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13
生物数学丛书

Zhang Qimin Li Xining Yue Hongge

Stochastic Age-Structured Population Systems

（随机年龄结构种群系统）

科学出版社

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《生物数学丛书》序

传统的概念：数学、物理、化学、生物学，人们都认定是独立的学科，然而在 20 世纪后半叶开始，这些学科间的相互渗透、许多边缘性学科的产生，各学科之间的分界已渐渐变得模糊了，学科的交叉更有利于各学科的发展，正是在这个时候数学与计算机科学逐渐地形成生物现象建模，模式识别，特别是在分析人类基因组项目等这类拥有大量数据的研究中，数学与计算机科学成为必不可少的工具. 到今天，生命科学领域中的每一项重要进展，几乎都离不开严密的数学方法和计算机的利用，数学对生命科学的渗透使生物系统的刻画越来越精细，生物系统的数学建模正在演变成生物实验中必不可少的组成部分.

生物数学是生命科学与数学之间的边缘学科，早在 1974 年就被联合国科教文组织的学科分类目录中作为与“生物化学”、“生物物理”等并列的一级学科.“生物数学”是应用数学理论与计算机技术研究生命科学中数量性质、空间结构形式，分析复杂的生物系统的内在特性，揭示在大量生物实验数据中所隐含的生物信息. 在众多的生命科学领域，从“系统生态学”、“种群生物学”、“分子生物学”到“人类基因组与蛋白质组即系统生物学”的研究中，生物数学正在发挥巨大的作用，2004 年*Science*杂志在线出了一期特辑，刊登了题为“科学下一个浪潮 —— 生物数学”的特辑，其中英国皇家学会院士 Lan Stewart 教授预测，21 世纪最令人兴奋、最有进展的科学领域之一必将是“生物数学”.

回顾“生物数学”我们知道已有近百年的历史：从 1798 年 Malthus 人口增长模型，1908 年遗传学的 Hardy-Weinberg“平衡原理”；1925 年 Voltera 捕食模型，1927 年 Kermack-Mckendrick 传染病模型到今天令人注目的“生物信息论”，“生物数学”经历了百年迅速地发展，特别是 20 世纪后半叶，从那时期连续出版的杂志和书籍就足以反映出这个兴旺景象；1973 年左右，国际上许多著名的生物数学杂志相继创刊，其中包括 Math Biosci, J. Math Biol 和 Bull Math Biol；1974 年左右，由 Springer-Verlag 出版社开始出版两套生物数学丛书：*Lecture Notes in Biomathermatics*（二十多年共出书 100 部）和*Biomathematics*（共出书 20 册）；新加坡世界科学出版社正在出版*Book Series in Mathematical Biology and Medicine*丛书.

“丛书”的出版，既反映了当时“生物数学”发展的兴旺，又促进了“生物数学”的发展，加强了同行间的交流，加强了数学家与生物学家的交流，加强了生物数学学科内部不同分支间的交流，方便了对年轻工作者的培养.

从 20 世纪 80 年代初开始，国内对“生物数学”发生兴趣的人越来越多，他（她）

们有来自数学、生物学、医学、农学等多方面的科研工作者和高校教师, 并且从这时开始, 关于“生物数学”的硕士生、博士生不断培养出来, 从事这方面研究、学习的人数之多已居世界之首. 为了加强交流, 为了提高我国生物数学的研究水平, 我们十分需要有计划、有目的地出版一套“生物数学丛书”, 其内容应该包括专著、教材、科普以及译丛, 例如: ① 生物数学、生物统计教材; ② 数学在生物学中的应用方法; ③ 生物建模; ④ 生物数学的研究生教材; ⑤ 生态学中数学模型的研究与使用等.

中国数学会生物数学学会与科学出版社经过很长时间的商讨, 促成了“生物数学丛书”的问世, 同时也希望得到各界的支持, 出好这套丛书, 为发展“生物数学”研究, 为培养人才作出贡献.

陈兰荪

2008 年 2 月

Preface

This book is intend to give an introduction to the theory of stochastic age-structured population dynamic system which has received strong attention in recent years because of its interesting structure and its usefulness in various applied fields.

The motivation for studying stochastic age-structured population dynamic system comes originally from biomathematics theory. Biomathematics, a newly emerged interdisciplinary subject, researches the quantitive feature and spatial pattern of life body and bio-system. Biomathematics also resolve problem of bioscience by theories and methods in mathematics. The 21st century is the century of bioscience, the century of information science and the century of microelectronics technology, which is also the century of all sciences gradual transform from quantitive research to qualitative research. With the features of bioscience and quantitative science, the basis of a great deal of information and computer technique as tool, the mathematical modeling of biological systems is evolving into an integral part of biological experiments.

The study of biomathematics started in early 20s, which was known as the golden age of theoretical ecologis solutions, so the discussion of the numerical method of stochastic population system to become the field of mathematical biological hot issue. The book mainly study the numerical method of the population with a variety of noise models. Including the author's research. The given numerical methods and the conclusions provide new ideas and theoretical basis for stochastic evolution-type partial differential equations numerical calculation, but also to the value of the stochastic population system provides a reliable method. Provide a strong basis to explore epidemics, ecological environment and the protection of the population. Since the dynamics method study of the life sciences was first proposed, and the logistic model, prey model and infectious diseases model was considered to be the most famous model. Subsequently, on the basis of the modeling, age structure, time-lag, migration, random interference of environment, intraspecific competition for resources, interspecific competition for resources have been considered. With the development of computer technology of the sixties and seventies years, the awareness of the seriousness of the ecological crisis to promote the further development of mathematical biology. To solve the five major worldwide problem: resources, energy, environment, population and food are also related on Ecology. For example, to predict behavior of the system and make the best decisions of governance, also need the help of systems analysis and computer simulation. Thus, using computer simulation to study the complex ecosystem has become the main method of systems ecology research. The dynamic model of population and community has also been further developed, taking into account the detailed behavior of space consistency, stability, and vagueness, disagreement. Biomathematics' field of study is carried out on the human will face such as the lack of natural resources and energy, environmental pollution, overpopulation, disease and health is complex and difficult problem in the 21st century. Therefore, how to establish appropriate ecological model and numerical simulation are particularly important in the field of biomathematics.

In recent years, the study of taking into account the random disturbance on the population model system have been being attracted widely attention. In general, the majority of stochastic population system have no exact analytical solution This book focuses on the latest research results of the numerical calculation of biomathematical research in stochastic population system, which is characterized by considering the effect of random noise to the the population system, and the numerical method and convergence is studied. To maintain systematic and easy to read, we first give the existence and uniqueness of the solution; followed by numerical calculation. Combined with their own research, by the main line to the value of single-species model, numerical methods of the various types of single-species model is clarified. Discuss the numerical calculation for stochastic population system with the Brownian motion, fractional Brownian motion, Markovian process and Poisson process.

The book is organized as follows. As an introduction, we present several theory of stochastic processes, Brownian motion, Itô's formula and basic inequalities in the first part. The second part have three chapters, which introduced populations system model systemically, study the existence and uniqueness of solutions of stochastic age dependent population. The third part of a total of seven chapters, focuses on the numerical calculation of stochastic age dependent population. According to the Euler method and the semi-implicit Euler method, stochastic age-dependent population was discussed, a sufficient condition for the convergence of the numerical approximation solution to the analytic solution and Exponential stability is given,and by the large number of numerical examples to verify the effectiveness of the algorithm, solving construct a stable algorithm for random population development system. There are three chapters of Part IV, mainly discussed the value of stochastic delay neural networks.

We would like to thank all those who have helped us with their support and contributions in the preparation of this book. Special thanks are due to Professor Jin Zhen, Zhang Fengqin for their invaluable guidance and assistance. Special thanks are also due to our colleagues and friends in Ningxia University.

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Contents

Chapter 1

Introduction

1.1 Introduction

Systems in many branches of science and industry are often perturbed by various types of environmental noise. For example, conside the simple population growth model

$$\frac{dN(t)}{dt} = a(t)N(t) \tag{1.1}$$

with intinal value $N(0) = N_0$, where $N(t)$ is the size of the population at time t and $a(t)$ is the relative rate of growth. It might happen that $a(t)$ is not completely known, but subject to some random environmental effects. In other words,

$$a(t) = r(t) + \sigma(t)\text{“noise”},$$

so equation (1.1) becomes

$$\frac{dN(t)}{dt} = r(t)N(t) + \sigma(t)N(t)\text{“noise”}.$$

That is, in form of integration,

$$N(t) = N_0 + \int_0^t r(s)N(s)ds + \int_0^t \sigma(s)N(s)\text{“noise”}ds. \tag{1.2}$$

The questions are: What is the mathematical interpretation for the “noise” term and what is the integration $\int_0^t \sigma(s)N(s)\text{“noise”}ds$?

It turns out that a reasonable mathematical interpretation for the “noise” term is the so-called white noise $\dot{B}(t)$, which is formally regarded as the derivative of a Brownian motion $B(t)$, i.e. $\dot{B}(t) = \frac{dB(t)}{dt}$. So the term “noise”$dt$ can be expressed as $\hat{B}(t)dt = dB(t)$, and

$$\int_0^t \sigma(s)N(s)\text{“noise”}ds = \int_0^t \sigma(s)N(s)dB(s). \tag{1.3}$$

If the Brownian motion $B(t)$ were differentiable, then the integral would have no problem at all. Unfortunately, we shall see that the Brownian motion $B(t)$ is nowhere differentiable hence the integral can not be defined in the ordinary way. On the other hand, if $\sigma(t)N(t)$ is a process of finite variation, one may define the integral by

$$\int_0^t \sigma(s)N(s)dB(s) = \sigma(t)N(t)B(t) - \int_0^t B(s)d[\sigma(s)N(s)].$$

However, if $\sigma(t)N(t)$ is only continuous, or just integrable, this definition does not make sense. To define the integral, we need make use of the stochasticnature of Brownian motion. This integral was first defined by K. Itô in 1949 and is now known as Itô stochastic integral. The main aims of this chapter are to introduce the stochastic nature of Brownian motion and to define the stochastic integral with respect to Brownian motion.

To make this book self-contained, we shall briefly the basic notations of probability theory and stochastic processes. We then give the mathematical definition theory and stochastic processes. We then give the mathematical definition of Brownian motions and introduce their important properties. Making use of Brownian motion, we proceed to define the stochastic integral with respect to Brownian motion, we proceed to define the stochastic integral with respect to Brownian motion and establish the well-known Itô's formula. As the applications of Itô's formula, we establish several moment inequalities e.g. the Burkholder-Davis-Gundy inequality for the stochastic integral as well as the exponential martingale inequality. We shall finally show a number of well-known integral inequalities of Gronwall type.

1.2 Basic notations of probability theory

Probability theory deals with mathematical models of trials whose outcomes depend on chance. All the possible outcomes—the elementary events are grouped together to form a set Ω with typical element $\omega \in \Omega$. Not every subset of Ω is in general an observable or interes-ting event. So we only group these observable or interesting events together as a family $\mathcal{F}$ of subsets of Ω. For the purpose of probability theory,such a family $\mathcal{F}$ should have the following properties:

(i) $\varnothing \in \mathcal{F}$, where $\varnothing$ denotes the empty set;

(ii) $A \in \mathcal{F} \Rightarrow A^C \in \mathcal{F}$, where $A^C = \Omega - A$ is the complement of A in Ω;

(iii) $\{A_i\}_{i\geqslant 1} \subset \mathcal{F} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$.

A family $\mathcal{F}$ with these three properties is called a σ-algebra. The pair $(\omega, \mathcal{F})$ is called a measurable space, and the elements of $\mathcal{F}$ is henceforth called $\mathcal{F}$-measurable sets instead of events. If $\mathcal{C}$ is a family of subsets of Ω, then there exists a smallest σ-algebra $\sigma(\mathcal{C})$ on Ω which contains $\mathcal{C}$. This $\sigma(\mathcal{C})$ is called the σ-algebra generated by $\mathcal{C}$. If $\Omega = R^d$ and $\mathcal{C}$ is the family of all open sets in R^d, then $\mathcal{B}^d = \sigma(\mathcal{C})$ is called the Bored σ-algebra and the elements of $\mathcal{B}^d$ are called the Borelsets.

A real-valued function $X : \Omega \to \mathcal{R}$ is said to be $\mathcal{F}$-measurable if

$$\{\omega : X(\omega) \leqslant a\} \in \mathcal{F} \ \text{ for all } \ a \in \mathcal{R}.$$

The function X is also called a real-valued ($\mathcal{F}$-measurable)random variable. An R^d-valued function $X(\omega) = (X_1(\omega), \cdots, X_d(\omega))^{\mathrm{T}}$ is said to be $\mathcal{F}$-measurable if all the elements X_i are $\mathcal{F}$-measurable. Similarly, a $d\times m$-matrix-valued function $X(\omega) = (X_{ij}(\omega))_{d\times m}$ is said to be $\mathcal{F}$-measurable if all the elements X_{ij} are $\mathcal{F}$-measurable. The indicator function I_A of a set $A \subset \Omega$ is defined by

$$I_A(\omega) = \begin{cases} 1, & \text{for } \ \omega \in A, \\ 0, & \text{for } \ \omega \notin A. \end{cases}$$

The indicator function I_A is $\mathcal{F}$-measurable if and only if A is an $\mathcal{F}$-measurable set, i.e. $A \in \mathcal{F}$. If the measurable function is then called a Borel measurable function. More generally, let $(\Omega', \mathcal{F}')$ be another measurable space. A mapping $X : \Omega \to \Omega'$ is said to be $(\mathcal{F}, \mathcal{F}')$-measurable if

$$\{\omega : X(\omega) \in A'\} \in \mathcal{F} \ \text{ for all } \ A' \in \mathcal{F}'.$$

The mapping X is then called an Ω'-valued ($\mathcal{F}, \mathcal{F}'$-meadsurable) (or simply, $\mathcal{F}$-measurable) random variable. Let $X : \Omega \to R^d$ be any function. The σ-algebra $\sigma(X)$ generated by X is the smallest σ-algebra on Ω containing all the sets $\{w : X(w) \in U\}$, $U \subset R^d$ open. That is

$$\sigma(X) = \sigma(\{w : X(w) \in U\} : U \subset R^d \text{ open}).$$

Clearly, X will then be $\sigma(X)$-measurable and $\sigma(X)$ is the smallest σ-*algebra* with this property. If X is $\mathcal{F}$-measurable, then $\sigma(X) \subset \mathcal{F}$,i.e. X generates a sub-σ-algebra of $\mathcal{F}$. If $X_i : i \in I$ is collection if R^d-valued functions , define

$$\sigma(X_i : i \in I) = \sigma\left(\bigcup_{i\in I} \sigma(X_i)\right)$$

which is called the σ-algebra generated by $X_i : i \in I$. It is the smallest σ-algebra with respect to which every X_i is measurable. The following results is useful. It is a special case of a result sometimes called the Doob-Dynkin lemma.