Quantum Field Theory in Condensed Matter Physics

凝聚态物理学中的量子场论

ALEXEI M. TSVELIK

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Alexei M. Tsvelik
*Department of Physics,
University of Oxford*

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Preface

The objective of this book is to familiarize the reader with the recent achievements of quantum field theory (henceforth abbreviated as QFT). The book is oriented primarily towards condensed matter physicists but, I hope, can be of some interest to physicists in other fields. In the last fifteen years QFT has advanced greatly and changed its language and style. Alas, the fruits of this rapid progress are still unavailable to the vast democratic majority of graduate students, post-doctoral fellows, and even those senior researchers who have not participated directly in this change. This cultural gap is a great obstacle to the communication of ideas in the condensed matter community. The only way to reduce this is to have as many books covering these new achievements as possible. A few good books already exist; these are cited in the General bibliography at the end of the Preface. Having studied them I found, however, that there was still room for my humble contribution. In the process of writing I have tried to keep things as simple as possible; the amount of formalism is reduced to a minimum. Again, in order to make life easier for the newcomer, I begin the discussion with such traditional subjects as path integrals and Feynman diagrams. It is assumed, however, that the reader is already familiar with these subjects and the corresponding chapters are intended to refresh the memory. I would recommend those just starting their research in this area to read the first chapters in parallel with some introductory course in QFT. There are plenty of such courses, including the evergreen book by Abrikosov, Gorkov and Dzyaloshinsky.†

Why study QFT? For a condensed matter theorist as, I believe, for other physicists, there are several reasons for studying this discipline. The first is that QFT provides some wonderful and powerful tools for

† I was trained with this book and thoroughly recommend it.

our research. The results achieved with these tools are innumerable; knowledge of their secrets is a key to success for any decent theorist. The second reason is that these tools are also very elegant and beautiful. This makes the process of scientific research very pleasant indeed. I do not think that this is a coincidence; it is my strong belief that aesthetic criteria are as important in science as empirical ones. Beauty and truth cannot be separated, because 'beauty is truth realized' (Vladimir Solovyev). The history of science strongly supports this belief: all great physical theories are at the same time beautiful. Einstein, for example, openly admitted that ideas of beauty played a very important role in his formulation of the theory of general relativity, for which any experimental support remained minimal for many years. Einstein is by no means alone; the reader is advised to read the philosophical essays of Werner Heisenberg, whose authority in the area of physics is hard to deny. Aesthetics deals with forms; it is not therefore suprising that a smack of geometry is felt strongly in modern QFT. For example, the idea that a vacuum, being an apparently empty space, has a certain symmetry, i.e., has a geometric figure associated with it. In what follows we shall have more than one chance to discuss this particular topic and to appreciate the fact that geometrical constructions play a major role in the behaviour of physical models.

The third reason for studying QFT is related to the first and the second. QFT has the power of *universality*. Its language plays the same unifying role in our times as Latin played in the times of Newton and Leibnitz. Its knowledge is the equivalence of literacy. This is not an exaggeration: the equations of QFT describe phase transitions in magnetic metals and in the early universe, the behaviour of quarks and fluctuations of cell membranes; in this language one can describe equally well both classical and quantum systems. The latter feature is especially important. From the very beginning I shall make it clear that from the point of view of calculations, there is no difference between QFT and classical statistical mechanics. Both these disciplines can be discussed within the same formalism. Therefore everywhere below I shall unify QFT and statistical mechanics under the same abbreviation of QFT. This language helps one

To see a World in a grain of sand
And a heaven in a wild flower,
Hold infinity in the palm of your hand
And eternity in an hour.
(William Blake)

I hope that by now the reader is sufficiently inspired for the hard work ahead, for which we must switch back to prose!

Let me now discuss the content of the book. One of its goals is to help the reader to solve future problems in condensed matter physics. These are more difficult to deal with than past problems, all the easy ones having already been solved. What remains is difficult, but is interesting nevertheless. The most interesting, important and complicated problems in QFT are those concerning strongly interacting systems. Indeed, most of the progress over the past fifteen years has been in this area. One widely known and related problem is that of quark confinement in quantum chromodynamics (QCD). This still remains unresolved, as far as I am aware. A lesser known example is the problem of strongly correlated electrons in metals near the metal-insulator transition. The latter problem is closely related to the problem of high-temperature superconductivity. Problems with the strong interaction cannot be solved by traditional methods, which are mostly related to perturbation theory. This does not mean, however, that it is not necessary to learn the traditional methods. On the contrary, complicated problems cannot be approached without a thorough knowledge of more simple ones. Therefore Part I of the book is devoted to such traditional methods as the path integral formulation of QFT and Feynman diagram expansion. It is not supposed, however, that the reader will learn these methods from this book. As I have said before, there are many good books which discuss the traditional methods, and it is not the purpose of Part I to be a substitute for them, but rather to recall what the reader has learnt elsewhere. Therefore discussion of the traditional methods is rather brief, and is targeted primarily at the aspects of these methods which are relevant to non-perturbative applications.

The general strategy of the book is to show how the strong interaction arises in various parts of QFT. I do not discuss in detail all the existing condensed matter theories where it occurs; the theories of localization and the quantum Hall effect are omitted and the theory of heavy fermion materials is discussed only very briefly. Well, one cannot embrace the unembraceable! Though I do not discuss all the relevant physical models, I do discuss all the possible scenarios of renormalization: there are only three of them. First, it is possible that the interactions are large at the level of a bare many-body Hamiltonian, but effectively vanish for the low-energy excitations. This takes place in quantum electrodynamics in (3+1) dimensions and in Fermi liquids, where scattering of quasiparticles on the Fermi surface changes only their phase (forward scattering). Another possibility is that the interactions, being weak at the bare level, grow stronger for small energies introducing profound changes in the low-energy sector. This type of behaviour is described by so-called 'asymptotically free' theories; among these are QCD, the theories describing scattering of conducting electrons on magnetic impurities in metals (the Anderson and the s-d models, in particular), models of two-dimensional magnets, and many others. The third scenario leads us to critical behaviour. In this case the interactions between low-energy excitations remain finite. Such situations occur at the point of a second-order phase transition. The past few years have been marked by great achievements in the theory of two-dimensional second-order phase transitions. A whole new discipline has appeared, known as conformal field theory, which provides us with a potentially complete description of all types of possible critical points in two dimensions. The classification covers two-dimensional theories at a transition point and those quantum $(1 + 1)$-dimensional theories which have a critical point at $T = 0$ (the spin $S = 1/2$ Heisenberg model is a good example of the latter).

In the first part of the book I concentrate on formal methods; at several points I discuss the path integral formulation of QFT and describe the perturbation expansion in the form of Feynman diagrams. There is not much 'physics' here; I choose a simple model (the $O(N)$-symmetric vector model) to illustrate the formal procedures and do not indulge in discussions of the physical meaning of the results. As I have already said, it is highly desirable that the reader who is unfamiliar with this material should read this part in parallel with some textbook on Feynman diagrams. The second part is less dry; here I discuss some miscellaneous and relatively simple applications. One of them is particularly important: it is the electrodynamics of normal metals, where on a relatively simple level we can discuss violations of the Landau Fermi liquid theory. In order to appreciate this part, the reader should know what is violated, i.e., be familiar with the Landau theory itself. Again, I do not know a better book to read for this purpose than the book by Abrikosov, Gorkov and Dzyaloshinsky. The real fun starts in the third and the fourth parts, which are fully devoted to non-perturbative methods. I hope you enjoy them!

Finally, those who are familiar with my own research, will perhaps be surprised by the absence in this book of exact solutions and the Bethe ansatz. This is not because I do not like these methods, but because I do not consider them to be a part of the *minimal* body of knowledge necessary for any theoretician working in the field.

Alexei Tsvelik
Oxford

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Contents

Part one

Introduction to methods

1
QFT: language and goals

Under the calm mask of matter
The divine fire burns.
(Vladimir Solovyev)

The reason why the terms 'quantum field theory' and 'statistical mechanics' are used together so often is related to the essential equivalence between these two disciplines. Namely, a quantum field theory of a D-dimensional system can be formulated as a statistical mechanics theory of a $(D+1)$-dimensional system. This equivalence is a real godsend for anyone studying these things. Indeed, it allows one to get rid of non-commuting operators and to forget about time-ordering, which seem to be characteristic properties of quantum mechanics. Instead one has a way of formulating the quantum field theory in terms of ordinary commuting functions, more or less conventional integrals, etc.

Before going into formal developments I shall recall the subject of quantum field theory (QFT). Let us consider first what classical fields are. To begin with, they are entities expressed as continuous functions of space and time coordinates $(\mathbf{x}, t)$. A field $\Phi(\mathbf{x}, t)$ can be a scalar, a vector (like the electromagnetic field represented by the vector potential $(\phi, \mathbf{A})$), or a tensor (like the metric field g_{ab} in the theory of gravitation). Another important thing about fields is that they can exist on their own, i.e., independent of their 'sources' – charges, currents, masses, etc. Translated to the language of theory, this means that a system of fields has its own action $S[\Phi]$ and energy $E[\Phi]$. Using these quantities and the general rules of classical mechanics one can write down equations of motion for the fields.

Example

As an example consider the derivation of Maxwell's equations for an electromagnetic field in the absence of any sources. I use this example in order to introduce some valuable definitions. The action for an electromagnetic field is given by

$$S = \frac{1}{8\pi} \int dt d^3x [\mathbf{E}^2 - \mathbf{H}^2] \tag{1.1}$$

where **E** and **H** are the electric and the magnetic fields, respectively. These fields are not independent, but are expressed in terms of the potentials:

$$\begin{aligned} \mathbf{E} &= -\nabla\phi + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{H} &= \nabla \times \mathbf{A} \end{aligned} \tag{1.2}$$

The relationship between $(\mathbf{E}, \mathbf{H})$ and $(\phi, \mathbf{A})$ is not unique; $(\mathbf{E}, \mathbf{H})$ does not change when the following transformation is applied:

$$\begin{aligned} \phi &\to \phi + \frac{1}{c}\frac{\partial \chi}{\partial t} \\ \mathbf{A} &\to \mathbf{A} + \nabla\chi \end{aligned} \tag{1.3}$$

This symmetry is called *gauge* symmetry. In order to write the action as a single-valued functional of the potentials, we need to specify the gauge. I choose the following one:

$$\phi = 0$$

Substituting (1.2) into (1.1) we get the action as a functional of the vector potential:

$$S = \frac{1}{8\pi} \int dt d^3x \left[\frac{1}{c^2}(\partial_t \mathbf{A})^2 - (\nabla \times \mathbf{A})^2 \right] \tag{1.4}$$

In classical mechanics particles move along trajectories with minimal action. In field theory we deal not with particles, but with configurations of fields, i.e., with functions of coordinates and time $\mathbf{A}(t, \mathbf{x})$. The generalization of the principle of minimal action for fields is that fields evolve in time in such a way that their action is minimal. Suppose that $\mathbf{A}_0(t, \mathbf{x})$ is such a configuration for the action (1.4). Since we claim that the action achieves its minimum in this configuration, it must be invariant with respect to an infinitesimal variation of the field:

$$\mathbf{A} = \mathbf{A}_0 + \delta\mathbf{A}$$

Substituting this variation into the action (1.4), we get:

$$\delta S = \frac{1}{4\pi}\int \mathrm{d}t\mathrm{d}^3x[c^{-2}\partial_t \mathbf{A}_0 \partial_t \delta\mathbf{A} - (\nabla \times \mathbf{A}_0)(\nabla \times \delta\mathbf{A})] + O(\delta\mathbf{A}^2) \quad (1.5)$$

The next essential step is to rewrite δS in the following canonical form:

$$\delta S = \int \mathrm{d}t\mathrm{d}^3x \delta\mathbf{A}(t,\mathbf{x})\mathbf{F}[\mathbf{A}_0(t,\mathbf{x})] + O(\delta\mathbf{A}^2) \quad (1.6)$$

where $\mathbf{F}[\mathbf{A}_0(t,\mathbf{x})]$ is some functional of $\mathbf{A}_0(t,\mathbf{x})$. **By definition**, this expression determines the function

$$\mathbf{F} \equiv \frac{\delta S}{\delta \mathbf{A}}$$

– the *functional* derivative of the functional S with respect to the function $\mathbf{A}$. Let us assume that $\delta\mathbf{A}$ vanishes at infinity and integrate Eq. (1.5) by parts:

$$\delta S = -\frac{1}{4\pi}\int \mathrm{d}t\mathrm{d}^3x\{c^{-2}\partial_t^2 \mathbf{A}_0(t,\mathbf{x}) - [(\nabla \times \nabla) \times \mathbf{A}_0(t,\mathbf{x})]\}\delta\mathbf{A}(t,\mathbf{x}) \quad (1.7)$$

Since $\delta S = 0$ for any $\delta\mathbf{A}$, the expression in the curly brackets (i.e., the functional derivative of S) vanishes. Thus we get the Maxwell equation:

$$c^{-2}\partial_t^2 \mathbf{A} - (\nabla \times \nabla) \times \mathbf{A} = 0 \quad (1.8)$$

Thus Maxwell's equations are the Lagrange equations for the action (1.4).

From Maxwell's equations we see that the field at a given point is determined by the fields at the neighbouring points. In other words the theory of electromagnetic waves is a mechanical theory with an infinite number of degrees of freedom (i.e., coordinates): these degrees of freedom are represented by the fields which are present at every point and coupled to each other. In fact it is quite correct to define classical field theory as the mechanics of systems with an infinite number of degrees of freedom. By analogy, one can say that QFT is just the quantum mechanics of systems with an infinite number of coordinates.

There is a large class of field theories where the above infinity of coordinates is trivial. In such theories one can redefine the coordinates in such a way that the new coordinates obey independent equations of motion. Then an apparently complicated system of fields decouples into an infinite number of simple independent systems. This is certainly possible to do for so-called linear theories, a good example of which is the theory of the electromagnetic field (1.4); the new coordinates in this

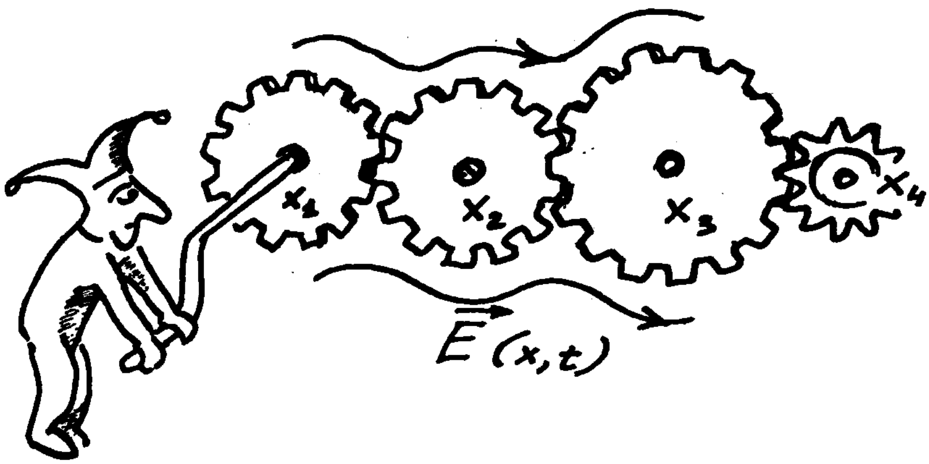

Fig. 1.1. Maxwell's equations as a mechanical system.

case are just coefficients in the Fourier expansion of the field **A**:

$$A(\mathbf{x}, t) = \frac{1}{V}\sum_{\mathbf{k}} \mathbf{a}(\mathbf{k}, t)e^{i\mathbf{kx}} \tag{1.9}$$

Substituting this expansion into Eq. (1.8) we obtain equations for the coefficients, which are just the Newton equations for harmonic oscillators with frequencies $\pm c|\mathbf{k}|$:

$$\partial_t^2 \mathbf{a}_i(\mathbf{k}, t) - (c\mathbf{k})^2 \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2}\right) \mathbf{a}_j(\mathbf{k}, t) = 0 \tag{1.10}$$

where $\mathbf{a} = (a_1, a_2, a_3)$.

The meaning of this transformation becomes especially clear if we confine our system of fields in a box with linear dimensions $L_i (i = 1, ..., D)$ and with periodic boundary conditions. Then our **k**-space becomes discrete:

$$k_i = \frac{2\pi}{L_i} n_i$$

(n_i are integer numbers). Thus the continuous theory of the electromagnetic field in real space looks like a discrete theory of independent harmonic oscillators in **k**-space. The quantization of such a theory is quite obvious: one should quantize the above oscillators and get a quantum field theory from the classical one. Things are not always so simple, however. Imagine that the action (1.4) has quartic terms in derivatives of **A**, which is the case for electromagnetic waves propagating through a