Directions for Mathematics Research Experience for Undergraduates

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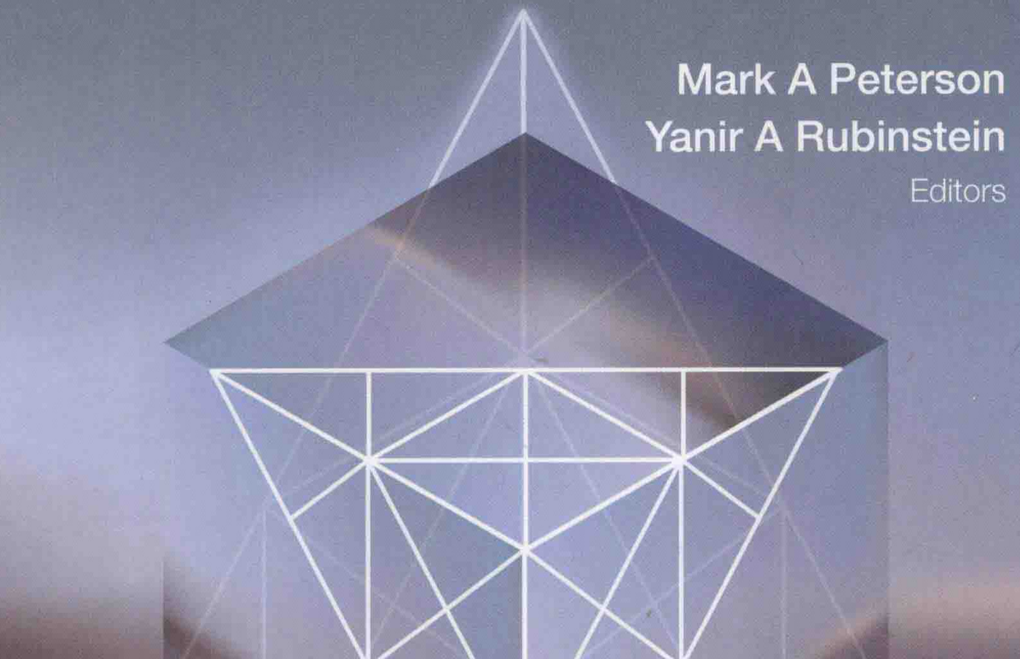

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Editors

Mark A Peterson
Mount Holyoke College, USA

Yanir A Rubinstein
University of Maryland, USA

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Introduction

This volume stems from the conference called "New Directions for Mathematics Research Experiences for Undergraduates," held at Mount Holyoke College in June, 2013, celebrating 25 years of the Research Experience for Undergraduates (REU) program of the NSF. The majority of the articles reflect talks given at the conference. The conference itself is described by the organizers in the last article. The main impression left by the conference, documented and explored in these articles, is that the REU program in mathematics has stimulated innovations in mathematics education that go far beyond the original intent of the program. It is even possible to imagine that the sustained support of this program by the NSF over such a long period has begun to transform US mathematics itself.

The idea that undergraduates might do mathematical research as a part of their education has a long history, but in the early years it was a lonely position to take. A conference on this topic was held in the summer of 1961 at Carleton College, documented in *Undergraduate research in mathematics; report of a conference held at Carleton College, Northfield, Minnesota, June 19 to 23, 1961*, edited by May and Schuster [4]. A second conference on undergraduate research in mathematics, held in July of 1988, also at Carleton College, gave rise to a second volume, *Models for Undergraduate Research*, edited by

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Senechal [5]. In a section called "Voices from Long Ago" it reprints selections from the first volume.

In the years between these two conferences, several things happened that changed the terms of undergraduate research and its role in mathematics education. In the mid-1960s, as part of a continuing response to the Sputnik launch, the NSF created a program called Undergraduate Research Participation (URP), recalled by Lynn Steen in a contribution to an AMS publication *Proceedings of the Conference on Summer Undergraduate Mathematics Research Programs* (2000), edited by Gallian [2, pp. 331–336]. The aim of the URP was explicitly to accelerate the production of research scientists and mathematicians. Not long after, in the early Reagan years, there seemed to be a glut of research scientists, and the program was abruptly ended. By this time, though, the spark had caught. In 1977, Gallian initiated a summer program for undergraduate mathematics research that is still continuing today, and his example was widely noticed. In 1987, the NSF created the REU program.

The AMS volume cited above [2] contains articles from 35 REU and REU-like programs as of the year 2000, describing in detail how they operate. Increasingly, undergraduates have begun attending national meetings and giving talks, first in their own special sessions and conferences like Mathfest, but now also in general sessions. By 2006, Gallian could report "377 undergraduates attend annual joint meetings at San Antonio with 44 of them giving talks," as the culmination of a chronology in *Proceedings of the Conference on Promoting Undergraduate Research in Mathematics*, AMS (2007), edited by Gallian [3, pp. 267–272].

One thing that is not obvious in the above account is that the mathematics community as a whole was not especially eager to embrace these developments. Lynn Steen notes that in 1967, out of more than 500 URP grants, only 10 were in mathematics, and not because the NSF was discriminating against mathematics, but because most mathematicians were skeptical that undergraduate research made much sense. Every research mathematician works at the limits of his or her abilities, and these abilities have been hard-won over a lifetime. What would an undergraduate do? The

REU program — a program that spans all the sciences — aims "to attract a diversified pool of talented students into research careers" (NSF 96-102), but for many it was easier to imagine how this might happen in a laboratory science than in mathematics.

Arguably, if one sets the bar high enough, none of us does important research. In the bleak words of Alfred Adler, "Each generation has its few great mathematicians, and mathematics would not even notice the absence of the others. They are useful as teachers, and their research harms no one, but it is of no importance at all. A mathematician is great or he is nothing." [1]. Everyone understands what Adler is saying, but must also understand that this exalted notion of research is not the whole point, and that the interval between greatness and nothingness is far from empty.

The antecedents of the REU program were typically one brave professor and a few promising undergraduate students working on challenging problems. The early REUs continued this model for the most part, except that they were typically more like three brave professors and perhaps ten students working individually or in small teams. The undergraduates made interesting progress with reassuring consistency, and the basic idea began to take root. This history is considered in more depth, with observations about what it may mean for the culture of mathematics, in the first article of this volume, "Undergraduate Research and the Mathematics Profession," by Donal O'Shea.

The express objective of the REU program was to attract undergraduate students into research careers, but in practice the program produced other benefits as well, sometimes much more immediate. Directors of REU programs frequently noticed unanticipated dividends in their own research. This observation prompts the suggestion of the second article, "FURST — A symbiotic approach to research at primarily undergraduate institutions," by Tamás Forgács. It points out that many mathematics PhDs, trained in research, find themselves in teaching positions at small institutions that are essentially isolated from the research community. A slightly de-localized REU, organized around some suitable topic and including these small institutions, can reconnect these mathematicians to a community of like-minded professionals, with the undergraduates providing the "symbiotic" glue that holds the network together. This idea highlights the benefit not just to the undergraduates but also to their mentors.

Another benefit of the REU program, not expressly a goal but still an obvious consequence of it, is the enhanced learning that takes place in a research setting as opposed to a classroom. Modern research on learning and pedagogy makes clear the importance of active student involvement in learning generally, of all kinds, not just mathematics, and strategies for making learning more active are ubiquitous in innovative approaches to education in all subjects. Mathematics has a distinguished early contribution of this kind to point to in the "Moore method." It is thus no surprise that in the years of the REU program's existence there have also been strategies that put something like undergraduate mathematics research into the classroom. A good example of such a course, now beginning to be copied at other places, is MIT's, described in the third article of this volume, "A Laboratory Course in Mathematics," by Kathy Lin and Haynes Miller. The reader will notice much in common with the REU idea, even if the problems posed in the course are not precisely research problems. The importance of effective communication of mathematics both in speaking and in writing is heavily emphasized in this course, just as it is a big part of a typical REU.

The first experiments with undergraduate research in mathematics were typically at small undergraduate institutions, and this trend continues, by and large, in the REU program. On the one hand, it is not surprising that undergraduate colleges would be the ones most committed to an expressly undergraduate movement in education, but on the other hand it seems a little odd that research universities (R1s), the locus of most research in mathematics, would not be better represented. The fourth article in this volume, "REUs with limited faculty involvement, 'underrepresented' subjects in the undergraduate curriculum, and the culture of mathematics," by Yanir A. Rubinstein and Ravi Vakil, describes a program at Stanford that was, at least in part, a response to undergraduate demand. The new element, and the basis for the "limited faculty involvement," is graduate students. The reference to the "culture of mathematics" in the title is, in part, a consideration of the essential openness and generosity necessary for a research team of faculty member, graduate student, and undergraduates to function, possibly contrasting with the more private one-on-one interaction of a thesis advisor and a graduate student.

The role that graduate students might play in undergraduate research programs is further developed in the fifth article, "'The Berkeley Summer Research Program for Undergraduates': One model for an undergraduate summer research program at a doctorate-granting university," by Daniel Cristofaro-Gardiner. Both this article and the previous one point out that the areas of mathematics best represented in traditional REU programs are not typically the most active research areas at R1 universities. If the involvement of R1 universities and their graduate students in formulating problems and guiding undergraduate research is a growing trend, the mathematics of the REU programs may broaden to include subjects that are currently underrepresented there.

The sixth article, "Fifteen Years of the REU and DRP at the University of Chicago," by J. Peter May, points out another possibility for the REU at an R1 university: large scale. The presence of a thriving R1 research culture alongside the liberal arts college of the university has produced a remarkable fusion, in which graduate students and undergraduates work together on problems that sometimes constitute research, but always contribute to an active and energetic mathematical environment, if the statistics on the growth of the mathematics major is any indication. The DRP of the title is the Directed Reading Program, voluntary pairings of graduate students and undergraduates to read about some topic chosen out of pure interest, a format beginning to be copied at other places as well. The graduate students are compensated slightly, and the undergraduate students receive only the private personal benefit of the work that they put in, yet the program thrives. A DRP may sometimes metamorphose into a summer REU project, and the reverse may also happen. The success of the program, essentially restricted to

University of Chicago students for logistical reasons, may be judged by the size of the REU, about 80 students. REU students have considerable freedom to choose what they do, attending lecture courses that introduce research areas, choosing a problem, and meeting with graduate student mentors. Students are required to write up their work by the end of the summer.

The freedom of participants to choose their own problems is almost a defining feature of a long running program called the Mathematical and Theoretical Biology Institute (MTBI), founded at Cornell in 1996, now at Arizona State University, described in the seventh article, "Why REUs Matter," by Carlos Castillo-Garsow and Carlos Castillo-Chavez. One aim of this program, consistent with the long-term trend that continually finds new applications for the REU idea, is to recruit underrepresented minorities into mathematics. This means that it has to target students in their first or second year of study, at a level of mathematical experience considerably less than traditional REU students. The REU introduces, through lectures and exercises, the idea of modeling with differential and difference equations, but leaves it up to the participants to think of a problem that might be susceptible to modeling using these tools. The result is what the directors call a "reversal of hierarchy." Students choose situations for modeling that are more familiar to them than they are to their mentors, making them, in effect, the experts. Of course, the mentors will know more about the mathematical methods that the students will use. The results are impressive, with many mathematics PhDs resulting eventually over the years, undoubtedly traceable in part to this REU experience. The authors call attention also to the importance for future schoolteachers of experience using mathematics to investigate an applied problem of immediate interest.

The goal of attracting underrepresented minority students into mathematics is also one of the goals of the eighth article, "Integrating mathematics majors into the scientific life of the country," by William Vélez, but the strategy described there is ingeniously indirect. The author describes a mathematics major program designed to facilitate the participation of its students in the REUs of other sciences(!) It is pointed out that students of mathematics are frequently desirable participants in research programs outside mathematics, and that with a little forethought, such as course preparation in something like biology or geology, together with some computer experience, these students will move easily into applied research areas where their mathematical skills are immediately useful and valued. They may double major in mathematics and another science, or may even move into a research career in another science, leaving mathematics behind. From the point of view of the NSF and the REU program, this is still a success.

It is an historical fact that mathematics has lagged behind the other sciences in promoting undergraduate research for perhaps understandable reasons. Something of the sort may be taking place at present on a scale that, if the programs of the last several papers could be called "large," should now be called "vast." "The Gemstone Honors Program: Maximizing Learning through Team-based Interdisciplinary Research," by Frank J. Coale, Kristan Skendall, Leah Kreimer Tobin, and Vickie Hill, the ninth paper in this volume, describes a four-year program within the Honors College of the University of Maryland. Nurtured through a program that begins even before the opening day of their first term, teams of honors students learn to approach research, formulate their own problem, and bring it to a polished completion by the time they graduate. Building on years of experience, the program includes not only mentors expert in the subject matter of the research problems, but also specially designated librarians, and coaches for learning and practicing the skills of teamwork.

Mathematics, however, does not play much of a role in this structure, otherwise so seemingly comprehensive. One can think of reasons: undergraduates formulating an applied research problem that has special appeal for them will probably not think of mathematics playing much of a role. Once the problem is formulated in an unmathematical way, the absence of a role for mathematics becomes tautologically true. It is hard to imagine, though, that there should not be more place for mathematics, and that the problems and the education that students derive from this experience would not be thereby enriched. It would take initiative by mathematicians close to the scene, and also on the part of the directors, perhaps emulating the technique of the MTBI described above, introducing methods of mathematical modeling in an outline fashion before the research problems take their final shape.

An experiment of this kind has actually begun at the University of Texas at Austin in the program described in the tenth article of this volume, "The Freshman Research Initiative as a model for addressing shortages and disparities in STEM engagement," by Josh T. Beckham, James Farre, Gwendolyn M. Stovall, and Sarah L. Simmons. Like the Gemstone Program, the Freshman Research Initiative (FRI) is a very large scale program engaging undergraduates in research from the moment they arrive on campus. The FRI consists of around 25 "research streams" involving about 800 students for the first three semesters of their education. Despite the size of the program, spread across the sciences generally, there is little explicit involvement of mathematicians, and one cannot help but think that this is a missed opportunity for mathematics. The FRI, displaying remarkable agility, has recently taken steps to try integrating mathematics into its structure. The first experimental steps in this direction are described in this article, and the FRI itself is described to make it possible to imagine various approaches to integrating mathematics into such programs. The FRI and Gemstone are sure to be emulated elsewhere, and mathematics might play a different role if it were present at the creation.

The eleventh article is "Determining the Impact of REU Sites in the Mathematical Sciences" by Jennifer Slimowitz Pearl, writing from her experience as an NSF officer, and not as an official voice of the NSF. It is a remarkable fact that it is hard to know with any precision what the effect of the REU program in mathematics has been, despite its long and well-documented history, and despite the intriguing, informed speculations that are to be found throughout this volume. Students participate and fade away, often to re-appear as faculty colleagues. Directors of individual programs would naturally like to know how successful their programs have been, both in the short and the long term. The NSF even requires some such assessment each year, although the format is left open. The author proposes a flexible but still somewhat standardized tool for assessment (adapted from the geologists!) that might help bring clarity to the question of how effective the mathematics REU is.

The volume ends with a summary of the Mount Holyoke College conference.

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