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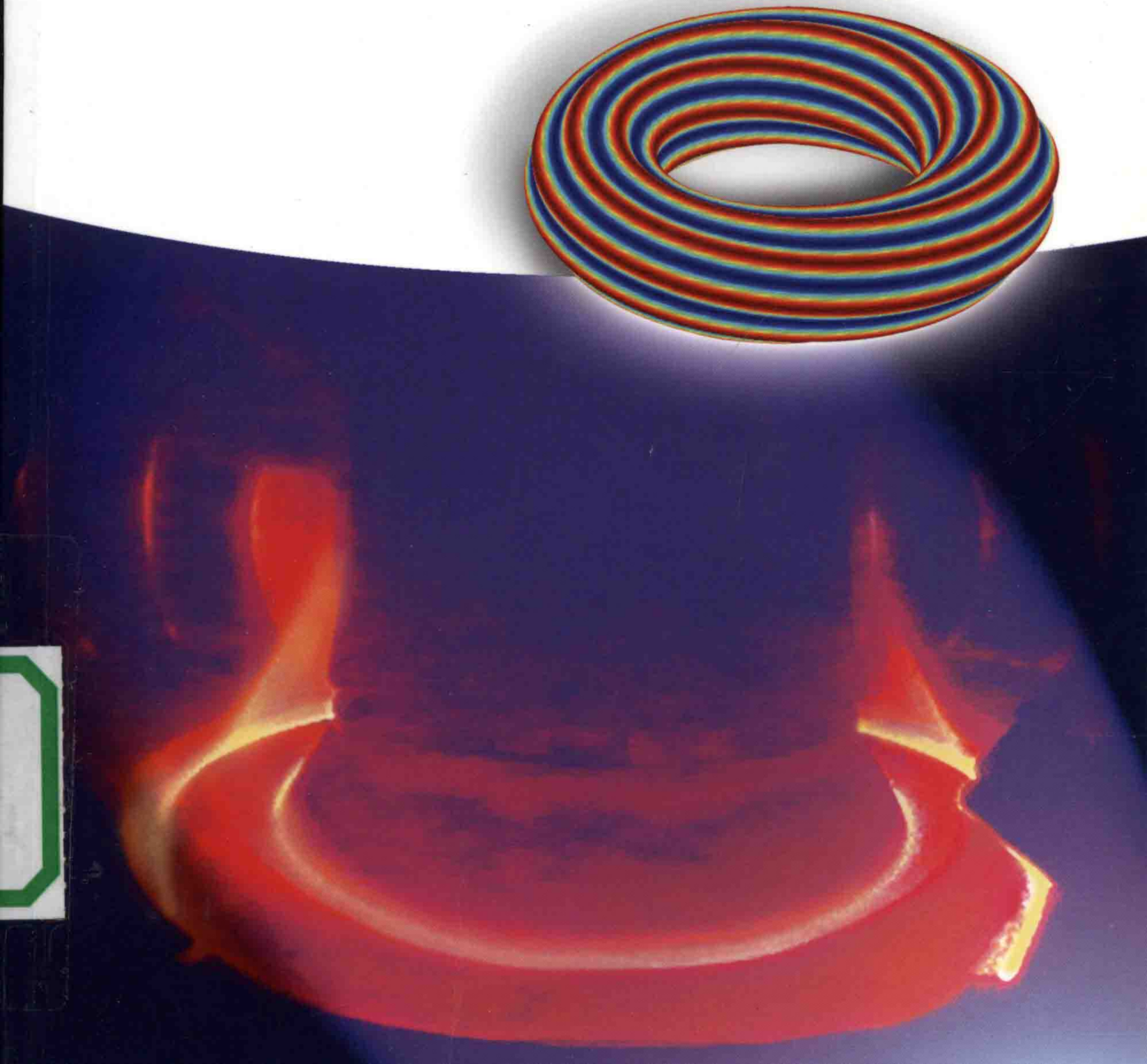
Magnetohydrodynamic Stability of Tokamaks



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Magnetohydrodynamic Stability of Tokamaks

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Author

Hartmut Zohm
MPI für Plasmaphysik
Garching
Germany
zohm@ipp.mpg.de

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Preface

This textbook is based on a University course on MHD stability of tokamak plasmas that I have been developing since 1996. MHD stability of tokamaks is an evolving field, and while a lot of specialist's knowledge exists, I found it very important for students to see that there is a solid theoretical foundation in common with other areas of plasma physics from which the understanding of tokamak MHD stability is developed. The book should hence serve to bridge the gap between a basic plasma physics course and forefront research in MHD stability of tokamaks. This means that there are some elements of review of the field's present status in it that will certainly develop over the coming years, but I found it important to point out where we have reached basic understanding and where there are still open ends at the time of writing the book.

At this point, I want to thank all the individuals who have contributed directly or indirectly to this book. First I would like to thank Karl Lackner for all the enlightening physics discussions and especially for always having time for me whenever I entered his office. Some of the most insightful physics arguments presented in this book actually originate from these discussions. Then, I want to thank the colleagues who have worked with me on MHD stability on ASDEX Upgrade for the last 20 years or so, namely Marc Maraschek, Anja Gude, Sibylle Günter and Valentin Igochine, trying to figure out where the experiment knows about the theory. It is also a pleasure to acknowledge the very helpful discussions with Per Helander on stellarator physics and Steve Sabbagh on RWM stability. In preparing the book, Hans-Peter Zehrfeld helped me through all the troubles of editing, formatting and organizing the text. He also contributed greatly to Chapter 2. A special thanks goes to Sina Fietz for her help with the figures and to Emanuele Poli for the thorough proofreading. Last but not least, a major part of the text was written during a stay at University of Wisconsin, Madison, and I want to thank Cary Forest and Chris Hegna for their hospitality and the very useful discussions about the topics of this book.

Hartmut Zohm
Garching, August 2014

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1
The MHD Equations

1.1
Derivation of the MHD Equations

In this book, we will treat the description of equilibrium and stability properties of magnetically confined fusion plasmas in the framework of a fluid theory, the so-called Magnetohydrodynamic (MHD) theory. In this chapter, we are going to derive the MHD equations and discuss some of their basic properties and the limitations for application of MHD to the description of fusion plasmas. The derivation follows the treatment given in [1]. For a more in-depth discussion of the MHD equations, the reader is referred to [2]. Non-linear aspects of MHD are treated in [3]. A good overview of general tokamak physics can be found in [4].

1.1.1
Multispecies MHD Equations

As a magnetized plasma is a many-body system, its description cannot be done by solving individual equations of motion that would typically be a set of, say, 10^{20} equations[1] that are all coupled through the electromagnetic interaction. Hence, some kind of mean field theory is needed.

Starting point of our derivation is the kinetic equation known from statistical physics. It describes the many-body system in terms of a distribution function f_α in six-dimensional space $d^3x d^3v$, where

$$f_\alpha(\mathbf{x}, \mathbf{v}, t)\; d^3x\; d^3v \tag{1.1}$$

is the probability to find a particle of species α at $\mathbf{x}$ with velocity $\mathbf{v}$ at time t. Here, $\mathbf{x}$ and $\mathbf{v}$ are independent variables that, in the sense of classical mechanics, fully describe the system.

The basic assumption of kinetic theory is that fields and forces are macroscopic in the sense that they have already been averaged over a volume containing many particles (say, a Debye-sphere[2]) and the microscopic fields and forces at the exact

1) Here, we think of a typical fusion plasma of density 10^{20} particles per cubic metre.

2) The Debye length λ_D is the typical distance on which the electric field in a plasma is shielded so that its action is limited to a sphere of radius λ_D.

particle location can be expressed through a collision term giving rise to a change of f_α along the particle trajectories in six-dimensional space. We note that this has reduced the microscopic problem of the 10^{20} interactions to the proper choice of the collision term.

Evaluating the total change of f_α along the trajectories and keeping in mind that along these, $d\mathbf{x}/dt = \mathbf{v}$ and $d\mathbf{v}/dt = \mathbf{F}_\alpha/m_\alpha$, where $\mathbf{F}_\alpha$ is the force acting on the particle and m_α its mass, the kinetic equation can be expressed as

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\cdot\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{coll}} \tag{1.2}$$

where we have assumed that the only relevant force is the Lorentz force and hence explicitly neglected gravity (which is a good approximation for magnetically confined fusion plasmas, but generally not true in Astrophysical applications). According to the above-mentioned description of mean field theory, the fields $\mathbf{E}$ and $\mathbf{B}$ will have to be calculated from Maxwells equations using the charge density and current resulting from appropriate averaging over the distribution function in velocity space as will be described in the following.

The kinetic equation is used to describe phenomena that arise from f_α not being a Maxwellian, which is the particle distribution in thermodynamic equilibrium to which the system will relax through the action of collisions. In fusion plasmas, this frequently occurs as the mean free path is often large compared to the system length as is for example the case for turbulence dynamics in a tokamak along field lines. Another important example is when the relevant timescales are short compared to the collision time, such as in RF (radio frequency) wave heating and current drive that can occur by Landau damping rather than collisional dissipation. Here, a description using the Vlasov or Fokker–Planck equation is needed.

However, in situations where f_α is close to Maxwellian, one can average the kinetic equation over velocity space to obtain hydrodynamic equations in configuration space. When doing so, one encounters so-called moments of f_α. The kth moment, which is related to the velocity average of $\mathbf{v}^k$, is given by

$$\int \mathbf{v}^k f_\alpha d^3v = n_\alpha\langle\mathbf{v}^k\rangle \tag{1.3}$$

These moments are related to the hydrodynamic quantities used to describe the plasma in configuration space. For the zeroth moment, we obtain

$$n_\alpha(\mathbf{x},t) = \int f_\alpha(\mathbf{x},\mathbf{v},t)d^3v \tag{1.4}$$

which is the number density in real space. The first moment of f_α is related to the fluid velocity in the centre of mass frame by

$$\mathbf{u}_\alpha(\mathbf{x},t) = \frac{1}{n_\alpha}\int \mathbf{v}f_\alpha(\mathbf{x},\mathbf{v},t)d^3v \tag{1.5}$$

For the second moment, it is of advantage to separate the particle velocity into the fluid velocity and the random thermal motion $\mathbf{w}$ according to

$$\mathbf{v} = \mathbf{u}_\alpha + \mathbf{w} \tag{1.6}$$

It is easy to show that $\langle \mathbf{w} \rangle = 0$, as expected for thermal motion, as

$$\langle \mathbf{w} \rangle = \langle \mathbf{v} \rangle - \langle \mathbf{u}_\alpha \rangle = \mathbf{u}_\alpha - \mathbf{u}_\alpha = 0 \tag{1.7}$$

However, the quadratic average is non-zero, representing the thermal energy via

$$\frac{1}{2} m_\alpha \int \mathbf{w}^2 f_\alpha d^3 v = \frac{3}{2} n_\alpha k_B T_\alpha = \frac{3}{2} p_\alpha \tag{1.8}$$

where k_B is the Boltzmann constant and we have used the definition of the thermal energy density and its relation to the pressure p_α for an ideal plasma. We note that this definition relies on the previous assumption that f_α is close to Maxwellian. More generally, the second moment is defined as a tensor of rank 2, the pressure tensor

$$\mathbf{P}_\alpha = m_\alpha \int \mathbf{w} \otimes \mathbf{w} f_\alpha d^3 v, \tag{1.9}$$

where $\otimes$ denotes the dyadic product. The non-diagonal terms of this tensor are related to viscosity, whereas from Eq. (1.8), it is clear that the trace of $\mathbf{P}_\alpha$ is equal to $3p_\alpha$, that is for an isotropic system, the diagonal elements of $\mathbf{P}_\alpha$ are just equal to the scalar pressure. Therefore, the pressure tensor is also often written as

$$\mathbf{P}_\alpha = p_\alpha 1 + \Pi_\alpha, \tag{1.10}$$

where 1 is the unit tensor and Π_α the anisotropic part of $\mathbf{P}_\alpha$.

We now integrate the kinetic equation (Eq. 1.2) over velocity space[3] to obtain

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = 0 \tag{1.11}$$

which is the equation of continuity for species α. Here, we have assumed that the velocity space average of the collision term is zero, meaning that the total number of particles is conserved for each species. Should this not be the case (e.g. by ionization or fusion), the right-hand side would consist of a source term $S(\mathbf{x}, t)$.

The next moment is obtained by multiplying the kinetic equation by $\mathbf{v}$ and integrating over velocity space. This yields the momentum balance

$$\begin{aligned} m_\alpha \frac{\partial (n_\alpha \mathbf{u}_\alpha)}{\partial t} &= -\nabla \cdot (m_\alpha n_\alpha \mathbf{u}_\alpha \otimes \mathbf{u}_\alpha + \mathbf{P}_\alpha) \\ &\quad + n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) + \mathbf{R}_{\alpha\beta}, \end{aligned} \tag{1.12}$$

where the friction force $\mathbf{R}_{\alpha\beta}$ is the first moment of the collision term for collisions with species β. We note that only collision with unlike particles lead to a net friction force while collisions within one species, which are important for thermalization, do not transfer net momentum to that species. This form is also called the *conservative* form as, like the equation of continuity, it relates the temporal derivative of a quantity (in this case, the momentum) to the divergence of a flux.

3) When integrating over velocity space, it is useful to remember that t, $\mathbf{x}$ and $\mathbf{v}$ are independent so that the derivative with respect to t and x can be taken out of the integral. In addition, terms containing a v derivative are integrated partially and the surface term vanishes as $f_\alpha \to 0$ faster than any power of v for $v \to \infty$.

However, this equation can be rearranged using the continuity equation into a form in which the dyadic product of the velocity can be absorbed in the derivative on the left-hand side:

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{u}_\alpha}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \mathbf{u}_\alpha \right) = -\nabla \cdot \mathbf{P}_\alpha + n_\alpha q_\alpha (\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) + \mathbf{R}_{\alpha\beta}, \tag{1.13}$$

which is usually called the *force balance*. Here, the operator

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u}_\alpha \cdot \nabla \tag{1.14}$$

is called the *substantial* or *convective derivative* and measures the change along the trajectory of a fluid element in the laboratory frame. In ordinary hydrodynamics, Eq. (1.13) is called the *Euler equation* while equations in the co-moving frame are referred to as the *Lagrange description*.

The system of equations so far is not closed as a second moment appears in the first moment equation, just as the velocity as first-order moment occurs in the zeroth-order continuity equation. It is clear that this problem cannot be solved by adding the second moment of the kinetic equation as a third moment will appear. This is the closure problem of MHD, where at each step, an additional relation will be required to close the system. If we want to stop here, we obviously need a relation for the pressure, that is an equation of state. This could be the adiabatic equation

$$\frac{d}{dt} \left(\frac{p_\alpha}{{\rho_\alpha}^{\gamma_\alpha}} \right) = 0, \tag{1.15}$$

where γ_α is the adiabatic coefficient and we have assumed that we only deal with the scalar pressure in Eq. (1.13). Together with Maxwell's equations for the fields $\mathbf{E}$ and $\mathbf{B}$, we now have indeed a closed system. However, we will still simplify this system for a two-component plasma in Section 1.1.2.

1.1.2
One-Fluid Model of Magnetohydrodynamics

For the case of a two-component plasma consisting of one ion species and electrons, the system of two-fluid equations can be combined to give a set of one-fluid equations. Here, owing to the large mass difference between the two species, the mass and momentum are more or less contained in the ions, whereas the electrons guarantee quasineutrality and lead to an electrical current if their velocity is different from that of the ions. In the following, we will assume a hydrogen plasma, that is charge number $Z = 1$. Specifically, the one-fluid variables are the mass density

$$\rho = n_i m_i + n_e m_e \approx n m_i \tag{1.16}$$

where we have used charge neutrality ($n_e = n_i = n$), the centre of mass fluid velocity

$$\mathbf{v} = \frac{1}{\rho}(m_i n_i \mathbf{u}_i + n_e m_e \mathbf{u}_e) \approx \mathbf{u}_i \tag{1.17}$$

and the electrical current density

$$\mathbf{j} = en_i\mathbf{u}_i - en_e\mathbf{u}_e = en(\mathbf{u}_i - \mathbf{u}_e). \tag{1.18}$$

The one-fluid equations are obtained by adding or subtracting the continuity and force balance equations for the individual species and expressing them in the one-fluid variables, neglecting terms of the order m_e/m_i. In this process, addition will give a one-fluid equation for the velocity, whereas the subtraction will yield one for the current density.

Adding the continuity equations yields

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{1.19}$$

that is a one-fluid continuity equation while subtracting them leads to

$$\frac{\partial \rho_{\text{el}}}{\partial t} + \nabla \cdot \mathbf{j} = 0 \tag{1.20}$$

which is the continuity equation for the electrical current. As we assume the plasma to be quasi-neutral, the electrical charge density $\rho_{\text{el}} = en_i - en_e$ vanishes and the equation just reads $\nabla \cdot \mathbf{j} = 0$.

Adding the force equations leads to

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla \cdot \mathbf{P} + \mathbf{j} \times \mathbf{B} \tag{1.21}$$

the Euler equation, where $\mathbf{P} = \mathbf{P_i} + \mathbf{P_e}$ as in an ideal plasma, the total pressure is the sum of the partial pressures of the individual species. As pointed out earlier, the fluid velocity is mainly the ion velocity. To determine the role that the electrons play, we can re-write the electron equation of motion in terms of the one-fluid velocity $\mathbf{v}$ to obtain

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \frac{1}{\sigma}\mathbf{j} + \frac{1}{en_e}(\mathbf{j} \times \mathbf{B} - \nabla p_e) - \frac{m_e}{e}\frac{d\mathbf{u}_e}{dt} \tag{1.22}$$

which is Ohm's law for a plasma. One can see that here, not all two-fluid variables could be eliminated from the equation through $m_e \ll m_i$. However, we will argue in the following that the last two terms are usually small for our applications and can be neglected so that this problem will not appear in what follows.

Assuming that we will deal with the scalar pressure only, we can use the adiabaticity equation

$$\frac{d}{dt}\left(\frac{p}{\rho^\gamma} \right) = 0. \tag{1.23}$$

In ordinary hydrodynamics, neglecting the viscous part of the pressure tensor corresponds to infinite Reynolds number and hence the use of the Euler instead of the Navier Stokes equation that rules out a proper description of fluid turbulence. However, if we keep finite conductivity in Ohm's law, there is still dissipation in the system and the relevant dimensionless number becomes the magnetic Reynolds number (Chapter 8).

Finally, we use Maxwell's equations for **E** and **B**

$$\nabla \cdot \mathbf{B} = 0, \tag{1.24}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}, \tag{1.25}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1.26}$$

and we have a closed system of equations to describe a plasma as a single fluid. We note that we have neglected the polarization current in Ampere's law, eliminating phenomena with (phase) velocity close to that of the speed of light, such as electromagnetic waves that arise from this term. In addition, we do not need to solve explicitly an equation for $\nabla \cdot \mathbf{E}$ in the quasi-neutral plasma. Finally, counting the number of variables and equations reveals that one scalar equation seems obsolete; this is related to the fact that any solution that satisfies $\nabla \cdot \mathbf{B}$ in the beginning will always do so and hence this is rather a boundary condition than a separate equation.

1.1.3
Validity of the One-Fluid Model of Magnetohydrodynamics

The system of equations derived earlier relies on a number of assumptions that have been made during the derivation. Here, we briefly review them and point out the restrictions arising.

- By assuming that we can use a continuum description, we think of the plasma described as fluid elements that are infinitesimally small such that individual particles are not distinguished. This means that the typical 'extension' of the particle orbit, that is the Larmor radius r_L, is small compared to a typical system length L:

$$r_{Li} = \frac{\sqrt{m_i k T_i}}{eB} \ll L. \tag{1.27}$$

  This is also known as the condition for a *magnetized plasma* and is usually very well fulfilled in the fusion plasmas under study here where typical ion Larmor radii are of the order of millimetres and the electron Larmor radius is even smaller by a factor $\sqrt{m_e/m_i}$, which is the reason why we have used the ion Larmor radius earlier. For the MHD instabilities treated in this book, it is important to remember that the validity of our results will break down for very small scales, and finite Larmor radius (FLR) effects set the limit to the applicability in the limit $L \to 0$.
- Defining a local temperature requires that f_α is close to a Maxwellian. This relies on considering timescales that are long compared to the collision time

$$\tau_{coll} \sim T^{3/2}/n \ll \tau$$

  or, in terms of spatial scales, the mean free path λ_{mfp} being small compared to the system length:

$$\lambda_{\mathrm{mfp}} \sim T^2/n \ll L.$$