CLASSICAL MECHANICS

VOLUME II

EDWARD A. DESLOGE

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Volume 2

EDWARD A. DESLOGE
Department of Physics
Florida State University

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Preface

This book is the product of teaching classical mechanics on both the undergraduate and the graduate levels intermittently over the past 20 years. It covers mechanics in a unified fashion from the foundations, through elementary and intermediate mechanics, to a moderately advanced graduate level. A knowledge of calculus is presumed. Any other mathematics needed is provided in the appendices.

Volume 1 can be used as a text for an undergraduate course. Volume 2 can be used as a text for a graduate course. By judicious deletion of material, courses of almost any length can be accommodated. The breadth and detail of the coverage is such that the book can also be used by students wanting to learn mechanics on their own, or by instructors wanting to direct students through self-paced programs. All the topics customarily found in an undergraduate or graduate text are covered, though frequently, in greater depth or detail, or from a slightly different point of view. In addition there are many interesting and useful topics that are seldom found in the standard texts. Hence this book can also serve as a source book for an instructor in mechanics, or as an aid to a researcher whose need for mechanics exceeds what is provided by the usual text.

Much material is covered, but no attempt has been made to write an encyclopedic text on classical mechanics; rather, the subject is arranged in such a way that additional material can be inserted easily and naturally.

A knowledge of mechanics that will continue to mature beyond the termination of a formal course requires a clear and accurate grasp of the organization of the subject. Throughout this book, I have tried to stress organization, clarity, and accuracy. This emphasis may sometimes appear to result in an approach that is overly stiff, detailed, and formal. The niceties of charm, elegance, and warmth, where lacking, can be supplied by a good instructor. A weakness in organization is almost impossible to repair.

One of the most crucial stages in the exposition of any subject in physics is the choice of notation. A good notation is comprehensive; that is, it carries all the information that is required to avoid misinterpretation, and yet it is simple.

Slight differences are often quite significant. For example it is both more meaningful and more useful to express the components of a vector **A** with respect to a frame S' as $A_{i'}$ rather than A'_i. I have thought a great deal about the notation. If I have erred, it is more often than not in using notation that is somewhat overloaded. The appearance of many of the discussions and derivations could be improved by stripping the notation of some of its appendages. The loss however would generally outweigh the gain. For example, the appearance of arguments involving partial derivatives, such as $(\partial f/\partial x)_y$ or equivalently $\partial f(x, y)/\partial y$ could be improved by writing such terms simply $\partial f/\partial x$. I have at times done this myself. However, anyone who has taught a course in thermodynamics will verify that the consequences can be disastrous, if one is not careful.

In writing this book, I have used a number of organizational and pedagogical devices that I have found over the years to be particularly helpful. (1) *The material is highly subdivided*, to give emphasis to the organization and to facilitate reorganization. (2) *The chapters are short*, to make assimilation easier and to aid in the addition, deletion, or rearrangement of material. (3) *Formal definitions, postulates, and theorems are frequently used*, to emphasize and clarify important concepts and to foster exactness. (4) *Most chapters contain one or more examples* designed to illustrate an idea, or to provide general techniques for the solution of problems, rather than to serve as models to be slavishly imitated. (5) *There are a large number of problems*, since the ability to systematically solve a wide variety of problems is one of the goals of a course in mechanics. (6) *All mathematical developments needed are provided in appendices*. In this way the continuity of a physical argument is not broken, and yet the mathematical tools are readily available.

A number of benefits, other than problem solving, can be derived from a course in classical mechanics. Three in particular have influenced me strongly in writing this book.

1. Since physics as we know it today had its origins in classical mechanics, the very structure and language of physics is permeated by ideas that come from classical mechanics; hence a *knowledge of the foundations of mechanics can provide a student with a deeper grasp of all of physics*. I have therefore devoted considerable effort to developing the foundations of classical mechanics.
2. *A course in classical mechanics is not only a course in physics but a course in applied mathematics*. The inclusion of certain topics, and the extensive mathematical appendices in this book, reflect this aspect of mechanics. As much consideration has been given to the writing of the appendices as to the text proper. Though relegated to appendices, the mathematical topics are an integral part of the text.
3. The human brain contains two hemispheres whose characters have been shown to be different but complementary. In most individuals, the right

hemisphere, which is associated directly with the left hand, the left field of vision, and so forth, is superior in handling geometrical concepts, and the left hemisphere, which is associated directly with the right hand, the right field of vision, and so forth, is superior in handling formal analytical concepts. Learning physics involves an interesting interplay between these two abilities. The left hemisphere provides the analytical map that takes us from one point to another, while the right hemisphere provides the geometrical vision necessary to see the goal and landmarks along the way. *A course in classical mechanics offers a marvelous vehicle for developing and integrating both the geometrical and the analytical powers of the brain.* The Newtonian approach to mechanics is strongly geometrical, whereas the Lagrangian and Hamiltonian approaches are strongly analytical. It follows that a course designed to exercise both sides of the brain should, as this text does, include a thorough foundation in Newtonian mechanics before proceeding to Lagrangian and Hamiltonian mechanics. Too many modern courses in classical mechanics are built on a weak foundation of Newtonian mechanics, with the resultant complaint by instructors that a particular student is good at mathematics but does not seem to have any physical intuition.

To make Volume 1 adequate for an undergraduate course and Volume 2 adequate for a graduate course, certain topics are covered in both volumes. Central force motion, the differential scattering cross section, and small oscillations are introduced in Volume 1, and reviewed and extended in Volume 2. Rigid body motion is covered in great detail from the Newtonian point of view in Volume 1, and briefly reviewed and treated from the Lagrangian point of view in Volume 2. An introduction to Lagrangian and Hamiltonian mechanics is given in Volume 1, to prepare the student for the full treatment in Volume 2.

To conclude this preface, I point out some aspects of the book that are unique or are, in my opinion, treated better or more thoroughly here than elsewhere.

In Chapters 1–4, 85, and 86 the basic principles of Newtonian and relativistic kinematics are derived starting from the assumption of the existence and equivalence of inertial frames of reference. With this assumption, the law of transformation between inertial frames arises naturally and contains only one undetermined parameter, which is identified as the upper speed with which a particle can move with respect to an inertial frame. By choosing this parameter to be infinite we obtain the Galilean transformation, and by choosing it to be finite we obtain the Lorentz transformation.

In Chapters 8, 9, 88, and 89 an investigation of quantities that might be conserved in a collision leads naturally to the momentum conservation laws of Newtonian and relativistic mechanics, and from these laws to the definitions of force in both Newtonian and relativistic mechanics.

The treatment of the foundations of Newtonian and relativistic mechanics contained in the two above-mentioned sets of chapters clearly brings out the close relation between Newtonian and relativistic mechanics, and the primacy of the law of conservation of momentum over Newton's equation of motion and its counterpart in relativistic mechanics. It is the most thorough treatment that can be found anywhere. Many of the details are new and unique.

Chapter 6 presents very carefully and thoroughly the definition and properties of the angular velocity of one coordinate system with respect to another. By starting with the analytical definition of angular velocity rather than the geometrical definition, as is customarily done, many of the difficulties associated with this concept are avoided. Even though a good grasp of this concept is a prerequisite to an ability to express the laws of motion in rotating frames, and to an understanding of the kinematics and dynamics of rigid bodies, it is amazing to me how many of the standard texts are weak on this subject, and frequently contain spurious definitions of angular velocity.

Chapter 23 contains one of the simplest and most organized treatments of central force motion of which I am aware.

Chapters 24, 26, 60, and 61 contain a very complete treatment of the differential scattering cross section. Chapter 61 derives the relationship between the center of mass and laboratory cross sections for a collision in which both particles are moving, the collision is inelastic, and the product particles differ from the incident particles. No other text that I know of contains this complete result.

The treatment of rigid body motion in Chapters 34–40, with the possible addition of Chapters 41, 63, and 64, is sufficiently complete and detailed to form a course by itself. Chapter 38 is a thorough treatment of the inertia tensor from both the analytical and the geometric points of view. Chapter 39 contains an extremely detailed treatment of Euler angles, and the kinematics of rigid body motion. The expression in Chapter 39 of the equations of transformation between different reference frames and the relative angular velocities between these frames in terms of Euler angles is a very useful aid to anyone interested in solving rigid body motion problems.

Chapters 47–54 give a thorough treatment of Lagrange's equations of motion and include a detailed treatment of constraints both holonomic and anholonomic. Since most undergraduate courses do not have the time for such a complete treatment, an introductory abbreviated treatment appears in Chapters 42–44.

The treatment of small vibrations in Chapters 45 and 65 contains a number of results that cannot be found elsewhere.

The material in Chapter 66 together with the material in Appendices 28–30 represents a complete introduction to group theory and its application to symmetrical vibrating systems. Although the complexity of the notation makes the going a little tedious at times, the text contains none of the gaps and guesswork that seem to mar the treatments I have found elsewhere.

Quasi-coordinates and the Gibbs-Appell equations of motion covered in Chapters 67–70 are probably unknown to most physicists. There are only a few mechanics books in which they are even mentioned. Because I think they deserve more attention and are quite useful in solving certain problems, I have included them.

The treatment of Hamilton's equations of motion, canonical transformations, and Hamilton-Jacobi theory in Chapters 71–78 is quite unusual in that no use is made of the calculus of variations. Although this results in many of the proofs being a little longer than usual, I feel that it provides a more secure foundation in the subject, since most students who are encountering this subject for the first time are not completely sure of themselves in the use of the calculus of variations. This unsureness is usually compounded by the casual and sometimes erroneous use of the calculus of variations by some authors.

Although I prefer not to use the calculus of variations in a student's first encounter with Hamiltonian mechanics, I certainly believe it to be an extremely useful tool, hence have devoted Part 8 and Appendix 32 to this subject. The initial separation of the calculus of variations from Hamiltonian mechanics has the added organizational advantage of allowing the instructor who so desires to pursue the subject beyond and apart from what is required in Hamiltonian mechanics.

The presentation of the foundation and basic principles of relativistic mechanics in Part 9 is I believe one of the most logical and straightforward to be found anywhere. Though some of the proofs are a little ponderous, the general flow of concepts does not require an extraordinary distortion of a student's imagination. As a consequence relativistic mechanics seems almost inevitable.

The nature and importance of constants of the motion is stressed time and again throughout the text, starting with Chapter 17 and proceeding through Chapters 33, 46, 56, 71, and 81. Chapter 56 considers in detail the relationship between constants of the motion and the invariance of the Lagrangian under certain transformations, and Noether's theorem is presented without making use of variational techniques as is usually done in those few sources where this theorem can be found.

The subject of impulse, which usually causes students a great deal of unnecessary grief, is developed in detail in Chapters 18, 41, and 57.

Many of the appendices are quite useful in themselves, apart from their value in the body of the text. However since this book is intended primarily as a mechanics text, and only secondarily as a course in applied mathematics, many theorems in the appendices are stated without proof. Proofs that are particularly pertinent or cannot easily be found elsewhere are given.

In Appendices 4–6, and 23, the reader is led systematically from the geometric concept of a vector as a directed line segment to the highly analytical concept of general tensors. With a little amplification and completion of proofs, the material would make a good course in vectors and tensors.

Similarly the material in Appendices 12 and 24–26, with the possible addition of the material on quadratic forms in Appendix 27, forms a good outline for a course in matrices.

Appendix 32 provides an excellent introduction to the calculus of variations.

Appendices 11, 15, 19, and 22 cover a number of topics very important to classical mechanics in a manner that is both simpler and clearer than can be found elsewhere.

While writing this book I have not had in mind a hypothetical audience, but rather have written as if I were to be the reader. The book is in a sense a reflection of myself. Its exposure to numerous students over the years has sharpened rather than altered this reflection. I suspect that this is how most texts are written. Interestingly, I am dominantly a right hemisphere thinker—that is, I think in terms of pictures—but the first impression one gains of the text is that it is dominantly analytical. The probable explanation of this apparent anomaly is that one tends to emphasize the things that are personally difficult while at the same time ignoring what comes easily, with the result that the material is more analogous to a photographic negative than to a positive print. In any case the success of this book will depend on how many others share the difficulties, problems, loves, and hates that I experienced in learning classical mechanics. I hope that there are many, and that through this book they will derive some of the pleasures I have found in my encounter with classical mechanics.

EDWARD A. DESLOGE

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