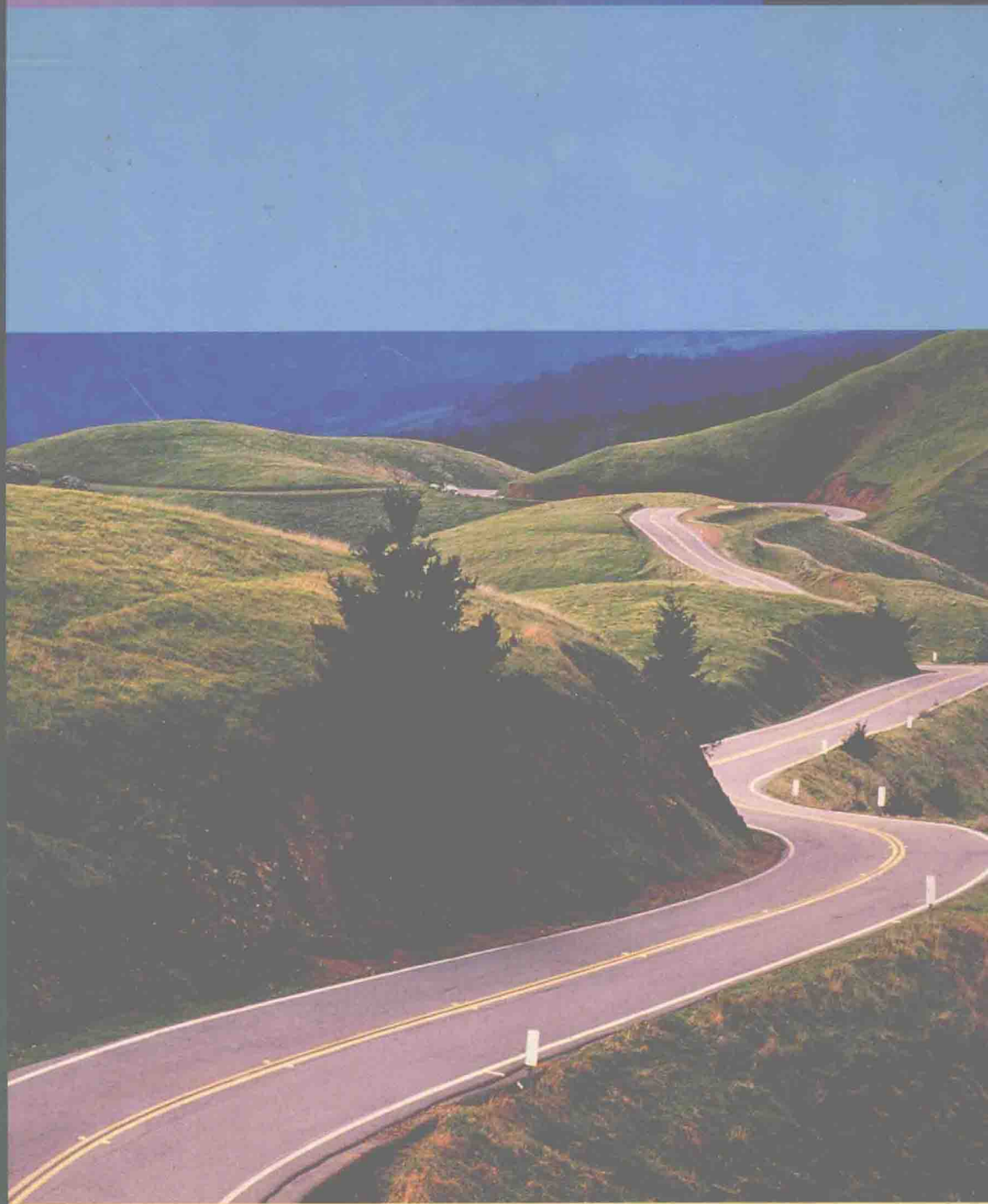
ELEMENTARY ALGEBRA

Annotated Instructor's Edition

fifth edition



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PREFACE

When preparing *Elementary Algebra*, Fifth Edition, I attempted to preserve the features that made the previous editions successful while at the same time incorporating a number of improvements suggested by reviewers. Some of this new material addresses the emerging reforms in the mathematics curriculum.

This text is for students who have never had an elementary algebra course, as well as for those who need a review before taking additional mathematics courses. The basic concepts of elementary algebra are presented in a simple, straightforward manner. Concepts are developed through examples, continuously reinforced through additional examples, and then applied in problem solving situations.

Algebraic ideas are developed in a logical sequence, but in an easy-to-read manner, without excessive technical vocabulary and formalism. Whenever possible, the algebraic concepts are allowed to develop from their arithmetic counterparts. The following are two specific examples of this development.

1. Manipulation with simple algebraic fractions begins early (Sections 2.1 and 2.2) when we review operations with rational numbers.
2. Multiplying monomials—without any of the formal vocabulary—is introduced in Section 2.4 when we work with exponents.

There is a common thread running throughout the book, namely, *learn a skill*, next *use the skill to help solve equations*, and then *use equations to solve word problems*. This thread also influenced the following additional decisions:

- Approximately 550 word problems are scattered throughout the text. Every effort was made to start with easy ones, thereby building confidence in solving problems. Numerous problem-solving suggestions are offered with special discussions in several sections. *The key is to work with various problem solving techniques and not to become overly concerned with whether all of the traditional types of problems are studied.*
- Newly acquired skills are used as soon as possible to solve equations and word problems. Therefore, the concept of solving equations is introduced early and developed throughout the text. The concepts of factoring, solving equations, and solving word problems are tied together in Chapter 6.

Approximately 700 worked-out examples demonstrate a large variety of situations while leaving some things for students to think about in the problem sets. Examples are used to guide students in organizing their work and to help

them decide when a shortcut may be tried. The progression from showing all the steps to demonstrating a suggested shortcut format is gradual.

As recommended by the American Mathematical Association of Two-Year Colleges, some geometric concepts are integrated in a problem solving setting that shows the connections between algebra, geometry, and the real world. Approximately 25 examples and 180 problems are designed to review basic geometric ideas. The following sections contain the bulk of the geometry material:

Section 2.5: Linear measurement concepts

Section 3.3: Complementary and supplementary angles; the sum of angles of a triangle equals 180°

Section 4.3: Area and volume formulas

Section 6.3: Pythagorean theorem

Section 10.1: More on the Pythagorean theorem, including work with isosceles right triangles and 30°–60° right triangles

New in This Edition

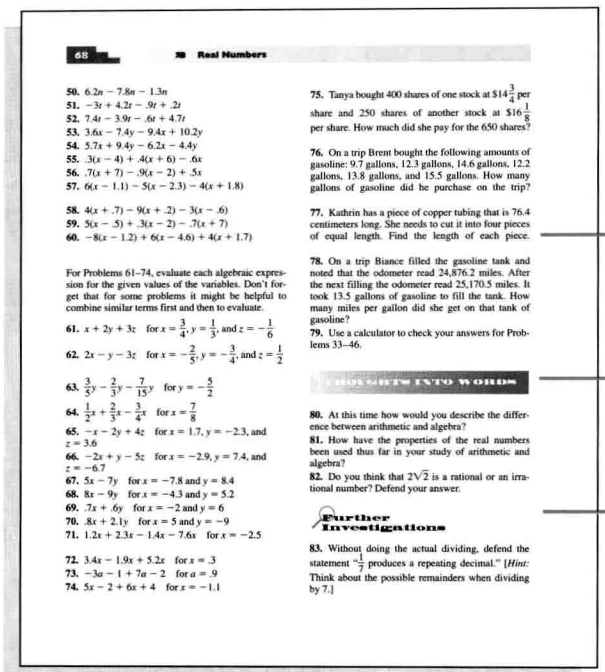
- Problems called **Thoughts into Words** are now included in every problem set except the review exercises. These problems are designed to give students an opportunity to express in written form their thoughts about various mathematical ideas. See, for example, Problem Sets 3.2, 3.4, 5.2, 5.4, and 6.4.
- Miscellaneous problem sections, now called **Further Investigations**, have been enhanced by adding more problems that lend themselves to small group work. These problems remain as "extras" but add flexibility for the instructor. See, for example, Problem Sets 1.2, 4.2, 5.3, and 9.2.
- The **chapter introductions** have been rewritten in an effort to provide more motivation for students to study algebra. Each introduction begins with at least one application that leads into the material of the chapter.
- A **Chapter Test** has been included at the end of each chapter. Along with the Chapter Review Problem Sets, these practice tests should provide the students with ample opportunity to prepare for the "real" tests.
- **Applications** have been added in several sections, including the following:
  1. Some applications have been added to sections of Chapters 1 and 2 that review arithmetic skills.
  2. In Chapter 5, examples and problems were added that connect geometry and the study of polynomials.
  3. In Section 8.3, several applications of slope were added as examples and problems.
  4. In Sections 9.1 and 9.5, applications involving radicals and radical equations were added.

5. In Section 10.1, some additional applications of the Pythagorean theorem were added.

- **Chapters 3 and 4** have been reorganized as suggested by some users of the previous edition. The work with ratios, proportions, and percents has been moved from the end of Chapter 3 to the beginning of Chapter 4. Inequalities and problem solving have been moved from the end of Chapter 4 to the end of Chapter 3. Old Section 4.6 (Equations and Inequalities Involving Absolute Value) has been moved to Chapter 11.
- At the request of users of the previous edition, a few more **motivating paragraphs** have been added. See, for example, the paragraphs at the ends of Sections 3.4, 4.4, and 7.6.

Other Special Features

- **Cumulative Review Problem Sets** appear at the ends of Chapters 2, 4, 6, 7, and 10. The one at the end of Chapter 10 could serve as a review of all of the basic topics of elementary algebra.
- All **answers** for Chapter Review Problem Sets, Chapter Tests, and Cumulative Review Problem Sets appear in the back of the text.
- **Problem Sets** have been constructed on an even-odd basis; that is, all variations of skill-development exercises are contained in both the even- and odd-numbered problems. Furthermore, problem sets are a focal point of every revision. We constantly add, subtract, and reword problems as suggested by users of the previous editions.
- **Chapter 11** was originally designed to be an "extra" chapter for those students who probably are going to take more mathematics. It remains that type of chapter with the addition of old Section 4.6 (Equations and Inequalities Involving Absolute Value) and old Section 8.8 (Graphing Linear Inequalities). Each section is a continuation of some topic presented in an earlier chapter.
- The text allows **flexibility in the continuity pattern**. For example, if an earlier introduction to graphing and solving systems of linear equations is desired, then Chapter 8 could be covered immediately after Chapter 4. This would provide the opportunity to use graphing and systems of linear equations to solve problems.



68 Real Numbers

50. $6.2n - 7.8n - 1.3n$
51. $-3t + 4.2t - .9t + .2t$
52. $7.4t - 3.9t - .6t + 4.7t$
53. $3.6x - 7.4y - 9.4x + 10.2y$
54. $5.7x + 9.4y - 6.2x - 4.4y$
55. $.3(x - 4) + .4(x + 6) - .6x$
56. $.7(x + 7) - .9(x - 2) + .5x$
57. $6(x - 1.1) - 5(x - 2.3) - 4(x + 1.8)$

58. $4(x + .7) - 9(x + .2) - 3(x - .6)$
59. $5(x - .5) + .3(x - 2) - .7(x + 7)$
60. $-8(x - 1.2) + 6(x - 4.6) + 4(x + 1.7)$

For Problems 61–74, evaluate each algebraic expression for the given values of the variables. Don't forget that for some problems it might be helpful to combine similar terms first and then to evaluate.

61. $x + 2y + 3z$ for $x = \frac{3}{4}$, $y = \frac{1}{3}$, and $z = -\frac{1}{6}$

62. $2x - y - 3z$ for $x = -\frac{2}{5}$, $y = -\frac{3}{4}$, and $z = \frac{1}{2}$

63. $\frac{3}{5}y - \frac{2}{3}y - \frac{7}{15}y$ for $y = -\frac{5}{2}$

64. $\frac{1}{2}x + \frac{2}{3}x - \frac{3}{4}x$ for $x = \frac{7}{8}$

65. $-x - 2y + 4z$ for $x = 1.7$, $y = -2.3$, and $z = 3.6$

66. $-2x + y - 5z$ for $x = -2.9$, $y = 7.4$, and $z = -6.7$

67. $5x - 7y$ for $x = -7.8$ and $y = 8.4$

68. $8x - 9y$ for $x = -4.3$ and $y = 5.2$

69. $.7x + .6y$ for $x = -2$ and $y = 6$

70. $.8x + 2.1y$ for $x = 5$ and $y = -9$

71. $1.2x + 2.3x - 1.4x - 7.6x$ for $x = -2.5$

72. $3.4x - 1.9x + 5.2x$ for $x = .3$
73. $-3a - 1 + 7a - 2$ for $a = .9$
74. $5x - 2 + 6x + 4$ for $x = -1.1$

75. Tanya bought 400 shares of one stock at $\$14\frac{3}{4}$ per share and 250 shares of another stock at $\$16\frac{1}{8}$ per share. How much did she pay for the 650 shares?

76. On a trip Brent bought the following amounts of gasoline: 9.7 gallons, 12.3 gallons, 14.6 gallons, 12.2 gallons, 13.8 gallons, and 15.5 gallons. How many gallons of gasoline did he purchase on the trip?

77. Kathrin has a piece of copper tubing that is 76.4 centimeters long. She needs to cut it into four pieces of equal length. Find the length of each piece.

78. On a trip Biance filled the gasoline tank and noted that the odometer read 24,876.2 miles. After the next filling the odometer read 25,170.5 miles. It took 13.5 gallons of gasoline to fill the tank. How many miles per gallon did she get on that tank of gasoline?

79. Use a calculator to check your answers for Problems 33–46.

THOUGHTS INTO WORDS

80. At this time how would you describe the difference between arithmetic and algebra?

81. How have the properties of the real numbers been used thus far in your study of arithmetic and algebra?

82. Do you think that $2\sqrt{2}$ is a rational or an irrational number? Defend your answer.

Further Investigations

83. Without doing the actual dividing, defend the statement "$\frac{1}{7}$ produces a repeating decimal." [*Hint:* Think about the possible remainders when dividing by 7.]

Well-crafted word problems reflect true-to-life situations.

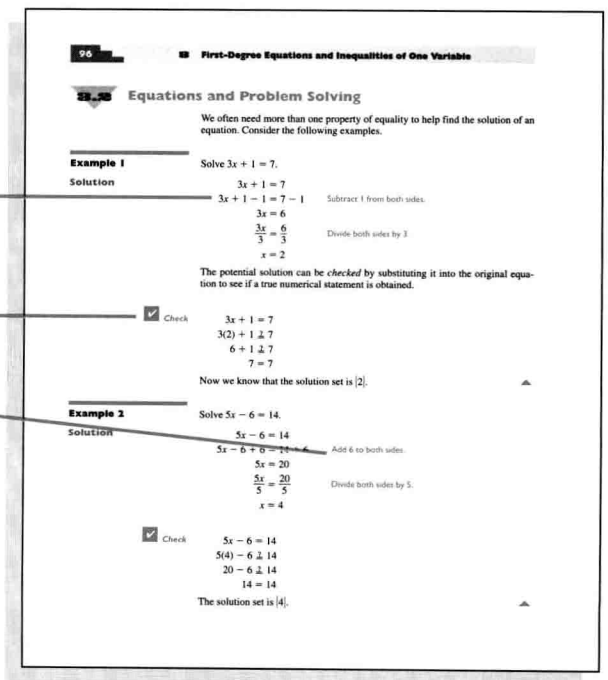
"Thoughts into Words" problems encourage students to express their mathematical understanding verbally.

"Further Investigations" problems, which require skills learned in the section, are especially appropriate for group work.

Many worked examples show careful, step-by-step problem solving.

The "Check" feature in worked examples and problems reminds students to complete this important problem-solving step.

Annotations make clear each step of the problem.



96 First-Degree Equations and Inequalities of One Variable

2.3B Equations and Problem Solving

We often need more than one property of equality to help find the solution of an equation. Consider the following examples.

Example 1

Solution Solve $3x + 1 = 7$.

$3x + 1 = 7$
$3x + 1 - 1 = 7 - 1$ Subtract 1 from both sides.
$3x = 6$
$\frac{3x}{3} = \frac{6}{3}$ Divide both sides by 3.
$x = 2$

The potential solution can be *checked* by substituting it into the original equation to see if a true numerical statement is obtained.

Check
$3x + 1 = 7$
$3(2) + 1 \stackrel{?}{=} 7$
$6 + 1 \stackrel{?}{=} 7$
$7 = 7$

Now we know that the solution set is $\{2\}$. ▲

Example 2

Solution Solve $5x - 6 = 14$.

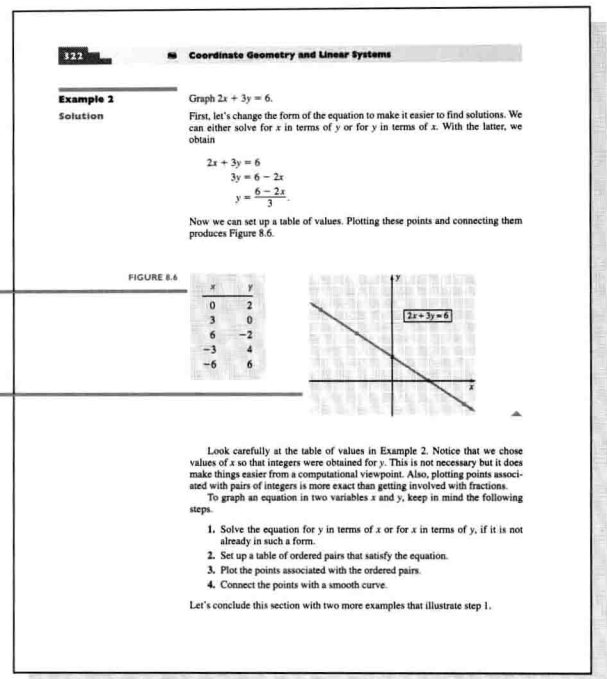
$5x - 6 = 14$
$5x - 6 + 6 = 14 + 6$ Add 6 to both sides.
$5x = 20$
$\frac{5x}{5} = \frac{20}{5}$ Divide both sides by 5.
$x = 4$

Check
$5x - 6 = 14$
$5(4) - 6 \stackrel{?}{=} 14$
$20 - 6 \stackrel{?}{=} 14$
$14 = 14$

The solution set is $\{4\}$. ▲

In sections on graphing, tables of values show numerical approaches to problems.

Large, clear graphs depict slopes accurately and restate the equation being graphed.



322 8 Coordinate Geometry and Linear Systems

Example 2

Solution

Graph $2x + 3y = 6$.

First, let's change the form of the equation to make it easier to find solutions. We can either solve for x in terms of y or for y in terms of x. With the latter, we obtain

$$2x + 3y = 6$$
$$3y = 6 - 2x$$
$$y = \frac{6 - 2x}{3}.$$

Now we can set up a table of values. Plotting these points and connecting them produces Figure 8.6.

FIGURE 8.6

x	y
0	2
3	0
6	−2
−3	4
−6	6

Look carefully at the table of values in Example 2. Notice that we chose values of x so that integers were obtained for y. This is not necessary but it does make things easier from a computational viewpoint. Also, plotting points associated with pairs of integers is more exact than getting involved with fractions.

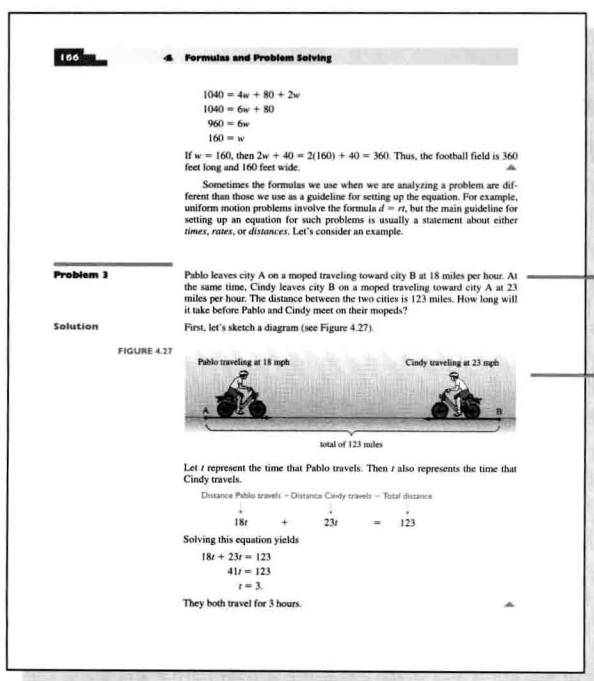
To graph an equation in two variables x and y, keep in mind the following steps.

1. Solve the equation for y in terms of x or for x in terms of y, if it is not already in such a form.
2. Set up a table of ordered pairs that satisfy the equation.
3. Plot the points associated with the ordered pairs.
4. Connect the points with a smooth curve.

Let's conclude this section with two more examples that illustrate step 1.

Many sample word problems are fully solved in sections specifically emphasizing problem solving.

Clearly rendered representational art lends interest and helps students visualize the problem.



166 4 Formulas and Problem Solving

$$1040 = 4w + 80 + 2w$$
$$1040 = 6w + 80$$
$$960 = 6w$$
$$160 = w$$

If $w = 160$, then $2w + 40 = 2(160) + 40 = 360$. Thus, the football field is 360 feet long and 160 feet wide.

Sometimes the formulas we use when we are analyzing a problem are different than those we use as a guideline for setting up the equation. For example, uniform motion problems involve the formula $d = rt$, but the main guideline for setting up an equation for such problems is usually a statement about either *times*, *rates*, or *distances*. Let's consider an example.

Problem 3

Pablo leaves city A on a moped traveling toward city B at 18 miles per hour. At the same time, Cindy leaves city B on a moped traveling toward city A at 23 miles per hour. The distance between the two cities is 123 miles. How long will it take before Pablo and Cindy meet on their mopeds?

Solution

First, let's sketch a diagram (see Figure 4.27).

FIGURE 4.27

Let t represent the time that Pablo travels. Then t also represents the time that Cindy travels.

Distance Pablo travels + Distance Cindy travels = Total distance

$$18t + 23t = 123$$

Solving this equation yields

$$18t + 23t = 123$$
$$41t = 123$$
$$t = 3.$$

They both travel for 3 hours.

Ancillaries for Instructors

The following supplements are available to adopters of this text:

- **Annotated Instructor's Edition** includes answers to all problems in the text—most printed adjacent to the problem.
- **Instructor's Solutions Manual** contains solutions for even-numbered problems and answers for all odd-numbered problems.
- **Test Bank with Chapter Tests** contains all questions and answers from the computerized test bank and three sample tests (two multiple choice, one open ended) for each chapter. These tests may be duplicated for student testing by instructors using the text.
- **Computerized testing software** is available for the IBM and compatibles and for the Macintosh. The computerized testing programs contain multiple-choice and open-ended questions that allow users to edit, rearrange, and add to the question bank.
- **Videotape Series** is text-specific, following the organization and style of the textbook. Video lectures include basic instruction and worked examples.

Ancillaries for Students:

- **Student's Solutions Manual** contains complete worked-out solutions for all odd-numbered exercises.
- **Worksheets and Study Guide** is a text-specific study resource in worktext format. It includes examples and exercises keyed to sections in the text so that students have the opportunity for additional practice and study assistance. The manual is designed to be integrated as an interactive component to lectures or for instructional use outside the classroom.
- **MathQuest Tutorial Software** is an interactive, text-specific intuitive tutorial that runs on both Windows and Macintosh platforms. The program provides fill-in, multiple-choice, and true/false questions. If a student answers a question incorrectly, the program will first respond with hints; if the student answers incorrectly a second time, the program will supply a step-by-step solution. Record-keeping capabilities enable students to monitor their progress.

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CONTENTS

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Elementary Algebra

FIFTH EDITION

Jerome E. Kaufmann

PWS Publishing Company

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CHAPTER 1

Some Basic Concepts of Arithmetic and Algebra

Karla started 1995 with $500 in her savings account and she planned to save an additional $15 per month for all of 1995. Without considering any accumulated interest, the numerical expression $500 + 12(15)$ represents the amount in her savings account at the end of 1995.

The numbers $+2$, -1, -3, $+1$, and -4 represent Woody's scores relative to par for five rounds of golf. The numerical expression $2 + (-1) + (-3) + 1 + (-4)$ can be used to determine how Woody stands relative to par at the end of the five rounds.

The temperature at 4 A.M. was $-14°$F. By noon the temperature had increased by 23°F. The numerical expression $-14 + 23$ can be used to determine the temperature at noon.

In the first two chapters of this text the concept of a numerical expression **is used as a basis for reviewing addition, subtraction, multiplication, and division of various kinds of numbers. Then the concept of a** variable **allows us to move from numerical expressions to** algebraic expressions**; that is, to start the transition from arithmetic to algebra. Keep in mind that algebra is simply a generalized approach to arithmetic. Many algebraic concepts are extensions of arithmetic ideas; your knowledge of arithmetic will help you with your study of algebra.**

1.1 Numerical and Algebraic Expressions

In arithmetic, we use symbols such as 4, 8, 17, and π to represent numbers. We indicate the basic operations of addition, subtraction, multiplication, and division by the symbols $+$, $-$, $\cdot$, and $\div$, respectively. Thus, we can formulate specific **numerical expressions**. For example, we can write the indicated sum of eight and four as $8 + 4$.

In algebra, **variables** allow us to generalize. By letting x and y represent *any* number, we can use the expression $x + y$ to represent the indicated sum of *any two* numbers. The x and y in such an expression are called variables and the phrase $x + y$ is called an **algebraic expression**. We commonly use letters of the alphabet such as x, y, z, and w as variables; the key idea is that they represent numbers. Our review of various operations and properties pertaining to numbers establishes the foundation for our study of algebra.

Many of the notational agreements made in arithmetic are extended to algebra with a few slight modifications. The following chart summarizes these notational agreements pertaining to the four basic operations. Notice the variety of ways to write a product by including parentheses to indicate multiplication. Actually the ab form is the simplest and probably the most used form; expressions such as abc, $6x$, and $7xyz$ all indicate multiplication. Also note the various forms for indicating division; the fractional form, $\frac{c}{d}$, is usually used in algebra although the other forms do serve a purpose at times.

Operation	*Arithmetic*	*Algebra*	*Vocabulary*
Addition	$4 + 6$	$x + y$	The *sum* of x and y
Subtraction	$7 - 2$	$w - z$	The *difference* of w and z
Multiplication	$9 \cdot 8$	$a \cdot b$, $a(b)$, $(a)b$, $(a)(b)$, or ab	The *product* of a and b
Division	$8 \div 2$, $\frac{8}{2}$, $2\overline{)8}$	$c \div d$, $\frac{c}{d}$, or $d\overline{)c}$	The *quotient* of c and d

As we review arithmetic ideas and introduce algebraic concepts, it is important to include some of the basic vocabulary and symbolism associated with sets. A **set** is a collection of objects, and the objects are called **elements** or **members** of the set. In arithmetic and algebra the elements of a set are often numbers. To communicate about sets, we use set braces, $\{\ \}$, to enclose the elements (or a description of the elements) and we use capital letters to name sets. For example, we can represent a set A, which consists of the vowels of the alphabet, as