Weak Interaction of Elementary Particles

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WEAK INTERACTION OF ELEMENTARY PARTICLES

FOREWORD

THIS book is based on lectures delivered by the author in 1960 and 1961 at the Institute of Theoretical and Experimental Physics of the Academy of Sciences of the USSR and at the Joint Institute of Nuclear Research. The book is meant for experimental physicists working in the field of elementary particles and high energies, and for young theoretical physicists specializing in this field.

The author has set himself two tasks: first, to make the reader familiar with the basic ideas and problems of the theory of the weak interaction of elementary particles; second, to make the reader familiar with the methods of calculation within the theory and to show him how the methods are to be applied.

The overall content of the book is concentrated about two pivotal hypotheses: the universality of the weak interaction, and the composite model of strongly interacting particles. These hypotheses allow the contents to be expounded in a more concise way and to retrace the connection between various problems in the theory of weak and strong interactions. Like every extrapolation, the hypotheses of the universality and of the composite model will undoubtedly be improved in the future and in part modified in the light of new experimental facts. In the form in which they are presented in the book, these hypotheses are to be considered as a "zero approximation".

The author is deeply grateful to A. I. Alikhanov and I. Y. Pomeranchuk, with whose initiative the lectures were delivered and the book was published, to I. Y. Kobzarev for his valuable advice, to V. B. Berestetskii who read the manuscript and made a number of useful remarks, and also to V. A. Kolkunov, E. P. Shabalin, V. V. Solovyev and N. S. Libova for their help in preparing the book for publication.

FOREWORD TO THE ENGLISH EDITION

In preparing this edition only minor amendments were made in the basic text of the book. To keep up, if only in part, with recent developments in this field, the chapter "Weak Interaction and Unitary Symmetry" was added. I take this opportunity to express my deep gratitude to V. B. Berestetskii, V. B. Mandeltsveig, I. Y. Pomerachnuk, J. Prentki, I. S. Shapiro, V. V. Sudakov, V. V. Vladimirskii and V. I. Zakharov for their discussions on various problems of unitary symmetry, which were very useful to me in writing the chapter quoted. I am in particular grateful to I. Y. Kobzarev, who read the manuscript and made a number of valuable remarks.

L. B. Okun'

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CHAPTER 1

PARTICLES. INTERACTIONS. MODELS

CLASSIFICATION OF ELEMENTARY PARTICLES

All matter around us is made of elementary particles. All known processes and interactions in nature are due to the interaction between elementary particles.

The present known elementary particles can be divided into four classes. The first class contains only one particle—the photon. The second consists of leptons: electron, muon, neutrino, and their antiparticles. The third one comprises the mesons: three π-mesons and four K-mesons. The fourth one contains baryons (nucleons, Λ-, Σ- and Ξ-hyperons) and antibaryons. All these particles are enumerated in Table 1.

Beside the mesons and baryons enumerated in Table 1 other particles are known which are not included in the table because of their extremely short lifetimes. These "particles" live for such a short time that they manifest themselves only in the form of resonances in reactions at high energies. Such resonances are often considered as excited states of the π-meson, K-meson, nucleon, Λ-hyperon and so on.†

TYPES OF INTERACTIONS

There are four types of elementary-particle interactions, sharply differing from one another: the gravitational, the electromagnetic, the strong and the weak.

The gravitational interaction has a very small coupling constant (it is very weak), and, if its character does not change sharply at small distances, its role is insignificant for the phenomena that we are going to consider. Indeed, the energy of gravitational interaction

† See tables of baryon and meson resonances on pp. 234 and 235 (note added in 1964).

of two protons set apart at a distance r is equal to

$$\varkappa \frac{m^2}{r},$$

where $\varkappa$ is the Newtonian constant, $\varkappa = 6 \times 10^{-39}/m^2$, while m is the proton mass.† When $r \sim 1/m$ this energy amounts to $\sim 10^{-38} m$

TABLE 1. MASSES AND MEAN LIVES OF STABLE ELEMENTARY PARTICLES

Class	Particle	Spin	Mass (MeV)	Mean life (sec)
Photon	γ	1	0	∞
Leptons	ν_e	$\frac{1}{2}$	$<2 \times 10^{-4}$	∞
	ν_μ	$\frac{1}{2}$	<4	∞
	e	$\frac{1}{2}$	$0{\cdot}511006 \pm 0{\cdot}000002$	∞
	μ	$\frac{1}{2}$	$105{\cdot}659 \pm 0{\cdot}002$	$(2{\cdot}2001 \pm 0{\cdot}0008) \times 10^{-6}$
Mesons	$\pi^\pm$	0	$139{\cdot}60 \pm 0{\cdot}05$	$(2{\cdot}551 \pm 0{\cdot}026) \times 10^{-8}$
	π^0	0	$135{\cdot}01 \pm 0{\cdot}05$	$(1{\cdot}80 \pm 0{\cdot}29) \times 10^{-16}$
	$K^\pm$	0	$493{\cdot}8 \pm 0{\cdot}2$	$(1{\cdot}229 \pm 0{\cdot}008) \times 10^{-8}$
	K_1^0	0	$498{\cdot}0 \pm 0{\cdot}5$	$(0{\cdot}92 \pm 0{\cdot}02) \times 10^{-10}$
	K_2^0	0	$498{\cdot}0 \pm 0{\cdot}5$	$(5{\cdot}62 \pm 0{\cdot}68) \times 10^{-8}$
	η^0	0	$548{\cdot}7 \pm 0{\cdot}5$	$\Gamma < 10$ MeV
Baryons	p	$\frac{1}{2}$	$938{\cdot}256 \pm 0{\cdot}005$	∞
	n	$\frac{1}{2}$	$939{\cdot}550 \pm 0{\cdot}005$	$(1{\cdot}01 \pm 0{\cdot}03) \times 10^3$
	Λ^0	$\frac{1}{2}$	$1115{\cdot}40 \pm 0{\cdot}11$	$(2{\cdot}62 + 0{\cdot}02) \times 10^{-10}$
	Σ^+	$\frac{1}{2}$	$1189{\cdot}41 \pm 0{\cdot}14$	$(0{\cdot}788 \pm 0{\cdot}027) \times 10^{-10}$
	Σ^0	$\frac{1}{2}$	$1192{\cdot}3 \pm 0{\cdot}3$	$< 1{\cdot}0 \times 10^{-14}$
	Σ^-	$\frac{1}{2}$	$1197{\cdot}08 \pm 0{\cdot}19$	$(1{\cdot}58 \pm 0{\cdot}05) \times 10^{-10}$
	Ξ^0	$\frac{1}{2}$	$1314{\cdot}3 \pm 1{\cdot}0$	$(3{\cdot}06 \pm 0{\cdot}40) \times 10^{-10}$
	Ξ^-	$\frac{1}{2}$	$1320{\cdot}8 \pm 0{\cdot}2$	$(1{\cdot}74 \pm 0{\cdot}05) \times 10^{-10}$
	Ω^-	?	1675 ± 3	$\sim 0{\cdot}7 \times 10^{-10}$

The table is taken from the article: A. ROSENFELD *et al.*, *Rev. Mod. Phys.* **36**, 977 (1964).

† Here and in what follows we shall use the system of units in which $\hbar = c = 1$. In this system the action and velocity are dimensionless quantities. Hence energy, momentum and mass have the same dimensions:

$$[E] = [p] = [m], \quad \text{since} \quad E = mc^2, \quad p = mv.$$

Dimensions of the time and length are also expressed in terms of those of mass:

$$[l] = [t] = [m^{-1}], \quad \text{since} \quad l = vt, \quad Et \sim \hbar, \quad pl \sim \hbar.$$

The reaction cross section has the dimensions $[\sigma] = [m^2]$, while the decay probability has the dimensions $[w] = [m]$.

It is easy to pass to ordinary units by taking into account the fact that $\hbar/mc \approx 2 \times 10^{-14}$ cm, $\hbar/mc^2 \approx 7 \times 10^{-25}$ sec, $1/m \approx 6 \times 10^{23}$ g^{-1}, m being the proton mass.

and, consequently, is negligibly small in comparison with the proton mass m.

The electromagnetic interaction, i.e. interaction of charged particles with photons and (in consequence of photon exchange) with each other, is characterized by the value of electric charge e. The energy of the Coulomb interaction of two protons set apart at a distance r is equal to α/r, where $\alpha = e^2 = \frac{1}{137}$, while e is the proton charge. For distances $r \sim m^{-1}$ between particles the energy of electromagnetic interaction amounts to αm and is small in comparison with the proper energy m of the particles. The relative weakness of electromagnetic interaction was used in constructing quantum electrodynamics, i.e. theory of interaction of electrons with photons. The smallness of the constant α allows one to consider electromagnetic interaction as a small perturbation and to develop in quantum electrodynamics the methods of perturbation theory. The mathematical methods of quantum electrodynamics enable calculations to be carried out with an accuracy exceeding that of contemporary experiments.

The strong interaction, i.e. the interaction between mesons and baryons, in contrast to the gravitational or electromagnetic, is short-range. The energy of the strong interaction between two particles at a distance larger than 10^{-13} to 10^{-12} cm is negligible, but at smaller distances ($r \sim 10^{-14}$ cm) the energy of strong interaction becomes of the same order of magnitude as the mass of the strongly interacting particles.†

If the photon emission by the electron is characterized by the dimensionless constant α, the pion emission by the nucleon can be characterized by the constant $g^2 \approx 14$. It is evident that such a strong interaction cannot be considered in terms of perturbation theory. Development of a theory of strong interaction has been, for about 30 years, one of the main problems in the physics of

† The term "strongly interacting particles" is cumbersome and inconvenient. Lack of a brief term for strongly interacting particles has led, for example, to the fact that decays into strongly interacting particles are called "non-leptonic". Such a term is inaccurate, since "non-leptonic" may also mean "photonic".

It is reasonable to call strongly interacting particles hadrons, and the corresponding decays—hadronic. In Greek the word hadros means "large", "massive", in contrast to the word leptos, which means "small", "light". The term hadron refers to long-lived mesons and baryons, as well as to resonances.

elementary particles. Recently there has been an intensive investigation of a dispersion approach to the theory of strong interaction, based on such general principles as causality and unitarity.

The weak interaction, responsible, in main, for elementary-particle decays, is very short-range: its effective radius is apparently substantially smaller than that of the strong interaction. It amounts to about 10^{-17} cm. The radius of the weak interaction and, consequently, its dependence on the momentum of the interacting particles, are only beginning to be investigated. We shall return to this later on. At low momenta the weak interaction can be characterized by the weak interaction constant $G = 10^{-5}/m^2$. The corresponding energy of the weak interaction of two protons set apart at a distance $1/m$ amounts to about $10^{-5}\ m$. This allows one to consider the weak interaction, in a number of cases, as a small perturbation, and to calculate slow processes due to it in the first order of a perturbation theory in the constant G. We shall discuss the validity of this procedure when we come back to the problem of the radius of the weak interaction (see Chapter 18).

ARE ELEMENTARY PARTICLES ACTUALLY ELEMENTARY?

It is now time to ask the question: are elementary particles actually elementary? This is quite natural when one takes into account the number of elementary particles that seem to exist at present. If, in addition, the number of extremely unstable excited states (resonances) is added in, the sum is not lower than that of elements Mendeleyev knew. It is therefore understandable why numerous attempts to "lower" the number of elementary particles have been made. The most radical of these attempts proceed from the assumption that all observed particles and interactions are manifestations of a unique non-linear spinor field whose non-linearity is characterized by a constant. Unfortunately, in the concrete development of this attractive idea (for example, in studies by the Heisenberg group) there are very serious difficulties. Such quantum numbers as electric charge, baryonic charge, strangeness, leptonic charge, leptonic strangeness (which distinguishes the muon from the electron) cannot as yet be obtained as characteristic numbers of some unique fundamental Lagrangian.

However, the question need not necessarily be formulated in the form that either all particles (fields) are elementary, or that there is only one fundamental field. One may adopt the standpoint that all elementary particles are characteristic states of a system of several fundamental fields interacting with one another. In this case individual fields act like carriers of a certain set of quantum numbers. This idea underlies numerous composite models of elementary particles. In such an approach it is natural to take as a principle of the theory the requirement that the number of fundamental fields and constants of interaction between them should be minimum.

COMPOSITE MODELS

Because the largest family of particles are strongly interacting, it is these that it is most important to reduce to a minimum number of fundamental ones. Besides, the large intensity of strong interactions leads one to expect, at least in principle, that the energies of interaction and, consequently, the mass differences between the particles, are of an order of magnitude comparable to the masses themselves. In 1949 Fermi and Yang pointed out that the π-meson may represent a bound state of the nucleon and antinucleon. In this case the binding energy must reach the enormous value of $940 \times 2 - 140 = 1740$ MeV. Data on the interaction of antinucleons with nucleons, obtained subsequently, do not contradict the assumption of such a strong attraction between these particles. With the discovery of strange particles a number of models appeared, in which some particles were chosen as fundamental ones, whereas the remaining particles were obtained as secondary or composite ones. The most economical model of such a type is the one proposed in 1956 by the Japanese physicist Sakata. In the Sakata model, fundamental particles are the three baryons: proton p, neutron n, and lambda hyperon Λ, and the corresponding antibaryons: antiproton $\tilde{p}$, antineutron $\tilde{n}$ and antilambda $\tilde{\Lambda}$. Thus, instead of more than twenty metastable mesons and baryons and about the same (or, maybe, a substantially larger) number of "excited" particles—resonances—the model contains three fundamental particles.

ISOTOPIC MULTIPLETS

One of the first successes of this model was that it allowed one to explain simply the classification of strongly interacting particles—the so-called scheme of isotopic multiplets, established by Gell-Mann and Nishijima few years earlier.

We have already mentioned that mesons are divided into the two groups π and K, while baryons are divided into the four groups: nucleons, Λ-hyperons, Σ-hyperons and Ξ-hyperons. These groups are called *isotopic multiplets*. The term is somewhat inaccurate, because the particles pertaining to a given multiplet are, according to nuclear physics, not isotopes but isobars, since they are of the same mass and different charge. It would be more correct to call these multiplets isobaric, as has been done, for example, in the journal *Nuclear Physics*. But, following tradition and custom, we shall, in what follows, use the term *isotopic*.

The particles pertaining to a given multiplet have not only almost the same masses but also analogous strong interactions. (The small mass differences within one multiplet are apparently of electromagnetic origin.) Hence different particles belonging to a given multiplet can be considered to be different charge states of the same particle. Thus, proton and neutron represent two charge states of the nucleon; Σ^+, Σ^- and Σ^0 represent three charge states of the Σ-hyperon, and so on. For describing isotopic multiplets it is convenient to make use of the isotopic-spin formalism (for more details see p. 83).

The number N of particles in a given multiplet is expressed directly in terms of the isotopic spin T characterizing this multiplet:

$$N = 2T + 1.$$

From this formula and from Table 1 it follows that the isotopic spin of Λ-hyperon is equal to zero (isosinglet); the isotopic spin of K-mesons, nucleons and Ξ-hyperons is equal to $\frac{1}{2}$ (isodublets), and the isotopic spin of π-mesons and Σ-hyperons is equal to 1 (isotriplets).

Not only metastable particles but also resonances possess a definite isotopic spin. The strong interaction conserves isotopic spin. This conservation law is due to the so-called isotopic invariance of strong interaction.

To the particles belonging to a given multiplet there correspond different values of the projection of isotopic spin onto the z-axis of a so-called isotopic space. Thus, to the proton and neutron there correspond, respectively, $T_3 = +\frac{1}{2}$ and $T_3 = -\frac{1}{2}$, while to the π^+, π^0 and π^- meson there correspond, respectively, $T_3 = +1$, $T_3 = 0$ and $T_3 = -1$.

The electric charge Q of a particle, its baryon number n and the third component of isotopic spin T_3 determine its strangeness S:

$$Q = T_3 + \frac{n}{2} + \frac{S}{2}.$$

It can easily be seen that $S = 0$ for π-mesons and nucleons, $S = +1$ for K-mesons, $S = -1$ for Λ- and Σ-hyperons and $\tilde{K}$-mesons, and $S = -2$ for Ξ-hyperons. Particles for which $S \neq 0$ are called in the literature strange particles. Strangeness conservation (equivalent to T_3 conservation) accounted for numerous specific properties of the reactions involving the creation and mutual conversion of strange particles. On the basis of the scheme of isotopic multiplets the existence of Σ^0- and Ξ^0-particles, unknown at the time, and dual properties of neutral K-mesons were predicted (see Chapter 15).

SCHEME OF ISOMULTIPLETS IN THE SAKATA MODEL

Assume that at small distances ($\sim 10^{-14}$ cm) there is attraction between an arbitrary fundamental baryon (p, n, Λ) and an arbitrary fundamental antibaryon ($\tilde{p}$, $\tilde{n}$, $\tilde{\Lambda}$), whereas there is repulsion between two baryons (or two antibaryons). In attracting each other the baryon and antibaryon form a bound state—a meson. Thus, a proton and an antineutron form a π^+-meson, a neutron and an antiproton form a π^--meson, p and $\tilde{\Lambda}$ form a K^+-meson, n and $\tilde{\Lambda}$ a K^0-meson, Λ and $\tilde{p}$ a K^--meson, Λ and $\tilde{n}$ a $\tilde{K}^0$-meson. π^0-meson is a combination of $p\tilde{p}$ and $n\tilde{n}$ with isotopic spin unity:

$$\pi^0 = \frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n}).\dagger$$

In analogy with mesons, "composite" hyperons are easy to obtain. They contain at least three particles: two baryons and an

† In essence, we operate here and in what follows with symbols of the type p and $\tilde{p}$ as with wave functions of corresponding particles and antiparticles. The states $p\tilde{p}$ and $n\tilde{n}$ may go over into each other. Their linear combinations $\frac{1}{\sqrt{2}}(p\tilde{p} - n\tilde{n})$ and $\frac{1}{\sqrt{2}}(p\tilde{p} + n\tilde{n})$ do not go over into each other. The first of these represents a component of an isotopic vector, while the second one represents an isotopic scalar (for more details see p. 94).