Kalamazoo Valley Community College

Math 096 Elementary Algebra Supplement

Warr

THOMSON LEARNING
CUSTOM PUBLISHING

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Custom Contents

From A First Course in Algebra

Chapter Zero

An Introduction to Learning Mathematics, Problem Solving, and Teamwork

The goal of this text may be different from that of others you have used in the past. Don't be surprised if you find the presentation of the material and the overall content unique as well. The purpose of this chapter is to communicate the goals of the text and to give you some insight into our thoughts about learning mathematics and the reason we approached the writing of this textbook as we did.

Learning Mathematics

One of the major goals of this textbook is to help you become a better problem solver. This means not only solving equations and typical mathematics problems but also being able to look at unfamiliar problems, choosing strategies, and applying mathematics to solve these problems.

A second goal of this textbook is to present mathematical ideas in meaningful ways. Mathematics is a useful tool in solving problems in many different fields, and we want you to see this value in mathematics.

Many of you may have learned mathematics in a lecture format. From our experience we have found several drawbacks to a mathematics course taught primarily from a lecture. Many students become bored by mathematics taught in this format and consider the subject matter as a collection of abstract, disjointed facts that they are unable to use in real-life situations. They therefore feel that mathematics is an unimportant hoop for them to jump through to receive a degree.

A more important issue is the quality of learning and understanding that students achieve through this approach. Many students who take mathematics in a lecture format comment that "It looks so easy when you do it, but I can't." In a lecture, an instructor typically shows the students many examples and works each of them out correctly. Of course it looks simple! The instructor knows exactly how to do the mathematics. It is much like watching carpenters work. They know exactly how to perform the skills and have hours of experience. It is an entirely different situation when an inexperienced person tries to perform the job. You do not know where you are going to have difficulty until you try. Mathematics is similar. Until you dig into the material, you may not know what you can do or where you have questions.

For this reason, this textbook expects you to actively participate in learning mathematics. In doing so you will not only learn the skills necessary to perform specific tasks, but also obtain an understanding of the concepts behind the mathematics and know when and where to apply the concepts and skills.

Learning anything new takes effort. Learning mathematics is no exception. In using this textbook you will be asked to work. You will be involved in doing mathematics both in and outside the classroom. You will be asked to discuss mathematics with your classmates and your instructor. To be successful in this course, you cannot be a passive recipient of mathematical facts. We are asking you to do more than put in time doing assignments. We are asking you to *think!* You will be asked to think about the mathematics that you are doing, as well as what is going on in your mind as you do mathematics. You will need to decide when to struggle with a mathematics problem and when to ask for help. In essence, we are asking you to take responsibility for your own learning.

In return, we promise that this textbook will provide you with the opportunity to learn mathematics in a meaningful way. If you use it properly and your course is consistent with the principles on which the textbook is written, then you will begin to claim these mathematical ideas as your own. Mathematics will no longer be a mystery or a mere collection of facts. It will be a collection of principles and concepts that you can understand. You will find yourself less frequently staring at a collection of symbols not knowing how to proceed. More often you will see many possibilities in choosing where to start a problem. Finally, we believe that you will begin to see the use, value, power, and beauty of mathematics.

Discovery Learning

In this textbook, we adopted a discovery approach to learning. Most sections begin with an Activity Set. These activities were written with the assumption that you will be working on them with other members of your class. Each activity has been written to provide you with the

opportunity to "dig into" the mathematics. Although we do not expect that you will learn all the mathematics through these activities, we do expect you to be actively involved with others in your class and to make your best effort in discussing your observations of patterns and in making conjectures.

Following a set of activities, your instructor will typically lead a discussion of the observations and conjectures made by the class. This discussion may include vocabulary, generalizations to formulas, or applications of the ideas learned in the activities. Even when you do not reach all of the conclusions intended in the activities, attempting the activities will make this discussion more meaningful and useful.

How to Use This Text

We have tried to produce a textbook that

- demands greater understanding and less routine manipulation
- covers less material in greater depth
- presents concepts numerically, graphically, and algebraically
- develops concepts through commonsense investigations rather than abstract definitions
- incorporates the use of technology and expects technology to be available at all times
- is written to be read
- is written for students to discover concepts, not as a reference for those who know the concepts

We wrote this textbook with you, the student, in mind. We believe that the combination of doing discovery activities, participating in class discussions, and actively reading the textbook will lead to your success in learning mathematics. Most sections of this textbook begin with activities, are followed by a text discussion, and end with a problem set. Some of the discussions read like a standard textbook, but there are some important differences. Many of the examples emphasize mathematics in context. We de-emphasize the rote learning of mathematics. Questions are raised and left to you, the reader, to answer. In addition, appendices in the back of the textbook include review material.

To successfully use this book, you must read it with a pencil and calculator in hand. The activities at the beginning of each section *should* be done first; however, you are not penalized for looking ahead. Once you have done the activities, you should participate in any class discussion that may follow, read the textbook in detail, answer any questions that are asked, work through the examples as you read them, and make notes in the margins. Most importantly, write down any questions that you have, and be sure that you get your questions answered.

Mathematics was created to solve real-world problems like balancing a checkbook or determining the load that a roof can bear. Because mathematics has many different applications, it is not possible to learn specific methods or templates to solve every type of application. Instead we need to learn problem-solving skills that will allow us to tackle all sorts of problems. For this reason, you will not find worked examples for every type of problem you will encounter in the problem sets. In addition, for a given problem, many different solution processes are often correct. We encourage you to discuss your solutions with your classmates. As you see different ways of approaching and solving problems, you will learn new strategies that can be used on future applications.

What Is a Good Problem Solver?

One of the goals of this course is to make you a better problem solver, but what is a good problem solver? The following is a list of suggestions for becoming a better problem solver written by students who successfully completed a course in problem solving.

How to Become a Better Problem Solver

- Accept the challenge of solving a problem.
- Take time to explore, reflect, think, . . .
- Look at the problem in a variety of ways.
- If appropriate, try the problem using simple numbers or break the problem into smaller steps.
- Develop good problem-solving helper skills. Don't give solutions; instead provide meaningful hints.
- Write up your solutions neatly and clearly enough so that you could understand your solution if you reread it in ten years.
- Help others by giving hints. You will find that you develop new insights.
- Don't hesitate to take a break. Many problems require an incubation period. But remember to return to try again!
- Be persistent. Don't give up!
- Don't just sit there, do something!

When solving a problem in mathematics (or elsewhere), you may begin by asking "How do I get started?" There is not one answer to this question but several possible ways to get started. The following is a list of strategies for trying to solve a problem. These are just the beginning. As you solve more and more problems, you will be adding to this list.

Write Down Everything You Know

Write down what you know about the problem and the question you are trying to answer. This can help organize the information.

Draw a Picture

Can you draw a picture of the problem situation? Often you may find that in the process of drawing and labeling a picture you gain some insight into relationships that can lead to the solution of the problem, or at least to the next step in the process.

Make a Table of Values

A table of values can give you some concrete information from which to discover patterns. In the textbook, you will see how to use this powerful tool.

Guess

If you don't know where to begin, go ahead and guess. Many problems have been solved by someone saying "What if . . .?" Once you make a guess, your next step is to determine whether or not your guess is correct. This can be done by substituting your guess into the problem situation. If you guessed correctly, great! If you did not, you may find the insight that you need to solve the problem, or you may be able to eliminate several possibilities.

Make a Physical Model

Many of us need to physically see the situation to understand what is happening. This may mean we need to represent the problem using concrete objects. For example, in Problem 2,

"Moving Dots" in the Problem Set at the end of this chapter, you may want to represent the shaded dots with pennies and the unshaded dots with dimes. Then you can physically try solving the problem using the coins. Other times you may want to build a model. For example, if the problem is about a swimming pool, it may be helpful to construct a model of the swimming pool out of cardboard.

Throughout this course, you will encounter many mathematical problems. Some you will be able to solve with what you already know or what you are learning. Others you will need to develop more ideas and tools to solve. The time to conclude that you are unable to solve a problem is *after you have given it a royal try—not before.*

As you struggle with various problems throughout the course, the most important thing for you to remember is not to think of these problems in terms of success and failure. When you are exploring new ideas and the results that you get do not completely solve the problem, think of this as an opportunity to learn and improve your understanding. Often a strategy that does not lead to the solution will lead to new strategies.

Why Work in Teams?

In this class, you may be expected to work in a team. Students often ask why they are being asked to do this. There are many reasons for working in a team in this class. Working in a team gives you a safe environment to try out new ideas and problem-solving strategies. By working with your team, you will learn to communicate more clearly your ideas and your understanding of concepts. Your facility with the language of mathematics will improve as you get more practice using it. You will also learn to listen carefully to other people's ideas. You will be challenged to explain concepts so others can understand them. The give and take of ideas will improve your own reasoning and critical thinking abilities.

In addition, many recent school-to-work studies call on schools to teach students more of the skills that business and industry find their employees need. The general ability to work with others and communicate clearly, orally and in writing, are always on the lists of desired skills. Other expectations are that workers know how to develop new skills, can manage themselves, and are responsible for their actions. Working in teams in class gives you one more place to develop and refine these skills. There is more to school than learning specific academic skills.

Production Team Versus Learning Team

Although many of the skills learned by working in teams are appropriate for both school and work settings, many important differences exist. In a workplace team, the team may have been formed by pulling together people with different skills and expertise to work together to produce a product. In this situation, the team may do some initial planning together. They may then go off to produce their individual part of the project before the team comes back together to integrate the separate pieces into the finished product. Each member of the team relies heavily on the differing expertise of the team members. Although the team members count on each other for support and trouble shooting, they may not be working closely together every day. This type of team could be called a **production team.**

In the classroom, everyone on the team is expected to learn the same concepts. The product they are producing is mutual understanding of the course material—not a consumer product for others. It is important that all members of the team gain a full understanding of all concepts. The main point of the team is to help in the learning process. We refer to this type of team as a

learning team. We need to keep in mind that in this class we are forming learning teams and not production teams.

Team Selection

The manner in which teams are selected will vary from class to class and instructor to instructor. Many instructors look for a balance of gender, previous experience, age, and motivation. Other considerations that may be taken into account are times you are available to study outside of class or what kind of graphing calculator you have. The object is to get a mix that allows all of you to have a good learning experience.

What Makes a Team Work?

For a team to be effective, it is important that each member knows what is expected of him or her. Often ground rules for team behavior are established to help with this process. A sample set of ground rules is included in the following box.

Ground Rules for Teams

- Attend class regularly, and come prepared.
- Stay focused on the team task.
- Work cooperatively with other team members.
- Reach a team decision for each problem.
- Make sure each person on the team understands the solution before the team moves on.
- Listen carefully to others, and try to build on their ideas.
- Share the leadership of the team.
- Make sure that everyone participates and no one dominates.
- Take turns recording team results.

It is important that team members agree on the ground rules for the class. It is the responsibility of team members to monitor each other in following the agreed-on rules. Although the instructor can help in this process, it is not possible for the instructor to be in all teams at all times.

Team Building

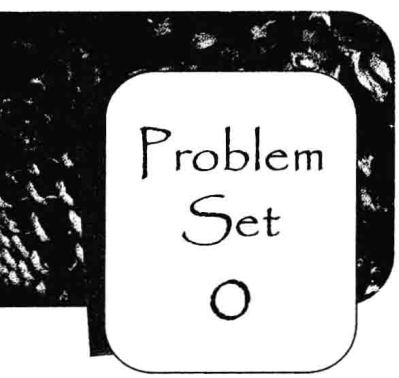
Being an effective team member takes some practice. The problems at the end of this section are designed for you to practice team-building skills in a nonthreatening environment. As you work on these activities, keep in mind the ground rules. Remember that the goal is to learn to work together and communicate clearly so that you can use these skills to help each other effectively learn mathematics.

Many of these activities may not seem mathematical to you. They are all in some way related to thinking and visualizing skills that are important in mathematics. Sometimes you will want to work together on a problem from the start—brainstorming approaches and strategies. At other

times, you may want to spend some time working individually before you share your ideas. Be aware of the needs of your team members as well as your own preferences. Some of the activities that follow will be easier with some concrete props—coins, or pieces of paper, for instance. Don't hesitate to use any tools that make the problem solving or explaining easier.

Last Words of Wisdom

Don't get discouraged if you don't always feel successful. Frustration is a natural part of any learning experience. Jump in, get involved, and enjoy yourself! Solving problems is a positive experience.



1. Coins If you have \$11.52 in your coin jar and there is an equal number of pennies, nickels, and dimes, how many of each coin do you have?

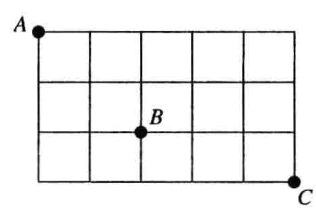
2. Moving Dots Start with the arrangement of dots shown here.

Try to produce this arrangement, given the following three restrictions.

- The solid dots can move only to the right, and the striped dots can move only to the left.
- A dot can move into an adjacent space.
- A dot can jump over *one* other dot—either solid or striped—into an empty space.

Once you can do the problem, work together on a way to clearly communicate your solution in writing. Assume that the solution is being written for someone who is familiar with the rules of the problem but has not necessarily solved it.

3. Paths
   a. How many paths are there from points A to C that are 8 units long?
   b. How many of the paths pass through the point B?



4. Last Digit What is the last digit of 4^{80}?

5. How Many Marbles? Carla placed five containers on a table at the front of the classroom. She said, "There are a total of 50 marbles in these containers. In the first and second containers there are a total of 20 marbles. The second and third containers hold a total of 23 marbles. In the third and fourth containers there are a total of 21 marbles. In the fourth and fifth containers there are a total of 15 marbles." How many marbles are in each container?

6. Following Directions

a. Print the words DON'T JUST SIT THERE without the apostrophe and without space between the words.

b. If a letter occurs more than once, reading from the left, delete all but the first one.

c. In your result from part b, exchange the first and the seventh letters.

d. In your result from part c, change the last letter to the letter that follows it in the alphabet.

e. In your result from part d, change the T to an E.

f. In your result from part e, exchange the third and the seventh letters.

g. In your result from part f, change the letter J to match the first and last letter.

h. In your result from part g, change the third vowel from the left to an I.

i. In your result from part h, change the last vowel to the first vowel in the alphabet.

j. In your result from part i, change the letter O to the next vowel in the alphabet.

k. In your result from part j, delete the second letter I, from the left.

l. In your result from part k, change the letter H to the letters CL.

m. In your result from part l, change the letter D to the letter that precedes it in the alphabet.

n. In your result from part m, if there is a vowel on both sides of a consonant, double that consonant.

o. In your result from part n, double the last letter.

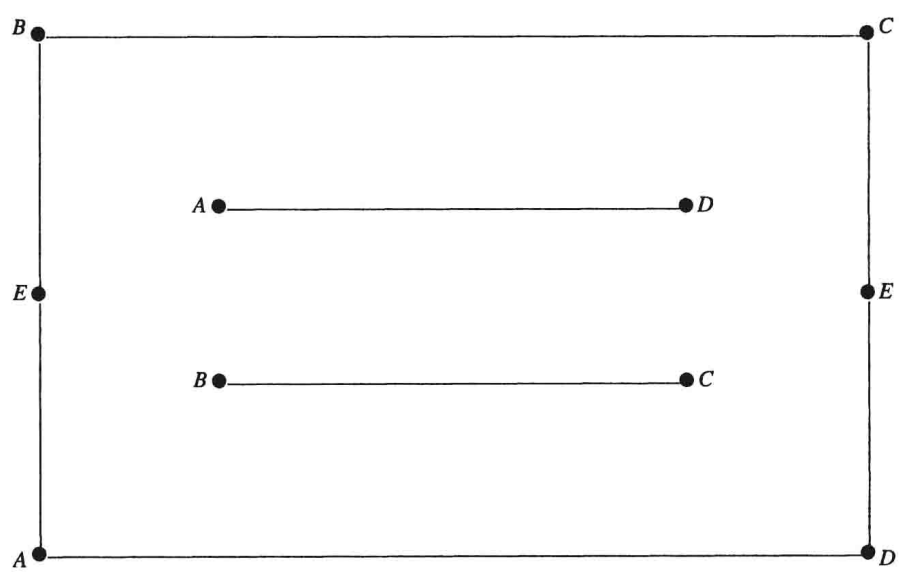
After following these directions, what will you have?

7. Occupations Alice, Brenda, Carl, and Daniel are all sitting around a square table. Their occupations are artist, biologist, chemist, and dentist, but the first letter of their first names and their occupations do not match.

- Alice and Brenda are female. Carl and Daniel are male.
- Brenda is sitting across the table from the dentist.
- The biologist is sitting across from Carl.
- The dentist and the artist are Brenda's mother and father (but not necessarily in that order).

Match each person with his or her occupation.

8. Connections Connect each pair of dots (*A* to *A*, *B* to *B*, and so on). You may not retrace or cross any drawn line. All lines must be contained within the boundary of the drawing.

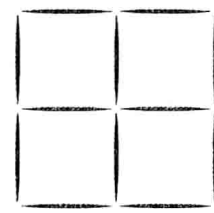


9. Toothpick Problems In these problems follow rules i and ii.

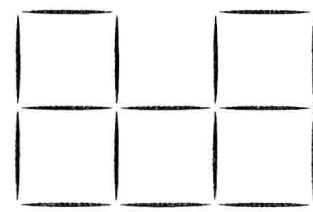
i. No toothpicks are to be broken.

ii. All of the toothpicks will be part of the figure described.

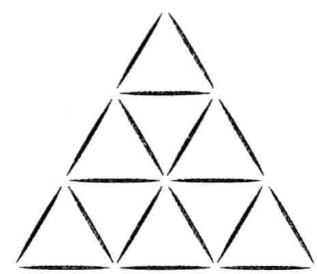
a. Use 12 toothpicks to make the pattern shown here. Move three toothpicks to form a pattern of three squares.



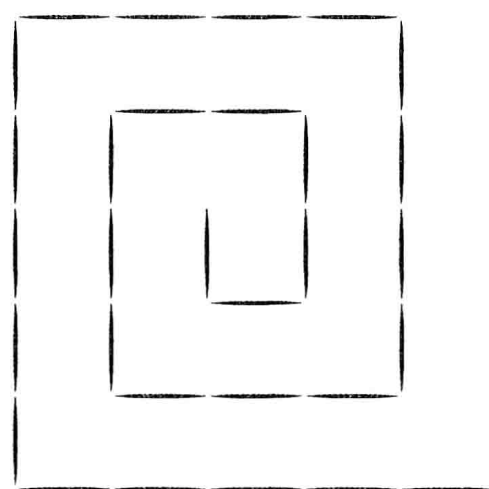
b. Use 16 toothpicks to make the following pattern. Change the position of three toothpicks to form a pattern of only four squares.



c. Use 18 toothpicks to make the following pattern. Remove four toothpicks so that five triangles remain.



d. Using 35 toothpicks, form the spiral pattern shown here. Move four of the toothpicks to form exactly three squares.



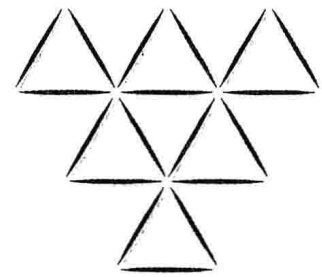
10. Pages To number the pages of a book, a printer used 3385 digits starting on page 1. How many pages were in the book?

11. More Toothpick Problems In these problems follow rules i and ii.

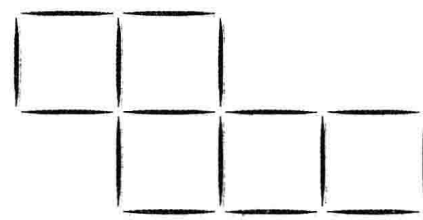
i. No toothpicks are to be broken.

ii. All of the toothpicks will be part of the figure described.

a. Seven identical triangles occur in the pattern shown. Move six toothpicks to form six identical rhombuses.



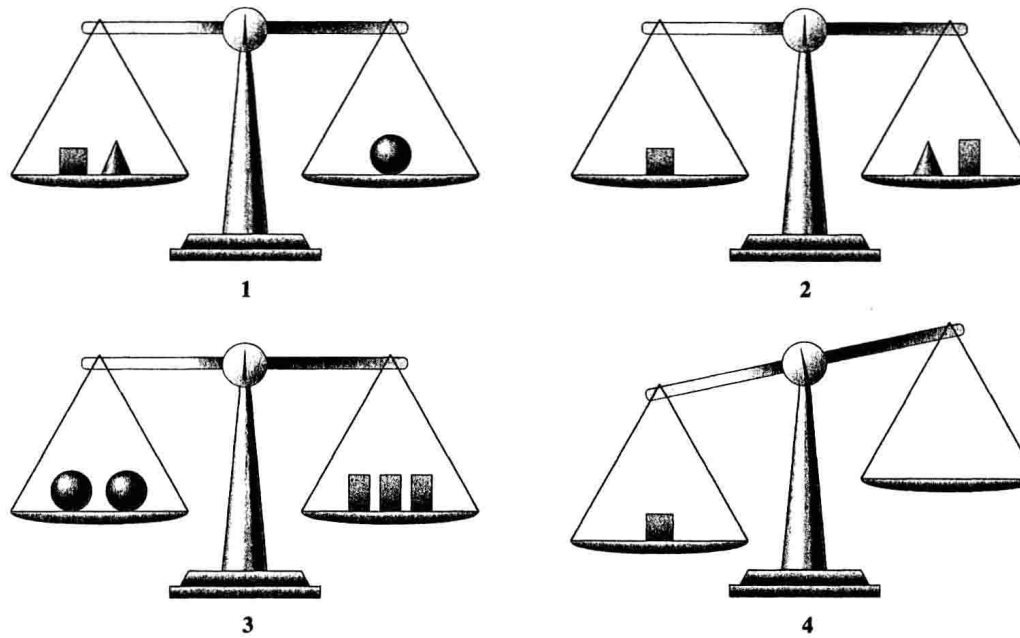
b. Sixteen toothpicks make the following pattern of five identical squares. By moving only two toothpicks, make an array of four identical squares.



c. Thirteen toothpicks are arranged to make the following six identical rectangular enclosures.

- Remove one of the toothpicks. Arrange the remaining 12 into six enclosures of the same size and shape.
- Remove another toothpick. Arrange the remaining 11 into six enclosures of the same size and shape.

12. Balance the Scale Assume that the first three scales shown here balance perfectly. How many of the same shaped solid objects (all balls, all rectangular blocks, or all cones) will it take to balance the fourth scale?



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2

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Chapter One

Numerical Concepts

A good understanding of numbers is necessary for the study of algebra and geometry. The goal of this chapter is to build a strong numerical understanding of mathematics. We will learn how to simplify and estimate expressions using terms. The ideas of precision, accuracy, and the correct rounding of numbers will be discussed. We will learn how to analyze units and convert between different units of measurements. The geometrical concepts of measurement, perimeter, and area will be included throughout the chapter. For additional background on geometry concepts, consult the appendices.