection form of the Navier-Stokes equations. If $R^{-1} = 0$,

$$\partial_t\mathbf{u} = -\mathbb{P}((\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} - \mathbf{f}).$$

We shall also be considering in these notes periodic domains, where $\mathbf{u}$ and grad p must be periodic, and infinite domains, where $\mathbf{w}$ must be square integrable and **u** must satisfy a decay condition at infinity. The kinetic energy of the flow is

$$E = \tfrac{1}{2}\int \mathbf{u}^2 d\mathbf{x}$$

($\mathbf{u}^2$ means $\sum u_i^2$, $d\mathbf{x} \equiv dx_1 dx_2 dx_3$). Its rate of change, assuming the external force $\mathbf{f} = 0$, is

$$\begin{aligned}\frac{dE}{dt} &= \int \mathbf{u}\,\frac{D\mathbf{u}}{Dt}\,d\mathbf{x} = \int (-\mathbf{u}\cdot\ \text{grad}\,p + R^{-1}\mathbf{u}\cdot\Delta\mathbf{u})d\mathbf{x}\\ &= R^{-1}\int (\mathbf{u}\cdot\Delta\mathbf{u})d\mathbf{x}\\ &= -R^{-1}\int (\nabla\mathbf{u})^2 d\mathbf{x}\,,\end{aligned} \tag{1.7}$$

the integration by parts being valid for any one of the boundary conditions we have considered ($\mathbf{u}\cdot n = 0$, periodic, decay at infinity). As one can readily work out, $(\nabla\mathbf{u})^2 \equiv \sum_{i,j}\left(\frac{\partial u_i}{\partial x_j}\right)^2$. If $R^{-1} = 0$ and **u** is smooth enough for the integral to be finite, $E =$ constant. If $R^{-1} \to 0$ and the integral grows more slowly than R, the same conclusion holds.

Note that we do not have an energy equation of the usual type, in which is asserted the conservation of the sum of the kinetic energy and the "internal energy", i.e., the energy associated with the microscopic vibrations of the fluid. The kinetic energy can decay, if $R^{-1} \neq 0$, and it presumably gets transformed into internal energy, which the equations do not take into account. On the other hand, there is no way for internal energy to become converted into kinetic energy since, by definition, an incompressible fluid does not allow for changes in the fluid's specific volume, and an inspection of the thermodynamic formula for the work done by a fluid shows that the work is zero. A change in the classical, molecular temperature of the fluid, the one that can be measured by a thermometer, can have only a limited effect on the motion of an incompressible fluid by changing the viscosity coefficient. (It will be seen below that the modifiers accompanying the word "temperature" in the preceding sentence make sense when a different temperature is introduced.) This effect of the temperature is, in general, small.

In the case of inviscid, smooth flow there is absolutely no coupling between the molecular motion of the fluid and its macroscopic motion as described by our velocity vector **u**. The molecular structure of the fluid, its microscopic temperature and entropy, have no effect at all on **u**, and vice versa.