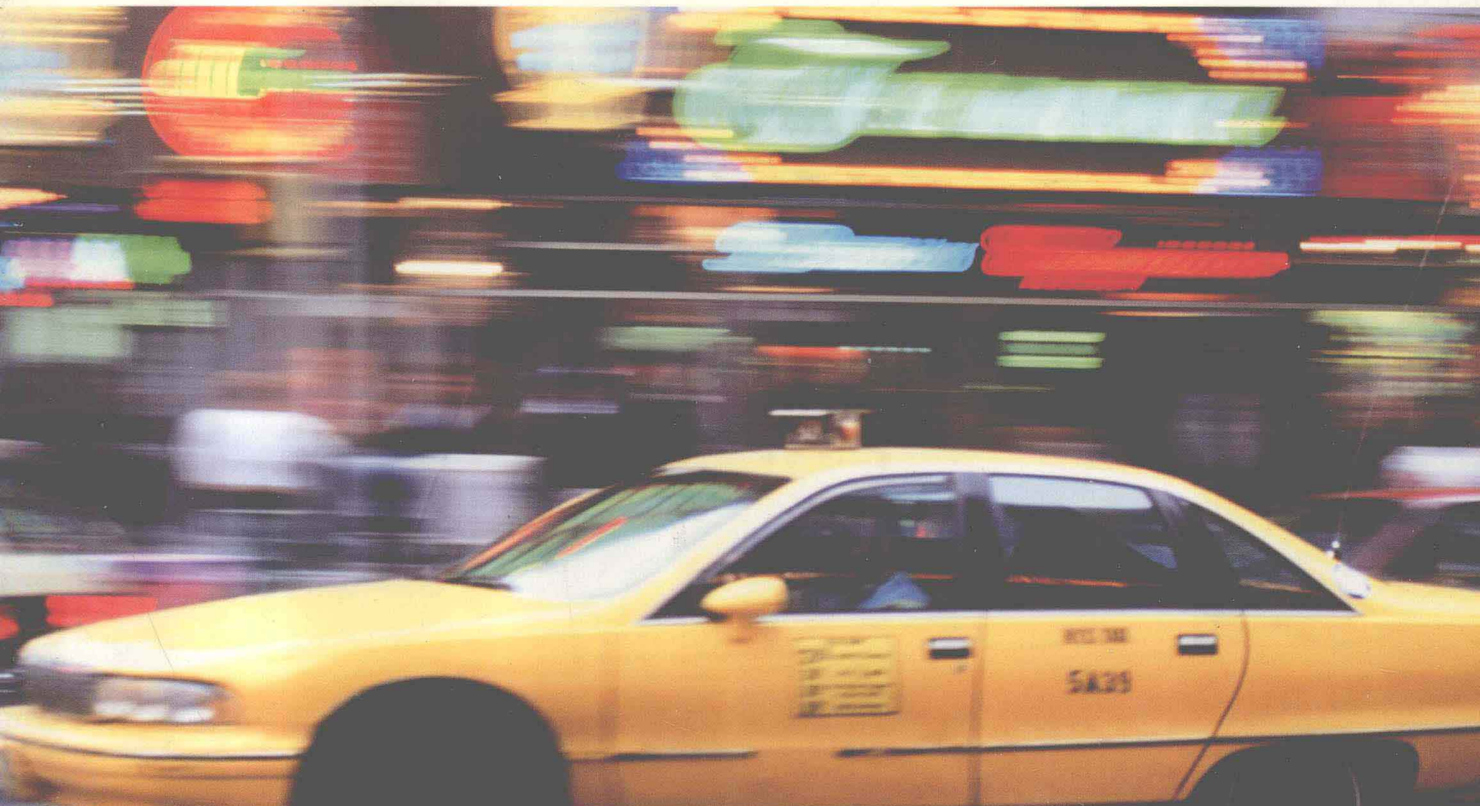
Brief Calculus

The Study of Rates of Change



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Brief Calculus

The Study of Rates of Change

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Preface

Audience and Prerequisites

Brief Calculus: The Study of Rates of Change is intended for a one or two-term calculus course for students majoring in business, economics, social or life sciences. Students who have completed an appropriate college algebra or precalculus course are prepared to study the topics presented in this textbook. The text is designed for both two-year and four-year schools.

Style and Approach

In contrast to many brief and applied calculus textbooks on the market, our book is written for the **student.** Our conversational writing style explains the mathematics clearly and simply and employs real data from the Internet to motivate and underscore the relevance of calculus in a wide range of applications. The book draws from the best ideas of reform mathematics, yet is built on a tradition of solid mathematics. As reviewer Rene Barrientos from Miami-Dade Community College put it, "This is the first post-reform calculus text that emphasizes applications without giving up the symbolic manipulations and coverage required of a rigorous calculus course."

The core concepts of calculus are introduced in applied settings using the concept commonly known as the Rule of Three (numerical, graphical, and algebraic). The Rule of Three is one tool we use to aid students' understanding of new ideas. The exploration of concepts and conjectures using graphing calculator technology has been seamlessly incorporated into examples and exercises where appropriate. We recognize that the graphing calculator is only a tool to aid in the understanding of mathematics. Nowhere does this technology overshadow the mathematics; it simply augments the mathematics and allows the power and relevance of calculus to shine through.

Content Highlights

Applications

The applications in *Brief Calculus: The Study of Rates of Change* are the heart of our textbook. Reviewers have lauded the quantity, quality, and variety of applications in our text calling them superior to that of competitors. Application examples and exercises usually end with the phrase "...and interpret" so that students are asked not only to determine the numerical solution to an application, but also to write the meaning of the solution in the context of the application itself.

Many application models such as U.S. Imports from China, the number of cases of tuberculosis in the U.S., and the percentage of 3 to 5 year olds enrolled in preschool are derived from real data gathered from sources such as the U.S. Census Bureau web site and are indicated by the "On the Web" icon. We incorporate real life applications taken from business and economics, the biological and life sciences, and the social and physical sciences throughout the entire text.

Rate of Change Theme

In Chapter 1, the basic algebraic and transcendental functions are reviewed in a concise, yet comprehensive manner. These functions are used to introduce the core concept of Chapter 1, that the average rate of change of a function over an interval is equivalent to the slope of a secant line over an interval. Appropriate units and proper interpretation of solutions are stressed. For example, a solution to an application may read, "This means during the period from 1987 to 1991, U.S. imports from China increased at a rate of 3.18 billion dollars per year." By introducing this idea in Chapter 1, we create a smooth transition to computing the instantaneous rate of change and the derivative in Chapter 2. The remaining chapters continue to emphasize the rate of change theme, along with the importance of correct units and a reasonable interpretation of the solution. If any student who uses *Brief Calculus: The Study of Rates of Change* is asked, "What is calculus?", the student will inevitably proclaim "It is the study of rates of change!"

To supplement the rate of change theme, the differential is introduced in Chapter 3 and is used as a mathematical tool to introduce new topics in later sections. After a simple introduction and explanation, the differential is used to introduce linear approximations, marginal analysis, and for measuring rates and errors. In Chapter 4, the differential is used to derive the elasticity of demand formula.

Rule of Three (plus one)

Concepts such as functions, rates of change, limits, derivatives, marginal analysis, optimization, integrals, differential equations, and partial derivatives are analyzed numerically, graphically, and algebraically. In addition, we stress the verbal approach through an emphasis on interpretation of solutions. The Rule of Three (plus one) is particularly powerful in Chapter 2 when limits and the derivative are introduced.

Graphs and Art

Our experience in the classroom indicates that many students at the brief calculus level are visual learners. Because of this observation, our text contains an abundance of graphs, pictures, charts, and tables. This visual approach acts to reinforce the mathematical concepts of calculus and to show that many real life applications begin with numerical data. Moreover, the charts and tables visually demonstrate to students that they are working frequently with real data taken from genuine sources.

Exercises

For many instructors, the exercise sets in a textbook are one of the most important components. We have taken great care to provide exercises that meet user demand in terms of quantity and quality. With greater than 3500 exercises from which to choose, we believe instructors and students will have no problem finding the types of problems they need ranging from routine skill and practice type problems to multi-step skill and concept based applications. We use a combination of real data-based problems, with their inherent complexities, and realistic problems that offer beneficial, but tidier solutions.

The exercises have been carefully written in matching odd-even pairs and are graded by level of difficulty. Care was given to write the exercises and solutions in a style and approach that is consistent with the text. Each exercise has been scrutinized in every draft of the manuscript to insure accuracy and appropriateness.

Chapter Features

Chapter Openers

The first page of each chapter includes a photo and a pair of graphs that foreshadow the fundamental ideas to be presented in the chapter. "What We Know" reiterates what information has been learned in previous chapters and "Where Do We Go" tells what new concepts will be studied. Taken together, these features help create a road map to guide students through the book and underscore the connections between topics.

Flashbacks

Selected functions and examples used earlier in the textbook are revisited to introduce new concepts and are denoted by the Flashback icon, . The Flashback concisely reviews an example from a previous section and then extends the problem by considering other questions that may be asked. In this manner, new topics are motivated in a natural way. Moreover, the Flashback often reviews skills and concepts from previous chapters that are needed. We believe that his pedagogical technique of using functions and applications previously introduced allows students to concentrate on new concepts using familiar applications.

On the Web

Many of the rich and varied applications in the textbook have been researched on the Internet. The "On the Web" icon, , denotes applications which use models based on data gathered from the Internet. This feature impresses on students the fact that calculus can be applied to real world problems. We have performed extensive research to insure that we have included applications from a variety of disciplines including problems taken from business and economics, the social sciences, the biological and life sciences, and the physical sciences.

Interactive Activities

Extensions to worked out examples are denoted by the "Interactive Activity" icon, . Many Interactive Activities are used to examine the problem from another perspective using the Rule of Three, while others explore additional properties of recently introduced topics. Others ask the student to do an exploration and make a conjecture to a completed example. Interactive Activities may serve many purposes: instructors may assign them as critical thinking exercises, they may be used as a springboard for classroom discussion, or may provide a vehicle for a collaborative activity. Solutions to selected Interactive Activities are given on the textbook's companion web site (www.prenhall.com/armstrong).

Checkpoints

At strategic points in each section, examples are followed by an exercise denoted with the "Checkpoint" icon, ✓. Each checkpoint asks students to work a particular, odd-numbered problem in the exercise set and helps to insure that a recently introduced skill or concept is understood. We have carefully chosen a parallel problem that requires a similar solution process to encourage students to check their grasp of the concept or skill. This pedagogical tool promotes better interaction between the text and student and encourages students to develop good study habits. Students who make use of the checkpoints will learn to take ownership of the course material.

Technology Notes

Additional tips and instructions for using graphing calculators are indicated with the "Technology Notes" icon, . These notes do not give keystroke commands, instead they offer tips based on common questions that students may have. Some Technology Notes refer to the **online calculator manuals** found at the textbook's companion web site (www.prenhall.com/armstrong).

From Your Toolbox

Whenever a previously introduced key definition, theorem, or property is needed, it is quickly reviewed and denoted by the "From Your Toolbox" icon, . This feature allows students to stay on task with the topic at hand without having to interrupt their reading to flip back to review previous material.

Notes

Immediately following many definitions, theorems, or properties, brief "Notes" are included to clarify a mathematical idea verbally, and to provide additional insights to help students understand the material.

Section Projects

At the end of each section, a *Section Project* presents a series of questions that ask students to explore the idea presented. We designed these questions to challenge, rather than discourage, the student. Many of the projects provide real data collected from the Internet and ask students to use the regression capabilities of their calculator to produce a model for the data, then to apply the recently introduced calculus concepts to the model. Instructors may use these section projects as a standard hand-in assignment or for a collaborative activity.

Supplements

Student's Solution Manual (0-13-085882-X)

Written by Matthew Hudock, Saint Philips College, San Antonio TX. This booklet contains complete, worked out solutions to all of the odd numbered exercises and review problems in the text.

Companion Web Site

This site (www.prenhall.com/armstrong) is designed to complement the text by offering a variety of teaching and learning resources including: a list of chapter objectives, a readiness quiz for each chapter to help students assess their preparedness for the chapter contents, solutions to many of the Interactive Activities, a set of destinations with links to other course related sites, the online graphing calculator manuals referenced in the Technology Notes, and a bulletin board for submitting and answering questions.

Instructor's Solution Manual (0-13-085885-4)

Written by Matthew Hudock, Saint Philips College, San Antonio TX. This booklet contains complete, worked out solutions to all even-numbered exercises and review problems in the text.

Test Item File (0-13-085881-1)

Written by Laurel Technical Service, Inc. This volume contains hardcopy of the test items available in PH Custom Test.

PH Custom Test: Windows (0-13-040295-8) Macintosh (0-13-040297-4)

PH Custom Test is a menu-driven random test generator available on either a Windows or Macintosh platform. The system incorporates a unique editing function that allows the instructor to enter additional problems, or alter existing problems in the test bank using a full set of mathematical notation. The test system offers free-response, multiple-choice, and mixed exams. An almost unlimited number of quizzes, review exercises, and chapter tests may be generated quickly and easily. The system will also save time by producing answer keys, student worksheets, and a gradebook for the instructor, if desired

ACKNOWLEDGMENTS

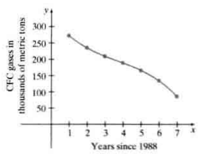
We owe a debt of gratitude to many individuals who helped us shape and refine *Brief Calculus: The Study of Rates of change*. Our first draft reviewers included: Martin Bonsangue, California State University at Fullerton, Fred Bakenhus, St. Philips College, Biswa Datta, Northern Illinois University, Matthew Hudock, St. Philips College, Anthony Macula, SUNY Geneseo, and Thomas Ordayne, University of South Carolina at Spartanburg. After thoughtful revision, we sought guidance from Rene Barrientos, Miami-Dade Community College, Mark Burtch, Arizona State University, Karabi Datta, Northern Illinois University, Adrienne Goldstein, Miami-Dade Community College, John Grima, Glendale

OVERVIEW

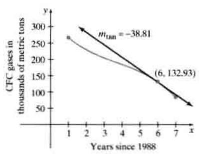
CHAPTER 2

Limits, Instantaneous Rate of Change, and the Derivative

The release of CFC's and other pollutants can cause smog.



The release of CFC's into the atmosphere by the U.S. from 1989–1995 can be modeled by $f(x) = -1.2x^3 + 13.37x^2 - 69.65x + 328.71$.



In 1994, the amount of CFC's released by the U.S. into the atmosphere was decreasing at a rate of $38.81 \frac{\text{thousand metric tons}}{\text{year}}$.

What We Know

In Chapter 1, we reviewed algebra and also learned about an important rate of change called an *average rate of change*. We saw how an average rate of change is equivalent to the slope of a secant line over an interval.

Where Do We Go

In this chapter, we will see how the limit concept is used to introduce a new type of rate of change called an *instantaneous rate of change*. We will see how an instantaneous rate of change is equivalent to the slope of a tangent line at a specific point.

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CHAPTER OPENERS

Each chapter begins with an **outline** and a **pictorial essay** that provides a visual orientation to the chapter contents.

What We Know reviews material learned in the previous chapter.

Where Do We Go introduces the major ideas that will be covered in this chapter. Taken together, they provide a roadmap to connect the topics and ideas presented in the book.

CHECKPOINTS

At strategic points in each section, examples are followed by a **check-point** icon. Each checkpoint tells the student to work an odd-numbered problem in the exercise set that reinforces the example just presented. Working the checkpoints is a good study habit that will result in better understanding of the material.

228 Chapter 4 • Additional Differentiation Techniques

✓ CHECKPOINT 3 Now work Exercise 33.

Applications

In our first application, we apply the Generalized Power Rule to a rational exponent function.

EXAMPLE 4 **Applying the Generalized Power Rule to Rational Exponent Functions**

The death rate caused by heart disease in the United States can be modeled by

$$f(x) = 336.18(x+1)^{-0.06}, \qquad 0 \le x \le 15$$

where x represents the number of years since 1980 and $f(x)$ represents the death rate (measured in deaths per 100,000 people) caused by heart disease. Determine $f'(x)$. Evaluate $f'(2)$ and interpret.

SOLUTION

Using the Generalized Power Rule, we get

$$\begin{aligned} f'(x) &= \frac{d}{dx}[336.18(x+1)^{-0.06}] \\ &= (336.18)\cdot(-0.06)(x+1)^{-0.06-1}\cdot(1) \\ &= -20.1708(x+1)^{-1.06} \end{aligned}$$

Notice that $x = 2$ corresponds to the year 1982. Continuing, we have

$$f'(2) = -20.1708(3)^{-1.06} \approx -6.29$$

So in 1982 the death rate caused by heart disease was decreasing at a rate of about

Community College, Michael Kirby, Tidewater Community College, Zhuangyi Lui, University of Minnesota at Duluth, and Martha Pratt, Mississippi State University. Our final panel of reviewers included Mark Burtch and Peter Casazza, Univ. of Missouri at Columbia.

We owe special thanks to Tony Palermino. Tony served not only as our Developmental Editor, but also our mentor, coach, advisor, and advocate. Tony's persistent and top quality professional work was essential in preparation of the manuscript. Without Tony's guidance, our text would not be where it is today. Matthew Hudock deserves thanks for his analysis and meticulous reviews and for preparation of exercise set solutions.

We thank Rollie Santos, Ph.D., from the Department of Economics at Lakeland Community College, who acted as advisor in many of the applications in business and economics; and Sue Hill, Ph.D., of the Department of Biology at Lakeland Community College, who provided materials and insight into many biological and life science applications. Teaching assistants Jamie Smolko and Denise Kerr were very helpful in the development of solutions to exercises in early drafts of the manuscript. The Lakeland Community College Applied Calculus students piloted the text in the manuscript and page proof phases and provided important student feedback.

Many people involved in the management and production of the text deserve recognition. We are grateful to Sally Yagan and Paige Akins for believing in us, Ann Heath for her scheduling and crisis management skills, Bob Walters for his formidable production abilities, Kathy Boothby Sestak for her can-do attitude, and to Patrice Jones for his innovative marketing techniques. We also thank the Prentice Hall sales staff for their sales efforts and enthusiasm. We appreciate the quality services of Laurel Technical Services who did a first-rate job in writing review and test bank exercises, and of Academy Artworks for their preparation of the graphs, tables, and art in the text, and Sara Beth Newell and Joanne Wendelken for copying the manuscript and routing our numerous telephone calls.

Finally, we owe personal thanks to our families. Our wives, Lisa and Melissa, were supportive and patient during the entire process. To our children Austin and Dylan; Randy, Rusty, and Ronnie who understood why sometimes Dad could not come out and play in the backyard.

Bill Armstrong
Don Davis

EXAMPLE 5 **Interpreting the Secant Line Slope**

The U.S. imports from China for the years 1987 to 1996 can be modeled by

$$f(x) = 0.32x^2 + 1.64x + 3.98, \qquad 1 \le x \le 10$$

where x represents the number of years since 1986 and $f(x)$ represents the dollar value, in billions, of goods imported.

(a) Make a table of function values for $x = 1, 2, 3, \dots, 10$. Use these values when calculating parts (b) and (c).

(b) Determine the average rate of change in U.S. imports from China from 1987 to 1991 and interpret. Include appropriate units.

(c) Determine the average rate of change in U.S. imports from China from 1991 to 1996 and interpret. Include appropriate units.

(d) Compare the results from parts (b) and (c).

SOLUTION

(a) A numerical table of values for the model is shown in Table 1.3.1.

Table 1.3.1

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	5.94	8.54	11.78	15.66	20.18	25.34	31.14	37.58	44.66	52.38

(b) First we see that the year 1987 corresponds to $x = 1$ and the year 1991 corresponds to $x = 1991 - 1986 = 5$. This means that the increment in x is $\Delta x = 5 - 1 = 4$. So we need to compute the difference quotient

$$m_{\text{sec}} = \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{f(1+4) - f(1)}{4} = \frac{f(5) - f(1)}{4}$$

The values from Table 1.3.1 give us

$$m_{\text{sec}} = \frac{20.18 - 5.94}{4} = \frac{14.24}{4} = 3.56 \quad \text{or} \quad 3.56\ \frac{\text{billion dollars}}{\text{year}}$$

This means that during the period from 1987 to 1991, U.S. imports from China increased at an average rate of 3.56 $\frac{\text{billion dollars}}{\text{year}}$. Notice that the amount of imports increased, since the secant line slope is

EXAMPLE—ON THE WEB

Every example in the book is titled and features a fully worked-out solution. Many of the applications in the textbook have been researched on the Internet. The **On the Web** icon denotes applications which use models based on real data gathered from the Internet.

FLASHBACK

Selected functions and examples used earlier in the textbook are revisited to introduce new concepts and are set off as a **Flashback.** Revisiting previously introduced material allows students to concentrate on the new concept and to see how more can be learned about a problem by investigating it with a different technique.

Flashback **U.S. Imports from China Revisited**

In Section 1.3, the U.S. imports from China for the years 1987 to 1996 were modeled by

$$f(x) = 0.32x^2 + 1.64x + 3.98,^* \qquad 1 \le x \le 10$$

where x represents the number of years since 1986 and $f(x)$ represents the dollar value, in billions, of goods imported. The graph of the model is shown in Figure 2.3.1.

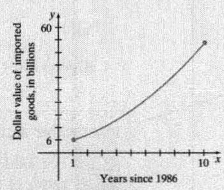


Figure 2.3.1 Graph of $f(x) = 0.32x^2 + 1.64x + 3.98$.

Determine the average rate of change for U.S. imports from China from 1990 to 1993 and interpret.

Flashback Solution

Recall that the average rate of change is simply equivalent to the *slope of the secant line* through two points. Since 1990 corresponds to an x-value of 4 and 1993 corresponds to an x-value of 7, we need to determine the corresponding y-values for these x-values to get the two points that the secant line passes through. We determine the y-value when $x = 4$ by simply substituting $x = 4$ into our model as follows:

$$y = f(x) = 0.32x^2 + 1.64x + 3.98$$
$$= 0.32(4)^2 + 1.64(4) + 3.98 = 15.66$$

We now know one point that the secant line passes through is (4, 15.66). Similarly, when $x = 7$, we get $y = 31.14$. Hence, the second point that the secant line passes through is (7, 31.14). See Figure 2.3.2. Now the secant line slope through these two points is simply

$$m_{\text{sec}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{31.14 - 15.66}{7 - 4} = 5.16$$

So our answer is that the average rate of change for U.S. imports from China from 1990 to 1993 was 5.16 $\frac{\text{billion dollars}}{\text{year}}$. Recall from Chapter 1 that this means that over the period 1990 to 1993 U.S. imports from China increased 5.16 $\frac{\text{billion dollars}}{\text{year}}$ on average.

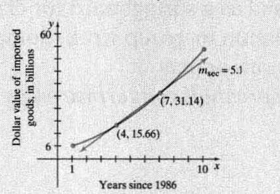


Figure 2.3.2 Slope of the secant line, m_{sec}, gives the average rate of change.

**The data and model given in exercises and examples preceded by the "On the Web" icon are based on information gathered at the U.S. Census Bureau web site. These exercises and examples appear throughout this textbook.*

There are a couple of important items in the Flashback that we must review:

- Since we found a *rate,* we must include *appropriate units.*
- The secant line has a positive slope indicating that from 1990 to 1993 U.S.

Interactive Activities

Extensions to worked-out examples appear as **Interactive Activities.** Many ask the student to complete the Rule of Three (plus one) or explore additional properties of recently introduced topics. Solutions to selected Interactive Activities appear on the companion Web site at *www.prenhall.com/armstrong.*

✓ CHECKPOINT 3

Now work Exercise 39.

EXAMPLE 5 **Analyzing a Limit Involving $\frac{0}{0}$**

Determine $\lim_{x\to 0} \frac{|x|}{x}$.

SOLUTION

If we try substituting, we get

$$\lim_{x\to 0} \frac{|x|}{x} = \frac{0}{0}$$

By letting $f(x) = \frac{|x|}{x}$, we can use a table and a graph to determine this limit. See Table 2.1.4 and Figure 2.1.12.

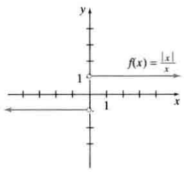


Figure 2.1.12

Table 2.1.4

	$x \to 0^-$					$x \to 0^+$			
x	−1	−0.1	−0.01	−0.001	0	0.001	0.01	0.1	1
$f(x)$	−1	−1	−1	−1		1	1	1	1

Interactive Activity

Recall that $|x|$ is defined to be

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}.$$

Use this definition to algebraically verify that $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.

As we can see numerically and graphically, since the left-hand limit does not equal the right-hand limit, we conclude that $\lim_{x\to 0} \frac{|x|}{x}$ does not exist. ■

For Example 6, we need to recall Limit Theorem 1, which states that the limit of a constant is that constant.

EXAMPLE 6 **Analyzing Limits Involving Two Variables**

Determine the following limits:

(a) $\lim_{h\to 0}(3x + 2h)$ (b) $\lim_{h\to 0} \frac{5xh + 2h^2}{h}$

(b) From the table and the graphs in Figures 1.6.1a and b, we see that any real number can be substituted for x, so the domain of the functions $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$ is $(-\infty, \infty)$.

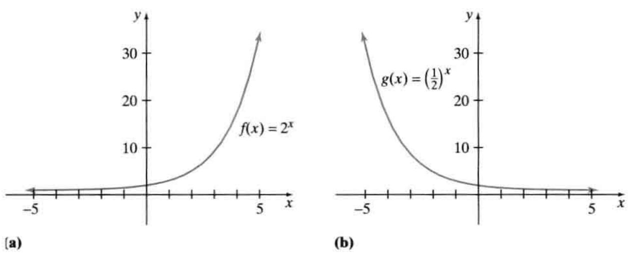


Figure 1.6.1 ■

Interactive Activities

Others ask the student to explore additional properties of recently introduced topics. Interactive Activities may be assigned as homework or may be used as a springboard for classroom discussion or group work. Some solutions appear at *www.prenhall.com/armstrong.*

Interactive Activity

For Table 1.6.1, make another column of the table that calculates the **successive ratios** of the function values by dividing each value by the preceding value. For example, for the function $g(x) = \left(\frac{1}{2}\right)^x$, the ratios would begin $\frac{16}{32}, \frac{8}{16}, \ldots$. What pattern do you notice?

Notice that $f(x) = 2^x$ is increasing on its domain and as $x \to \infty, f(x) \to \infty$. Consequently, we call this kind of function an **exponential growth function.** Whereas, notice that $g(x) = \left(\frac{1}{2}\right)^x$ is decreasing on its domain and, as $x \to \infty$, $g(x) \to 0$. We call this kind of function an **exponential decay function.** (See Properties of General Exponential Functions at the top of p. 71.)

For the function $f(x) = a \cdot b^x$, the value of a can affect the end behavior. First, we see that with any value of a, $f(0) = a \cdot b^0 = a \cdot 1 = a$. This means that the y-intercept of the graph of $f(x) = a \cdot b^x$ is at $(0, a)$. To see the effect of a, compare the graphs of $f(x) = 2 \cdot 3^x$ and $g(x) = -2 \cdot 3^x$ in Figure 1.6.2.

FROM YOUR TOOLBOX

Whenever a previously introduced key definition, theorem, or property is needed, it is reviewed in **From Your Toolbox.** This feature allows the student to stay on task without having to flip back to review previous material.

From Your Toolbox

1. The *price–demand function p* gives us the price $p(x)$ at which people buy exactly x units of product.
2. The cost of producing x units of product with variable costs m and fixed costs b is given by the *cost function:*

$$C(x) = mx + b = \begin{pmatrix}\text{variable}\\ \text{costs}\end{pmatrix} x + \begin{pmatrix}\text{fixed}\\ \text{cost}\end{pmatrix}$$

Note that since variable costs are often expressed as a function, $C(x)$ may be a higher-order polynomial function.

3. The total revenue R generated by producing and selling x units of product at price $p(x)$ is given by the *revenue function:*

$$R(x) = x \cdot [p(x)] = \begin{pmatrix}\text{quantity}\\ \text{sold}\end{pmatrix} \cdot \begin{pmatrix}\text{unit}\\ \text{price}\end{pmatrix}$$

4. The profit P generated after producing and selling x units of a product is given by the *profit function:*

$$P(x) = R(x) - C(x) = \text{revenue} - \text{cost}$$

TECHNOLOGY NOTES

The graphing calculator is incorporated into the text at appropriate junctures. Important tips and instructions for using graphing calculators are given in marginal **Technology Notes**. Many of the notes refer to the online calculator manual found on the companion Web site *www.prenhall.com/armstrong*.

TECHNOLOGY NOTE
Due to the limitations of graphing calculators, you may not see a hole in the graph at $x = 2$. Using ZDECIMAL or selecting the x-axis window so that $x = 2$ is the midpoint of the graphing interval should show the hole. See Figure 2.1.2.

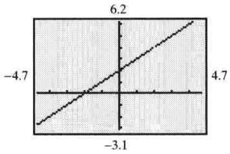


Figure 2.1.2

It appears from Table 2.1.1 that if we start to the left of $x = 2$ or to the right of $x = 2$, as we allow x to approach 2, our functional values are approaching 4. We can say that,

"The limit of $\frac{x^2-4}{x-2}$, as x approaches 2, is 4."

Using an arrow for the word *approaches* and *lim* as shorthand for the word limit, the mathematical notation for this English sentence is

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$$

You should interpret this limit notation to mean that as x gets closer and closer to 2, from both sides of 2, $\frac{x^2-4}{x-2}$ gets closer and closer to 4.

For a graphical perspective, let's graph $y = \frac{x^2-4}{x-2}$. Figures 2.1.2 and 2.1.3 show the graph of $y = \frac{x^2-4}{x-2}$, where Figure 2.1.3 is the result of utilizing the ZOOM IN command.

Notice in Figures 2.1.3b and c that we have used the TRACE command to get as close to 2 as possible from the left of 2 and from the right of 2, respectively. We now have graphical support for our numerical work. That is, numerically and graphically we believe that $\lim_{x \to 2} \frac{x^2-4}{x-2} = 4$.

$$\lim_{x \to 1} \frac{x^2-1}{x-1} = 2$$

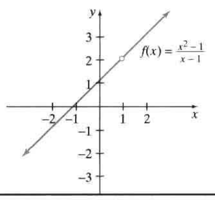
TECHNOLOGY NOTE
Many graphing calculators have a TABLE feature. Consult the on-line calculator manual at www.prenhall.com/armstrong

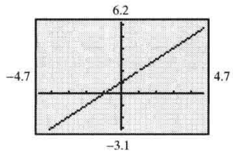
TABLE 2.1.2

	$x \to 1$ FROM LEFT					$x \to 1$ FROM RIGHT			
x	0	0.9	0.99	0.999	1	1.001	1.01	1.1	2
$f(x)$	1	1.9	1.99	1.999		2.001	2.01	2.1	3

Figure 2.1.4 shows the graph of $f(x) = \frac{x^2-1}{x-1}$. In Figure 2.1.5 we have used the ZOOM IN command to capture that part of the graph where $x = 1$, and Figure 2.1.6 is the result of our ZOOM IN.

We now use the TRACE command to get as close to $x = 1$ as possible, from both sides of $x = 1$, and observe the corresponding y-values. See Figures 2.1.7a and b.





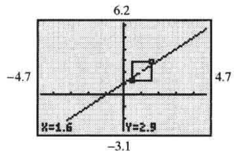


Figure 2.1.5

Section Summary

Each section ends with a brief **Summary** that reviews the key ideas and formulas presented. A bulleted list of important functions with their formulas facilitates a quick review.

Summary

In this section, we revisited the business functions and from them derived the marginal business functions. These functions are found by differentiation, and they determine the cost, revenue, or profit for producing one more item of a product. Then we discussed the average business functions, which were found by taking the business function and dividing by the independent variable. Finally, we discussed the marginal average business functions, which were the derivatives of the average business functions.

Important Functions

- **Marginal cost function:** $MC(x) = C'(x)$
- **Marginal revenue function:** $MR(x) = R'(x)$
- **Marginal profit function:** $MP(x) = P'(x)$
- **Average cost function:** $AC(x) = \frac{C(x)}{x}$
- **Average profit function:** $AP(x) = \frac{P(x)}{x}$
- **Marginal average cost function:** $MAC(x) = \frac{d}{dx}\left(\frac{C(x)}{x}\right)$
- **Marginal average profit function:** $MAP(x) = \frac{d}{dx}\left(\frac{P(x)}{x}\right)$

Section 3.2 Exercises

For Exercises 1–6, assume $C(x)$ is in dollars and complete the following:

(a) Determine the marginal cost function MC.

(b) For the given production level x, evaluate $MC(x)$ and interpret.

(c) Evaluate the actual change in cost by evaluating $C(x+1) - C(x)$ and compare with the answer to part (b).

1. $C(x) = 23x + 5200; \quad x = 10$
2. $C(x) = 14x + 870; \quad x = 12$

Section Exercises

Each section concludes with a comprehensive set of exercises that begins with basic skills and moves on to more conceptually challenging applications. The **Applications** feature both realistic and real data-based problems as noted by the On the Web symbol.

Section 4.2 Exercises

In Exercises 1–10, determine the derivative for the following functions.

1. $f(x) = 5\ln x$
2. $f(x) = -8\ln x$
3. ✓ $f(x) = \ln x^6$
4. $f(x) = \ln x^4$
5. $f(x) = 4x^3 \cdot \ln x$
6. $f(x) = 12x^3 \cdot \ln x$
7. $f(x) = \frac{3x^5}{\ln x}$
8. $f(x) = \frac{12}{\ln x}$
9. $f(x) = 10 - 12\ln x$
10. $f(x) = -2 + 8\ln x$

In Exercises 11–24, determine the derivative for the following functions.

11. $g(x) = \ln(x+7)$
12. $g(x) = \ln(2-x)$
13. $g(x) = \ln(2x-5)$
14. $g(x) = \ln(3x+4)$
15. $g(x) = \ln(x^2+3)$
16. $g(x) = \ln(3x^3-11)$
17. $g(x) = \ln(\sqrt{2x+5})$
18. $g(x) = \ln(\sqrt[3]{4x+2})$
19. $g(x) = (\ln x)^6$
20. $g(x) = (\ln x)^4$
21. $g(x) = \sqrt{x} \cdot \ln(\sqrt{x})$
22. $g(x) = 4x^5 \cdot \ln(3x^3)$
23. $g(x) = \frac{x^2+2x+3}{\ln(x+5)}$
24. $g(x) = \frac{4x^3-x+2}{\ln(x+7)}$

For Exercises 25–34, determine the derivative for the following functions.

25. $f(x) = \log_{10} x$
26. $f(x) = \log_5 x$
27. $f(x) = 6\log_3 x$
28. $f(x) = 11\log_4 x$
29. $f(x) = x^2 \log_9 x$
30. $f(x) = 2x^5 \log_8 x$
31. ✓ $f(x) = \log_2(5x+3)$
32. $f(x) = \log_5(3x+9)$
33. $f(x) = \log_{10}\left(\frac{x+3}{x^2+1}\right)$
34. $f(x) = \log_2\left(\frac{x^3}{x^2-1}\right)$

For Exercises 35–42, determine an equation for the line tangent to the graph of the function at the given point.

35. $f(x) = \ln x; \quad (2, \ln 2)$
36. $f(x) = \ln x; \quad (1, 0)$
37. $f(x) = \ln\sqrt{2x-1}; \quad (1, 0)$
38. $f(x) = \ln(3x); \quad (2, \ln 6)$

Applications

43. ✓ A research assistant in biology finds in an experiment that at low temperatures the growth of a certain bacteria culture can be modeled by

$$f(t) = 750 + 12\ln t, \qquad t \ge 1$$

where t represents the number of hours since the start of the experiment and $f(t)$ represents the number of bacteria present.

(a) Determine $f'(t)$.

(b) Evaluate and interpret $f(12)$ and $f'(12)$.

44. The city of Plantersville has enacted new zoning laws in order to curb the growth of the city's population. They find that the population can be modeled by

$$P(x) = 10{,}000 + 100\ln x, \qquad x \ge 1$$

where x represents the number of years since the laws were adopted and $P(x)$ represents the city's population.

(a) Determine $P'(x)$.

(b) Evaluate and interpret $P(20)$ and $P'(20)$.

45. Prescription drug companies have found that the popularity of the new drug Vectrum has dwindled and can be modeled by

$$f(x) = 150 + 5\log_2 x, \qquad x \ge 1$$

where x represents the number of years that the drug has been on the market and $f(x)$ represents the number of prescriptions written for the drug annually in thousands.

(a) Determine $f'(x)$.

(b) Evaluate $f'(2)$ and $f'(10)$ and interpret each.

46. The urban school district of Molisburg has started a new educational campaign in an attempt to reduce the increase in lice found in the elementary school student population. The number of children who contracted lice can be modeled by

$$g(t) = 200 + 8\log_3 t, \qquad t \ge 1$$

where t represents the number of years since the new educational campaign has been enacted and $g(t)$ represents the number of students who are diagnosed with lice annually.

(a) Determine $g'(t)$.

(b) Evaluate $g'(2)$ and $g'(7)$ and interpret each.

47. The life expectancy for African-American females in the United States can be modeled by

(c) Write the equation of the tangent line at $x = 3$, and determine y on the tangent line when $x = 15$. Interpret and compare to $f(15)$.

48. The life expectancy for white females in the United States can be modeled by

$$f(x) = 75.32 + 1.29 \ln x, \qquad 1 \le x \le 26$$

where x represents the birth year since 1969 and $f(x)$ represents the life expectancy in years.

(a) Determine $f'(x)$.

(b) Evaluate and interpret $f(3)$ and $f'(3)$. Compare to part (b) in Exercise 47.

(c) Write the equation of the tangent line at $x = 3$, and determine y on the tangent line when $x = 15$. Interpret and compare to $f(15)$. Compare to part (c) in Exercise 47.

49. The annual per capita consumption of light and skim milk in the United States can be modeled by

$$f(x) = 10.12 + 2 \ln x, \qquad 1 \le x \le 16$$

where x represents the number of years since 1979 and $f(x)$ represents the annual per capita consumption of light and skim milk in gallons.

(a) Graph f in the viewing window [1, 16] by [10, 17].

(b) Determine $f'(x)$.

(c) Evaluate and interpret $f'(5)$ and compare to $f'(10)$.

50. The average expenditure for a new domestic car in the United States can be modeled by

$$f(x) = 15{,}302.93 + 1685.66 \ln x, \qquad 1 \le x \le 7$$

where x represents the number of years since 1980 and $f(x)$ represents the average expenditure for a new domestic car in dollars.

(a) Graph f in the viewing window [1, 7] by [15,000, 19,000].

SECTION PROJECT

The revenue, in billions of dollars, generated in the solid waste management industry in the United States from 1980 to 1996 are displayed in Table 4.2.2.

TABLE 4.2.2

YEAR	x	REVENUE (IN $BILLIONS)
1980	1	52.0
1990	11	146.4
1994	15	172.5
1995	16	180.0
1996	17	184.3

Source: U.S. Census Bureau web site.

(a) Use your calculator to determine a logarithmic regression model of the form

$$f(x) = a + b \ln x, \qquad 1 \le x \le 17$$

where x represents the number of years since 1979 and $f(x)$ represents the revenue generated in the solid waste management industry, in billions of dollars.

(b) Determine $f'(x)$.

SECTION PROJECT

At the end of each section, a **Section Project** presents the student with a series of questions that challenge the student to explore the ideas presented. Many of the projects provide real data collected from the Internet and ask the students to use their calculator to produce a model for the data.

CHAPTER REVIEW EXERCISES

1. For $f(x) = \dfrac{1 - \sqrt{1 - 2x - x^2}}{x}$, complete the table to numerically estimate the following.

(a) $\lim_{x \to 0^-} f(x)$ (b) $\lim_{x \to 0^+} f(x)$ (c) $\lim_{x \to 0} f(x)$

x	−0.1	−0.01	−0.001	0	0.001	0.01	0.1
$f(x)$				?			

2. For $f(x) = (\sqrt{4 - x})^5$, complete the table to numerically estimate the following.

(a) $\lim_{x \to 4^-} f(x)$ (b) $\lim_{x \to 4^+} f(x)$ (c) $\lim_{x \to 4} f(x)$

x	3	3.9	3.99	4	4.01	4.1	5
$f(x)$				?			

For Exercises 3 and 4, use your calculator to graph the given function. Use the ZOOM IN and TRACE commands to graphically estimate the indicated limits. Verify your estimate numerically.

3. $f(x) = x^3 - 2x$; $\lim_{x \to 3.1} f(x)$

4. $f(x) = \sqrt{x} + \dfrac{1}{|x|}$; $\lim_{x \to 2.5} f(x)$

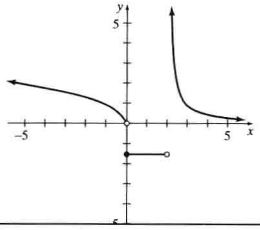
For Exercises 5–10, determine the indicated limit algebraically.

5. $\lim_{x \to 2} (7x^3 - 10x)$ 6. $\lim_{x \to -1} \dfrac{x^2 - 1}{x + 1}$

7. $\lim_{x \to -3} |x - 5|$ 8. $\lim_{x \to 0} \sqrt{36 - 8x}$

9. $\lim_{x \to 10} \dfrac{x + 10}{x^2 - 100}$ 10. $\lim_{x \to 2.2} (x + 9)^3$

11. Use the graph of f to find the following:



(a) $\lim_{x \to -4} f(x)$ (b) $\lim_{x \to 0^-} f(x)$ (c) $f(0)$

(d) $\lim_{x \to 2} f(x)$ (e) $\lim_{x \to 3} f(x)$ (f) $f(3)$

Determine the indicated limit in Exercises 12–14.

12. $\lim_{h \to 0} (x + 2h)^2$ 13. $\lim_{h \to 0} \dfrac{2x^2h - 9h}{h}$ 14. $\lim_{h \to 0} \dfrac{6x^3h^2 + h}{h}$

15. For $f(x) = 3x^2$, find

(a) $f(2 + h)$ (b) $\lim_{h \to 0} \dfrac{f(2 + h) - f(2)}{h}$

16. For $f(x) = \dfrac{9}{x}$, find

(a) $f(4 + h)$ (b) $\lim_{h \to 0} \dfrac{f(4 + h) - f(4)}{h}$

17. Acme Stuffed Animals, Inc, is introducing a new line of teddy bears. The total cost of producing Scare Bear (with glowing eyes) is projected to be $C(x) = 36{,}000 + \sqrt{10{,}000x}$, where x is the number of units made and $C(x)$ is the cost in dollars.

(a) Find and interpret $C(400)$.

(b) Find and interpret $\lim_{x \to 100} C(x)$.

(c) Find and interpret $AC(x) = \dfrac{C(x)}{x}$ for $x = 25$.

18. A toy speedboat moves away from a dock, and its distance from the dock is given by

$$d(t) = \begin{cases} 2t^2, & 0 \le t \le 3 \\ 12t - 18, & 3 < t \end{cases}$$

where $d(t)$ is in feet and t is in seconds.

(a) Graph d for $0 \le t \le 6$.

(b) Find $\lim_{t \to 3^-} d(t)$.

(c) Find $\lim_{t \to 3^+} d(t)$.

(d) Find $\lim_{t \to 3} d(t)$.

(e) Describe the behavior of the toy speedboat.

19. A pastry chef in a commercial test kitchen is fine-tuning a pie-filling recipe. Colleagues acting as tasters have rated various recipes on a scale of 1 to 10. The average rating $R(s)$ appears to be a function of the sugar content s.

$$R(s) = 8 - \frac{s^2 - 48s + 512}{50}$$

where s is the number of tablespoons of sugar.

(a) Graph R in the viewing window [0, 40] by [0, 10].

(b) Find $R(20)$, $R(25)$, and $R(30)$.

(c) Find $\lim_{h \to 0} \dfrac{R(24 + h) - R(24)}{h}$.

CHAPTER REVIEW EXERCISES

Rounding out each chapter is a group of more than 100 review exercises that test students' understanding of all of the topics covered in the chapter. The **Section and Review Exercises** provide the user with greater than 3500 exercises in total.

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