

Essential Textbooks in Physics



Vijay Tymmms

# Newtonian Mechanics for Undergraduates

 World Scientific

Essential Textbooks in Physics

# Newtonian Mechanics for Undergraduates



**Vijay Tymms**

*Imperial College London, UK*

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI • TOKYO

*Published by*

World Scientific Publishing (UK) Ltd.

57 Shelton Street, Covent Garden, London WC2H 9HE

*Head office:* 5 Toh Tuck Link, Singapore 596224

*USA office:* 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

**Library of Congress Cataloging-in-Publication Data**

Names: Tymms, Vijay, author.

Title: Newtonian mechanics for undergraduates / Vijay Tymms, Imperial College London, UK.

Description: New Jersey : World Scientific, [2016] | Series: Essential textbooks in physics | Includes bibliographical references.

Identifiers: LCCN 2015030925 | ISBN 9781786340078 (UK) (hc : alk. paper) | ISBN 9781786340085 (pbk : alk. paper)

Subjects: LCSH--Mechanics--Textbooks.

Classification: LCC QC127 .T85 2016 | DDC 531--dc23

LC record available at <http://lcn.loc.gov/2015030925>

**British Library Cataloguing-in-Publication Data**

A catalogue record for this book is available from the British Library.

Copyright © 2016 by World Scientific Publishing (UK) Ltd.

*All rights reserved. This book, or parts thereof, may not be reproduced in any form or by any means, electronic or mechanical, including photocopying, recording or any information storage and retrieval system now known or to be invented, without written permission from the publisher.*

For photocopying of material in this volume, please pay a copying fee through the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, USA. In this case permission to photocopy is not required from the publisher.

In-house Editors: Mary Simpson/Dipasri Sardar

Typeset by Stallion Press

Email: [enquiries@stallionpress.com](mailto:enquiries@stallionpress.com)

Printed in Singapore by Mainland Press Pte Ltd.

# **Newtonian Mechanics for Undergraduates**

# Essential Textbooks in Physics

ISSN: 2059-7630

---

*Published*

Vol. 1    Newtonian Mechanics for Undergraduates  
*by Vijay Tymms*

# Contents

1.	Overview	1
1.1	Introduction . . . . .	1
1.2	Why This Book is Needed . . . . .	1
1.3	Who Will Benefit From This Book? . . . . .	2
1.4	Assumed Prior Knowledge . . . . .	2
1.5	Structure and Topics . . . . .	3
	Feedback for the Author . . . . .	4
2.	Introductory Concepts	5
2.1	Quantities, Units, and Coordinate Systems . . . . .	5
2.1.1	Scalar and Vector Quantities . . . . .	5
2.1.2	When Vectors Will Be Used and What Knowledge Will Be Assumed . . . . .	6
2.1.3	Vector Notation in Print and in Handwriting	6
2.1.4	Knowing When a Quantity is Scalar or Vector	7
2.1.5	Units . . . . .	7
2.1.6	Standard SI Prefixes . . . . .	8
2.1.7	Coordinate Systems . . . . .	9
2.2	Time, Displacement, Velocity, and Acceleration . . .	10
2.2.1	Time . . . . .	10
2.2.2	What is Meant By “Time is Linear and Universal” and Some Musing on Time Travel? . . . . .	10
2.2.3	Displacement . . . . .	11

2.2.4	Velocity . . . . .	12
2.2.5	Acceleration . . . . .	14
2.3	Force, Mass (and Acceleration) . . . . .	16
2.3.1	Mass . . . . .	16
2.3.2	Force . . . . .	18
2.3.3	Relating Force, Mass, and Acceleration . . .	19
2.3.4	$F = ma$ as a Cause-to-Effect Ratio and Other Examples in Physics . . . . .	19
2.3.5	Watch out for Careless Alternative Definitions . . . . .	20
2.3.6	Definitions of the Second, Metre, and Kilogram . . . . .	20
3.	1D Motion . . . . .	21
3.1	The Equations for Constant Acceleration . . . . .	21
3.1.1	Setting up the Basic Situation . . . . .	21
3.1.2	Finding $x$ as a Function of $t$ . . . . .	25
3.1.3	Finding $v$ as a Function of $x$ . . . . .	26
3.1.4	Two More Equations . . . . .	27
3.1.5	Using the Equations for Constant Acceleration . . . . .	28
3.2	Time-Dependent Forces . . . . .	30
3.3	Displacement-Dependent Forces . . . . .	32
3.4	Velocity-Dependent Forces . . . . .	35
3.5	More Complicated Forces . . . . .	36
4.	Newton's First and Second Laws of Motion . . . . .	37
4.1	Newton's First Law of Motion . . . . .	37
4.1.1	The Law is Not Valid in Accelerating Reference Frames . . . . .	37
4.1.2	Nor is the Law Valid on Subatomic Scales . .	38

4.2	Introducing Linear Momentum Before Stating Newton's Second Law . . . . .	39
4.3	Newton's Second Law of Motion . . . . .	40
4.4	Derivation of $F = ma$ and the Definition of the Newton . . . . .	41
4.5	Simple $F = ma$ Examples for a Point Particle . . . .	42
4.5.1	No Velocity, Balanced Forces . . . . .	42
4.5.2	Constant Velocity, Balanced Forces . . . . .	43
4.5.3	Constant Acceleration, Unbalanced Forces . . . . .	43
4.5.4	Non-Constant Acceleration, Unbalanced Forces . . . . .	44
4.5.5	Force Implies Acceleration and Acceleration Implies Force; Deduction and Induction . . .	44
4.6	Alternative Statements of the Laws . . . . .	45
5.	Types of Force and Free Body Diagrams . . . . .	47
5.1	Free Body Diagrams . . . . .	47
5.2	Types of Mechanical Force . . . . .	48
5.2.1	Weight . . . . .	48
5.2.2	Normal Contact Force . . . . .	50
5.2.3	Friction . . . . .	52
5.2.4	Tension and Compression . . . . .	57
5.2.5	Upthrust . . . . .	60
5.2.6	Drag Force . . . . .	61
5.2.7	Lift . . . . .	61
6.	Newton's Third Law of Motion . . . . .	63
6.1	Newton's Third Law of Motion . . . . .	63
6.2	Newton's Third Law Pairs . . . . .	64
6.2.1	Type 1: Long Range Forces ("Action at a Distance") . . . . .	64



6.2.2	Type 2: Contact Forces . . . . .	66
6.2.3	Type 3: Fluid Pressure Difference Forces . .	71
6.3	Misuses and Apparent Paradoxes . . . . .	71
6.3.1	Action and Reaction . . . . .	71
7.	Linear Momentum . . . . .	73
7.1	Linear Momentum . . . . .	73
7.2	Change in Momentum: Impulse . . . . .	73
7.3	The Conservation of Linear Momentum . . . . .	74
7.3.1	Proof of the Conservation of Momentum for a General Two Particle System . . . . .	74
7.3.2	Conservation of Momentum for an $N$ -Particle System . . . . .	75
7.4	Using the Conservation of Linear Momentum . . . .	76
7.5	Splitting Momentum Into Components . . . . .	79
7.5.1	Situations with a Resultant External Force Along One Component . . . . .	80
7.6	Two Classic Physics Puzzles . . . . .	80
7.6.1	The Sailing Boat and The Hair Dryer . . . .	80
7.6.2	The Lorry Driver and the Geese . . . . .	82
8.	Work, Energy and Power . . . . .	85
8.1	Work . . . . .	85
8.1.1	Definition, Units, and Values . . . . .	85
8.1.2	More on the Angle between the Force and the Displacement . . . . .	86
8.1.3	Non-Constant Forces . . . . .	87
8.1.4	Is the Work Done by Friction Positive or Negative? Some Words on Terrestrial Locomotion . . . . .	89
8.2	Energy, its Conservation, and Types of Energy . . .	90
8.3	Kinetic Energy and the Work–Energy Theorem . . .	91

8.4	Power . . . . .	93
8.4.1	Does the Work Done When Lifting an Object Depend on How Fast it is Lifted? . .	94
9.	Potential Energy . . . . .	95
9.1	Gravitational Potential Energy . . . . .	95
9.1.1	More Familiar Interpretation . . . . .	98
9.1.2	Potential Energy is Shared between Two or More Objects . . . . .	98
9.2	General Case in 1D . . . . .	98
9.3	Elastic Potential Energy . . . . .	100
9.3.1	Stored $Energy = \frac{1}{2} \times Constant \times Variable^2$ Formulae Appear Quite a Lot in Physics . .	101
9.4	Conservative and Non-Conservative Forces . . . . .	102
9.4.1	Introduction . . . . .	102
9.4.2	Other Properties . . . . .	103
9.4.3	Lifting a Box . . . . .	103
9.5	Potential Wells . . . . .	104
9.6	Mass–Energy Equivalence and $E = mc^2$ . . . . .	106
9.6.1	Mass–Energy in General . . . . .	106
9.6.2	Stretching a Spring . . . . .	107
9.6.3	Charging a Battery . . . . .	108
9.6.4	Kinetic Energy, Dissipation of Heat, and Cups of Tea . . . . .	108
9.6.5	Climbing a Mountain . . . . .	109
9.6.6	Combustion, Breathing, and Weight Loss . .	111
9.6.7	Nuclear Reactions . . . . .	113
10.	Collisions and Rockets . . . . .	115
10.1	Collisions . . . . .	115
10.1.1	Elastic Collisions . . . . .	116
10.1.2	Inelastic Collisions . . . . .	122

10.1.3	Superelastic Collisions . . . . .	125
10.2	Reference Frames . . . . .	126
10.3	Particle–Wall Collisions . . . . .	128
10.4	Fluid Jet Pressure . . . . .	129
10.5	Rocket Propulsion . . . . .	131
10.5.1	The Basic Principle of Rocketry . . . . .	131
10.5.2	Rocket Propulsion for a Constant Velocity Fuel Ejection . . . . .	131
11.	Motion on a Curved Path . . . . .	135
11.1	Uniform Circular Motion . . . . .	135
11.1.1	General Kinematic Analysis . . . . .	135
11.1.2	What This Tells Us . . . . .	137
11.1.3	Example of An Object Travelling Around a Circular Banked Track . . . . .	138
11.2	Motion on a General Curve with Changing Speed . . . . .	140
11.2.1	More on the General Radius of Curvature and How to Use it with the Circular Motion Equation . . . . .	140
11.2.2	Example of an Object Sliding Off a Round, Frictionless Hill . . . . .	142
12.	Simple Harmonic Motion . . . . .	147
12.1	Amplitude, Period, Frequency and Angular Frequency . . . . .	147
12.2	Sinusoidal Oscillations . . . . .	148
12.2.1	A Simple Harmonic Oscillator Does not Necessarily Exhibit SHM . . . . .	149
12.3	Two Examples of SHM . . . . .	150
12.3.1	What Does “Small Angle” Mean? . . . . .	153
12.4	SHM and Uniform Circular Motion . . . . .	153
12.5	Energy in SHM . . . . .	153

12.5.1	Kinetic and Potential Energies . . . . .	153
12.5.2	The Constant, $k$ . . . . .	155
12.5.3	The Potential Well Approach . . . . .	155
12.5.4	Example with the Simple Pendulum Revisited . . . . .	156
12.6	Other Features of SHM . . . . .	156
13.	Gravitation . . . . .	157
13.1	Newton's Law of Gravitation . . . . .	157
13.1.1	The Gravitational Force is Weak . . . . .	158
13.1.2	Point Masses . . . . .	159
13.1.3	Example: Circular orbits about a planet (with a preface on Newton's cannon) . . . .	159
13.1.4	The Inaccuracy of the Term "Weightless" . .	162
13.2	Gravitational Field Strength . . . . .	163
13.2.1	Gravitational Field Strength and Weight . .	163
13.2.2	$g$ : Gravitational Field Strength in $Nkg^{-1}$ or Acceleration Due to Gravity in $ms^{-2}$ ? . . . .	164
13.2.3	Inertial and Gravitational Mass . . . . .	164
13.3	Gravitational Potential and Binding Energy . . . . .	165
13.3.1	Proof of Equation 13.3 . . . . .	166
13.3.2	Escape Velocity . . . . .	168
13.3.3	Black Holes and the Schwarzschild Radius .	169
13.4	Gravitational Effects of A Spherical Shell . . . . .	169
13.4.1	The Force on a Mass Outside a Hollow Sphere . . . . .	170
13.4.2	The Force on a Mass Inside a Hollow Sphere . . . . .	172
13.5	Planetary Variations in Field Strength . . . . .	174
14.	Rotational Analogues . . . . .	177
14.1	Angular Velocity . . . . .	177

14.2	Angular Acceleration . . . . .	178
14.3	Rotational Kinetic Energy and Moment of Inertia . .	179
14.3.1	Single Particle . . . . .	179
14.3.2	Several Particles . . . . .	179
14.3.3	Continuum of Particles . . . . .	180
14.3.4	Meaning of Moment of Inertia . . . . .	181
14.3.5	Common Examples . . . . .	181
14.4	Torque . . . . .	184
14.4.1	Rotational Equivalent of Newton's Second Law . . . . .	185
14.5	Angular Momentum . . . . .	186
14.6	A Bit More on Scalars, Vectors, and Tensors . . . .	186
14.6.1	Angular Velocity vs. Linear Velocity . . . . .	187
14.6.2	The Moment of Inertia Tensor . . . . .	188
15.	Equilibrium and Balance	189
15.1	Centre of Mass . . . . .	189
15.1.1	Discrete Particle System . . . . .	189
15.1.2	Continuum System . . . . .	190
15.1.3	L-Shaped Object . . . . .	191
15.1.4	Importance . . . . .	191
15.2	Centre of Gravity . . . . .	192
15.3	Centre of Buoyancy . . . . .	193
15.4	Equilibrium . . . . .	194
15.5	Examples of Equilibrium . . . . .	195
15.5.1	See-Saw . . . . .	195
15.5.2	Balancing Pencil . . . . .	195
15.5.3	Leaning Ladder . . . . .	196
16.	Unbalanced Objects	201
16.1	An Unbalanced Light See-Saw . . . . .	201
16.2	Rigid Object Toppling About A Pivot . . . . .	202

16.2.1	The Forces . . . . .	202
16.2.2	Unstable Equilibrium . . . . .	203
16.2.3	Stable Equilibrium . . . . .	203
16.2.4	Toppling . . . . .	204
16.2.5	Accelerations for a Uniform Rod (with a Note on Why Balancing a Pencil on Your Fingertip is Difficult But Balancing a Broom Handle is Easy) . . . . .	204
16.2.6	The Tangential Linear Acceleration and a Surprising Result . . . . .	206
16.2.7	Energy Approach . . . . .	207
16.2.8	Variation of Forces with Angle . . . . .	207
16.2.9	Oscillations About the Stable Equilibrium Point . . . . .	209
17.	Rolling and Sliding	213
17.1	The Condition for Rolling . . . . .	213
17.1.1	Think About Riding a Bicycle . . . . .	216
17.2	Rolling Friction — Why Rolling Objects Stop at All	217
17.3	Rolling Down an Inclined Plane . . . . .	218
17.3.1	Analysis Using Energy . . . . .	218
17.3.2	Analysis Using Dynamics . . . . .	220
17.3.3	The Condition for No Slipping . . . . .	221
17.4	An External Force Causing Rolling on a Flat Surface . . . . .	222
18.	Angular Momentum	227
18.1	Definition . . . . .	227
18.2	Torque and Angular Momentum . . . . .	227
18.3	Moment of Inertia and Angular Momentum . . . . .	229
18.4	The Conservation of Angular Momentum . . . . .	229

18.5	Examples of the Conservation of Angular Momentum . . . . .	230
18.5.1	The Ice Skater (Or Less Agile Person Sat on a Rotating Platform) . . . . .	230
18.5.2	The Bicycle Wheel Variant . . . . .	232
18.5.3	Turning Yourself Around Without Translational Motion on An Ice Rink . . . . .	232
18.5.4	The Physics of the Falling Cat . . . . .	233
18.5.5	Kepler's Second Law . . . . .	234
19.	Angular Momentum, Gyroscopes, and Precession	237
19.1	The Gyroscope . . . . .	237
19.2	Application of Torque about the Pivot to a Spinning Gyro . . . . .	238
19.3	Precession Formula . . . . .	241
19.4	Analogy with Linear Circular Motion . . . . .	242
19.5	Analysis of Precession in Terms of Forces and Velocities . . . . .	243
19.6	Precession is Nothing to do with the Conservation of Angular Momentum . . . . .	243
19.7	More Subtle Features of Gyroscopic Motion . . . . .	245
19.8	The Earth's Precession . . . . .	245
19.9	Examples and Uses of Gyroscopic Motion . . . . .	246
	<i>Bibliography</i>	247
	<i>Index</i>	249

# 1

## Overview

### 1.1. Introduction

I have been teaching physics for 16 years, starting with secondary school teaching, then later university lecturing where I taught the first year mechanics lecture course at Imperial College for four years from 2010–2014. Teaching this course has been one of the most enjoyable parts of my career thus far, giving me an opportunity to rein-spect some of the most fundamental concepts in the discipline for delivery to a demanding (though appreciative) audience, complete with multiple demonstrations plus interesting problems and puzzles. During these years I developed and refined a set of comprehensive course notes tailored for the students I was teaching. This textbook is an adaptation of the notes, altered to appeal to a broader audience.

### 1.2. Why This Book is Needed

School syllabuses are in a state of constant flux. The breadth and depth of core physics and mathematics curricula taught in schools varies a little from year to year and a lot from generation to generation. So while well-established subjects in physics remain the same, the level of knowledge and understanding of students that enter university to study the discipline varies. This means that lecturers have to constantly update their courses to suit their target audiences and make the transition from A-level to degree as smooth as possible.



Although there are already many mechanics textbooks out there, there is a need for producing up-to-date reference material to match the level of development of the target audience. Essentially, textbooks quickly become out of date and there will always be a need for new ones. This particular one is designed to be in line with the level of physics and mathematics that contemporary school leavers ready to start a physics or physics-related degree will have.

### **1.3. Who Will Benefit From This Book?**

The lecture course that led to the creation of this book was designed specifically for first year physics undergraduates at Imperial College and as such the direct target audience of this textbook are students making the transition from school to university.

The book should also appeal to advanced A-level students unsatisfied with the level they have reached, and especially those who are considering studying physics or physics-related subjects beyond school. It contains some A-level material that is delivered at university level of presentation and should strengthen such students' understanding while also providing a smooth introduction to subtopics beyond the syllabus.

A-level physics teachers and first year university lecturers should also find the book useful; as well as the basic subject matter, in-depth examples and problems, there are also suggestions as to basic demonstrations that can easily be recreated in the classroom at minimal expense.

### **1.4. Assumed Prior Knowledge**

Regarding mathematics, all the content that can feature in a standard A-level mathematics syllabus is assumed knowledge throughout