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Photoelasticity for Designers

R. B. Heywood

PHOTOELASTICITY FOR DESIGNERS

by

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VOLUME 2

PHOTOELASTICITY FOR DESIGNERS

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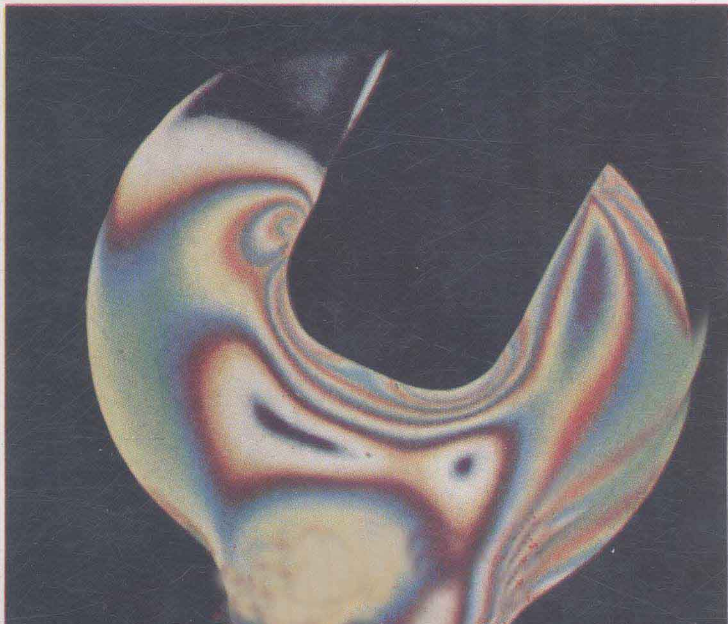

Isochromatics produced in model spanner on
tightening a nut.

To

MY WIFE AND FAMILY

in spite of whom this book
has reached publication

Preface

THE fundamental principles of photoelasticity were established many years ago, and in the past their application to the practical solution of design problems was restricted by the indifferent properties of the photoelastic materials that were then available. In the last decade or so this restriction has been removed by the discovery of epoxy resins, with their remarkable properties. This has enhanced the value of photoelasticity as an aid to design, and has accelerated progress in technique. The accuracy of the three-dimensional frozen stress method has been improved, and the use of a birefringent coating to determine the boundary stress in metal parts is now a practical proposition. The rapid advancement is illustrated by the fact that the author's earlier book on the subject published in 1952 made no reference to epoxy resins or to the birefringent coating technique. The present situation is that simple and powerful photoelastic techniques are available for the measurement of stress over the entire free boundary of a complicated part of a machine or structure, and for the evolution of a shape that will improve the strength/weight ratio; also more advanced techniques are available for a general two- or three-dimensional solution of stress, and for the determination of dynamic, transient or creep stresses.

Stress analysis is also possible by use of the finite-element technique, which does not require the physical construction of a model. Such methods will become of greater importance with the increasing availability of computers and with refinements in procedure. Nonetheless, the variety of problems that can be solved by use of photoelasticity, including those for the improvement of designs, its simplicity, and the advantages of an instantaneous pictorial representation of stress, make the photoelastic method of considerable value to the designer in industry or university, and to the research worker and student in the foreseeable future.

It is the purpose of this book to give the essential basis of photoelasticity, to describe techniques that are available for the designer, and to collate information published in the literature. Emphasis is placed on the unique value of photoelasticity as an aid to engineering design. Designing for high strength/weight or stiffness/weight ratio is of increasing importance in many fields, and unless the design is improved correspondingly, there will be an increased risk of failure by fatigue or other mode. Thus the maximum stress in particular, but also stress distributions, tend to become vital, and it is no longer sufficient to consider just the nominal stresses. Quite often the design is relatively complicated, not readily solved by straightforward calculations or by reference to standard solutions of the stress concentration factor. It is here that photoelasticity can be of value, for the stresses are found quite simply and accurately, variations in design are easily investigated, and routine checking of complicated designs becomes possible.

Whilst the book is intended to give the fundamental principles of photoelasticity, it is hoped that the student will be guided into the strength aspects of good design, the photoelastician will find useful information regarding techniques and properties of materials, and the designer will be interested in techniques for the optimization of design. A chapter has been added which describes an entirely new and complementary technique for the evolution of optimum design, only loosely related to the photoelastic techniques, but having similar objectives.

The author's initial experience in photoelasticity was gained some years ago at Rolls-Royce Ltd., Derby, in investigating ways of increasing the strength of reciprocating and gas turbine engine parts, and some aspects of this work were published in two papers presented to the Institution of Mechanical Engineers. The author gratefully acknowledges permission to reproduce material and he is also grateful for valuable comments made by Mr. V. M. Hickson of the Royal Aircraft Establishment who has kindly read the manuscript. Fellow members of the Stress Analysis Group of the Institute of Physics and Physical Society have generously given valuable infor-

mation in lectures and in informal discussion. Finally the author would like to thank associates in the United States and elsewhere for permission so readily given to quote from journals and papers.

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Definitions

Analyser. A filter for transmitting plane polarized light, placed farthest from the light source in a polariscope.

Birefringence; birefringent material. That property which causes a transparent material to be doubly refractive in polarized light, that is, the polarized light travels at different velocities in the two principal planes, so producing optical interference in a polariscope.

Circularly polarized light. Light in which the transverse vibration at any point is restricted to movement in a circle. It is equivalent to two plane polarized component rays of equal amplitude vibrating in perpendicular planes, with one ray a quarter of a wavelength ahead of the other. The vibration produces either a right-handed or a left-handed helix, wich is reversed by reflection at a mirror. Strictly, with known devices, only monochromatic light can be circularly polarized.

Compensator. A device for producing a measurable change in the relative retardation.

Cupic point. A point in a stressed model at which an isoclinic and a line of principal stress cross each other at right angles. That principal stress which acts along the line of principal stress is then at a maximum or minimum value.

Dichroism. The selective absorption of light on the planes of a crystal, to produce polarization of light.

Figure of merit, Q $(= E/f)$. The ratio of Young's modulus to the fringe-stress coefficient, or in uniaxial loading it is the number of fringes per unit thickness produced for unit change of direct strain.

Fringe $(\sigma_1 - \sigma_2 = \text{constant})$. A black band or line of constant relative retardation. The maximum shear stress, and therefore the difference between the (secondary) principal stresses in the plane normal to the direction of light, is constant at points along the fringe. Fringes are seen when a stressed transparent model is placed in a polariscope using *monochromatic* light.

Fringe order, n. The number of fringes counting from the zero-order fringe. This is the number of wavelengths (and fractions of a wavelength) of interference arising from the relative retardation produced in a model.

Fringe-strain coefficient, g $(= t(\varepsilon_1 - \varepsilon_2)/n)$. A material constant determining the strain-optical sensitivity, and defined as the difference between the two principal strains at a point in a two-dimensional model necessary to produce one fringe per unit thickness of material, for a specified monochromatic light.

Fringe-stress coefficient, material fringe value or fringe constant, $f\ (=t(\sigma_1-\sigma_2)/n)$. A material constant determining the stress-optical sensitivity. It is the difference between the two principal stresses at a point in a two-dimensional stressed model required to produce one fringe per unit thickness of material, for a specified monochromatic light.

Isochromatic ($\sigma_1-\sigma_2$ = constant). A coloured band or line of constant relative retardation along which the maximum shear stress, or the difference between the (secondary) principal stresses in the plane normal to the direction of light is constant. The maximum shear strain is also constant. Isochromatics are visible when a stressed model is placed in a polariscope using white light.

Isoclinic. A line along which the *directions* of the principal stresses in a two-dimensional model are constant. These directions coincide with the planes of polarization in a crossed plane polariscope. The direction of stress is not in general parallel to the isoclinic itself.

Isopachic ($\sigma_1+\sigma_2$ = constant). A line along which the sum of the two principal stresses in a two-dimensional model is constant.

Isostatic. See **Line of principal stress.**

Isotropic point ($\sigma_1=\sigma_2$). A point in a stressed two-dimensional model at which the two principal stress are equal. This produces zero fringe order.

Line of principal stress. A line whose direction at any point gives the direction of one of the principal stresses. Also termed an *isostatic* and a *principal stress trajectory.*

Photoelastic bench. A polariscope used for stress and strain analyses of transparent models.

Photoelasticity. The science of the measurement of stress or strain in a stressed transparent model by use of polarized light.

Plane polarized light. Light in which the transverse vibrations are restricted to movement in parallel planes. By convention the **plane of polarization** is assumed to be perpendicular to the plane of vibration of the electric force, that is, in the plane of the magnetic force.

Polariscope. An optical apparatus for polarizing and analysing light for the purpose of examining the optical properties of crystals and transparent materials.

Polarizer. A filter for producing plane polarized light, placed nearest the light source in a polariscope.

Principal stresses, σ_1, σ_2 and σ_3. At any point in a stressed two-dimensional model the direct stress attains a maximum and a minimum value in two mutually perpendicular directions, and these are termed the principal stresses. With three-dimensional models, a third principal stress acts in a direction perpendicular to the other two; thus one principal stress is the maximum stress, one the minimum, and the third is of intermediate value.

Principal stresses act in directions *normal* to their respective principal planes of stress. There is zero shear stress acting in the principal planes, but a maximum shear stress of $\frac{1}{2}(\sigma_1-\sigma_2)$, $\frac{1}{2}(\sigma_2-\sigma_3)$ or $\frac{1}{2}(\sigma_3-\sigma_1)$ in the three principal shear planes at 45° to the principal planes of stress.

Quarter wave plate ($\lambda/4$ plate). A transparent plate which produces a relative retardation of one-quarter of a wavelength between two perpendicular component polarized rays which pass through it. The quarter wave plate should normally be matched to the wavelength of the monochromatic light used.

Relative retardation, R. The length by which one polarized ray is retarded behind the other, in travelling through the two principal planes of stress in a model, or in travelling through a crystal. The retardation is almost independent of wavelength. Also termed the *optical path difference.*

Singular point ($\sigma_1 = \sigma_2 = 0$). A point in a stressed two-dimensional model at which the principal stresses are both of zero magnitude. The fringe order is zero. This is a special case of an isotropic point.

Strain-optical coefficient, $K\,(= R/t(\varepsilon_1-\varepsilon_2))$. A material constant determining the strain-optical sensitivity which is practically constant for light of differing wavelengths. It is the relative retardation produced for unit difference between the two pincipal strains (inches per inch) at a point in a two-dimensional model, per unit thickness.

Stress-optical coefficient, $C\,(= R/t(\sigma_1-\sigma_2))$. A material constant determining the stress-optical sensitivity which is practically constant for the different wavelengths of light. It is the relative retardation produced for unit difference in the two principal stresses at a point in a two-dimensional stressed model, per unit thickness of material.

Stress trajectory. **See Line of principal stress.**

List of Symbols

IN THE interests of standardization, the notation follows the recommendations of British Standard, B.S. 1991: Part 4: 1961 (p. 28), for terms used in photoelasticity where mentioned, except that subscripts 1, 2 and 3 are used instead of p, q and r to denote principal stresses or strains.

		British Units Pound-inch	SI Units Newton-metre
a	Instantaneous amplitude of light vibration	in	m
A	Maximum amplitude of light vibration	in	m
B	Brewster unit		cm^2/dyne
C	Stress-optical coefficient	in^2/lbf	m^2/N
E	Young's modulus	lbf/in^2	N/m^2
f	Fringe-stress coefficient or material fringe value	lbf/in.fr	N/m.fr
f^1, f_{eff}	As above, but for frozen stress	lbf/in.fr	N/m.fr
g	Fringe-strain coefficient	in/fr	m/fr
g^1, g_{eff}	As above, but for frozen stress	in/fr	m/fr
i	Angle of incidence	deg	rad
K	Strain-optical coefficient	in/in	m/m
n	Fringe order or number of fringes from zero		
Q	Figure of merit	fr/in	fr/m
r	Angle of refraction	deg	rad
R	Relative retardation, or optical path difference	in	m

t	Model thickness	in	m
T	Time	s	s
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Principal strains	in/in	m/m
λ	Wavelength	in	m
μ	Refractive index		
ν	Poisson's ratio		
$\sigma_1, \sigma_2, \sigma_3$	Principal stresses	lb/in^2	N/m^2
τ	Shear stress	lb/in^2	N/m^2
ϕ	Relative phase difference	deg	rad
ω	Angular velocity	deg/s	rad/s

Conversion Table

To convert	Non-SI Units into	SI Units	Multiply by
Length	1 Å	m	10^{-10}
Length	1 in	m	0·025 4
Area	1 in^2	m^2	$0{\cdot}645\ 16 \times 10^{-3}$
Volume	1 in^3	m^3	$16{\cdot}387\ 1 \times 10^{-6}$
Mass	1 lb	kg	0·453 592
Mass	1 ton (2240 lb)	kg	$1{\cdot}016\ 05 \times 10^3$
Density	1 lb/in^3	kg/m^3	$27{\cdot}679\ 9 \times 10^3$
Force	1 lbf	N	4·448 22
Force	1 tonf (2240 lbf)	N	$9{\cdot}964\ 01 \times 10^3$
Force	1 dyne	N	10^{-5}
Force	1 kgf	N	9·806 65
Moment, torque	1 lbf in	Nm	0·112 985
Moment, torque	1 kgf mm	Nm	$9{\cdot}806\ 65 \times 10^{-3}$
Stress	1 lbf/in^2	N/m^2	$6{\cdot}894\ 76 \times 10^3$
Stress	1 tonf/in^2	N/m^2	$15{\cdot}444\ 3 \times 10^6$
Stress	1 kgf/mm^2	N/m^2	$9{\cdot}806\ 65 \times 10^6$
Pressure	1 bar (14·504 lbf/in^2)	N/m^2	10^5
Stress-optical coefficient	1 in^2/lbf	m^2/N	$0{\cdot}145\ 038 \times 10^{-3}$
Strain-optical coefficient	1 in/in	m/m	1·0
Fringe-stress coefficient	1 lbf/in.fr	N/m.fr	175·127
Fringe-strain coefficient	1 in/fr	m/fr	0·025 4

Multiplying Factors

Factor by which the unit is multiplied	Prefix	Symbol
10^{12}	tera	T
19^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

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