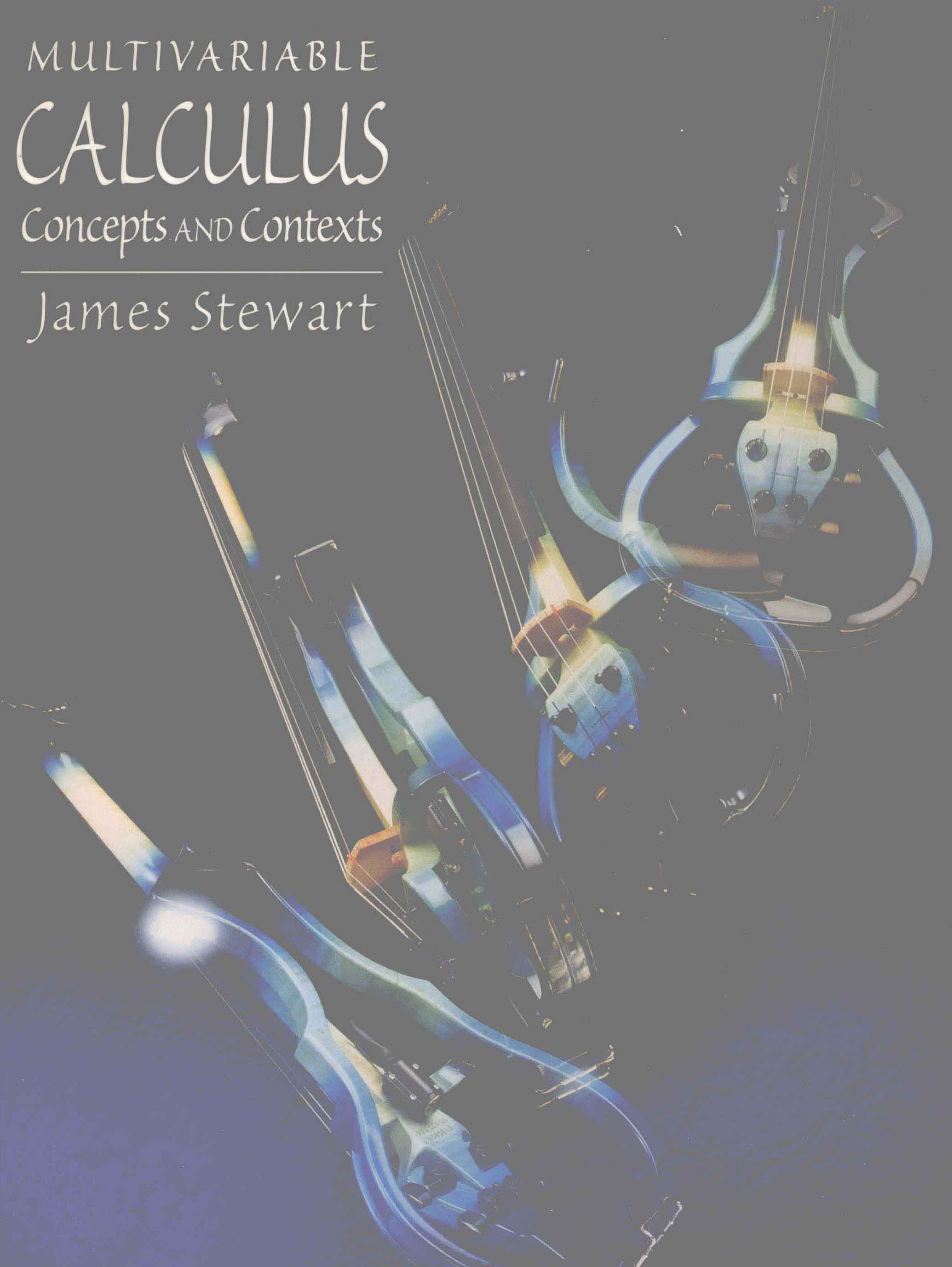
MULTIVARIABLE

CALCULUS

Concepts and Contexts

James Stewart



MULTIVARIABLE CALCULUS
Concepts AND Contexts

JAMES STEWART
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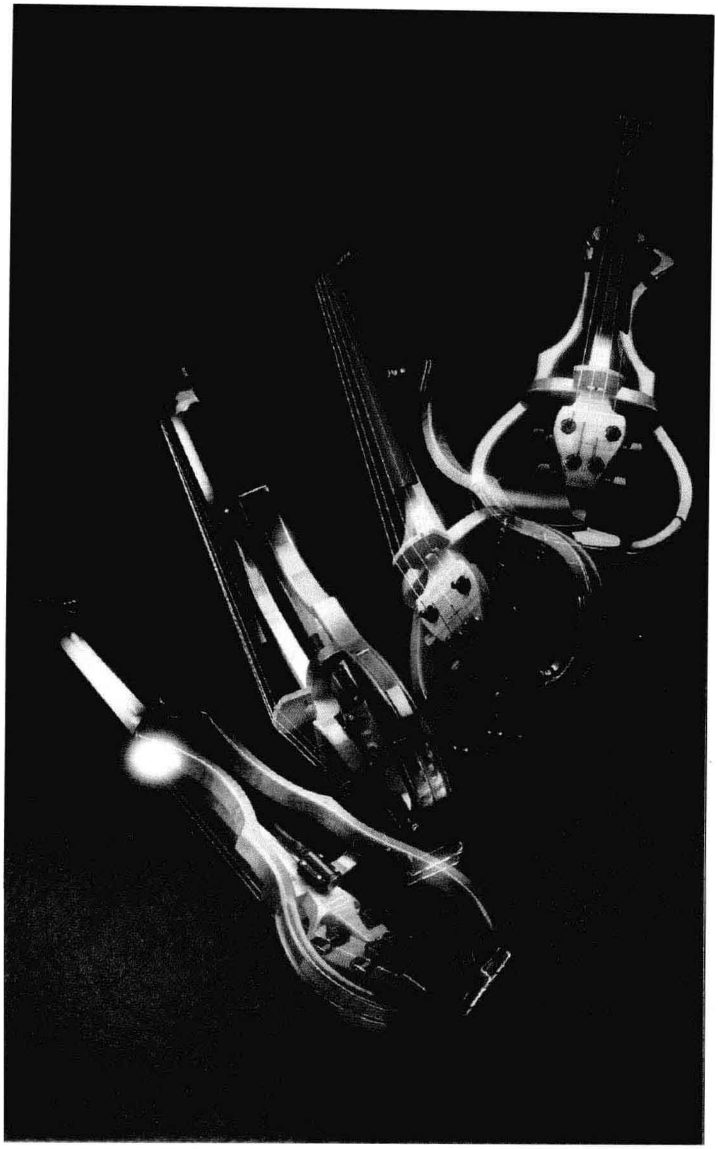
CALCULUS

Concepts AND Contexts



About the Cover

The photograph shows an electric violin made by David Bruce Johnson.

An acoustic violin, with its sound hole in the shape of an integral sign, became a symbol of James Stewart's previous calculus textbooks. Stewart plays both an 18th-century French violin and the blue electric violin that appears on the cover of this book.

The electric violin reflects the increased use of technology in calculus instruction, as well as a more informal approach to the subject. The quadruple image symbolizes the use of the Rule of Four throughout the book—four ways of looking at the same object.

For Geoff, Brad, Anna, and Jon.

For Debbie, Lorraine, Alan, and Matt.

For absent friends.

Preface

Mathematics departments are engaged in a debate over calculus reform. Our debate, of course, is part of an ongoing one that started 300 years ago when the first calculus textbook was published. But what do people *mean* today when they say they are in favor of calculus reform? As a result of talking with many instructors and reading hundreds of survey responses, I have learned that different people mean different things. They have passionately held opinions; there is common ground on some issues, but instructors are diametrically opposed on other issues. Let's look at some of the suggested key components of calculus reform.

Several survey respondents think that *technology* is the most important issue. Certainly, those of us who have watched our students use graphing calculators or computers know how enlivening this can be. We have seen from the looks on their faces how these devices engage our students' attention and enable them to become active learners. But these machines have been used by many schools with traditional curricula. For example, several traditional calculus texts (including my own *Calculus, Third Edition*) make extensive use of technology. Furthermore, I know of some very innovative reform calculus courses that use virtually no technology. So, while technology can be a critical component for implementing the goals of reform, I don't believe that technology itself characterizes reform.

Many people cite the *Rule of Three* as a key principle: "Topics should be presented geometrically, numerically, and algebraically." The implication is that, in the past, the algebraic point of view has been predominant and the graphical and numerical aspects have been given short shrift. More recently, the Rule of Three has been expanded to become the *Rule of Four* by emphasizing the verbal, or descriptive, point of view as well. But again, I think that my traditional book *Calculus, Third Edition* incorporates visualization and the Rule of Three. So I believe that the Rule of Three (or Four), important guiding principle though it is, still does not capture the most critical aspect of reform by itself.

Some respondents think that the enhanced attention to applications is a key feature and that instructors now have more freedom to choose applications for which they themselves have enthusiasm. While this aspect is certainly true, it is just as important in a traditional course.

So what *do* I think is the essence of calculus reform? In a word: *concepts.* We sometimes forget that the impetus for the current reform movement came from the Tulane Conference in January, 1986. I believe that the primary goal of reform should be what that conference formulated as their first recommendation:

Focus on conceptual understanding

What technology, the Rule of Four, and other aspects of reform have done is to enable instructors to use new tools and approaches to conceptual reasoning and skills. Visualization, numerical and graphical experimentation, and other approaches have changed how we teach conceptual reasoning in fundamental ways.

I think that nearly everybody—from the radical reformer to the staunch traditionalist—supports the central goal of focusing on conceptual understanding. But what is involved in implementing this goal? If we are serious about emphasizing conceptual understanding, then we have to expect faculty and students to give clear explanations of what symbols mean and why things work the way they do. That is simply not going to happen unless we take the time to work patiently with students. We need to slow down, provide multiple approaches, and not rush through the material when a new concept is introduced.

I have streamlined the coverage of some topics in order to free up time to achieve conceptual understanding. My premise in writing this book has been that it is possible to achieve conceptual understanding and still retain the best traditions of traditional calculus. I hope that this book will support a wider range of approaches to teaching calculus and improving students' conceptual understanding in diverse college and university settings.

Features

Conceptual Exercises The most important way to foster conceptual understanding is through the problems that we assign. To that end I have devised various types of problems. Some exercise sets begin with requests to explain the meanings of the basic concepts of the section. (See, for instance, the first couple of exercises in Sections 8.2, 11.2, and 11.3.) Similarly, review sections begin with a Concept Check and a True-False Quiz. Other exercises test conceptual understanding through graphs or tables. See Exercises 1–2 in Section 10.2, Exercise 25 in Section 10.3, Exercises 1–2 and 9–11 in Section 11.1, Exercises 1–2 in Section 11.6, Exercises 3–4 in Section 11.7, Exercises 1–2 in Section 11.8, Exercises 11–18 in Section 13.1, Exercises 13–14 in Section 13.2, Exercises 1, 2, 11, 23 in Section 13.3 and Exercises 7–9 in Section 13.5.

Pages 577, 765, 775
Pages 714, 723, 757
Pages 808, 818, 828, 923
Pages 934, 944, 960

Real-World Data My assistants and I spent a great deal of time looking in libraries, contacting companies and government agencies, and searching the Internet for interesting real-world data to introduce, motivate, and illustrate the concepts of calculus. As a result, many of the examples and exercises deal with functions defined by such numerical data or graphs. For instance, functions of two variables are illustrated by a table of values of the wind-chill index as a function of air temperature and wind speed (Example 1 in Section 11.1). Partial derivatives are introduced in Section 11.3 by examining a column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. This example is pursued further in connection with linear approximations (Example 3 in Section 11.4). Directional derivatives are introduced in Section 11.6 by using a temperature contour map to estimate the rate of change of temperature at Reno in the direction of Las Vegas. Double integrals are used to estimate the average snowfall in Colorado on December 24, 1982 (Example 4 in Section 12.1). Vector fields are introduced in Section 13.1 by depictions of actual velocity vector fields showing San Francisco Bay wind patterns.

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Projects One way of involving students and making them active learners is to have them work (perhaps in groups) on extended projects that give a feeling of substantial accomplishment when completed. *Applied Projects* involve applications that are designed to appeal to the imagination of students. The project on page 829, for example, uses Lagrange multipliers to determine the masses of the three stages of a rocket so as to minimize the total mass while enabling the rocket to reach a desired velocity. *Discovery Projects* explore aspects of geometry: tetrahedra (page 674), hyperspheres (page 894), and intersections of three cylinders (page 901). The *Laboratory Project* on page 697 uses technology to discover how interesting the shapes of surfaces can be and how these shapes evolve as the parameters change in a family. The *Writing Project* on page 978 explores the historical and physical origins of Green's Theorem and Stokes' Theorem and the interactions of the three men involved. Many additional projects are provided in the Instructor's Guide.

Technology The availability of technology makes it not less important but more important to clearly understand the concepts that underlie the images on the screen. But, when properly used, graphing calculators and computers are powerful tools for discovering and understanding those concepts. The icon indicates an exercise requiring the use of a device that can graph surfaces and curves in three dimensions (either a computer with graphing software or a graphing calculator with this capability), but that is not to say that technology can't be used on the other exercises as well. The symbol CAS is reserved for problems in which the full resources of a computer algebra system (like Derive, Maple, Mathematica, or the TI-92) are required. But technology doesn't make pencil and paper obsolete. Hand calculation and sketches are often preferable to technology for illustrating and reinforcing some concepts. Both instructors and students need to develop the ability to decide where the hand or the machine is appropriate.

Content

Chapter 8 Sequences and Series Tests for the convergence of series are considered briefly, with intuitive rather than formal justifications. Numerical estimates of sums of series are based on which test was used to prove convergence. The emphasis is on Taylor series and polynomials and their applications to physics. Error estimates include those from graphing devices.

Chapter 9 Vectors and the Geometry of Space The dot product and cross product of vectors are given geometric definitions, motivated by work and torque, before the algebraic expressions are deduced. To facilitate the discussion of surfaces, functions of two variables and their graphs are introduced here.

Chapter 10 Vector Functions The calculus of vector functions is used to prove Kepler's First Law of planetary motion, with the proofs of the other laws left as a project. In keeping with the introduction of parametric curves in Chapter 1, parametric surfaces are introduced as soon as possible, in this chapter. I think an early familiarity with such surfaces is desirable, especially with the capability of computers to produce their graphs. Then tangent planes and areas of parametric surfaces can be discussed in Sections 11.4 and 12.6.

Chapter 11 Partial Derivatives Functions of two or more variables are studied from verbal, numerical, visual, and algebraic points of view. In particular, I introduce partial derivatives by looking at a specific column in a table of values of the heat index (perceived air temperature) as a function of the actual temperature and the relative humidity. Directional derivatives are estimated from contour maps of temperature, pressure, and snowfall.

Chapter 12 Multiple Integrals Contour maps and the Midpoint Rule are used to estimate the average snowfall and average temperature in given regions. Double and triple integrals are used to compute probabilities, areas of parametric surfaces, volumes of hyperspheres, and the volume of intersection of three cylinders.

Chapter 13 Vector Fields Vector fields are introduced through pictures of velocity fields showing San Francisco Bay wind patterns. The similarities among the Fundamental Theorem for line integrals, Green's Theorem, Stokes' Theorem, and the Divergence Theorem and emphasized.

Ancillaries

Multivariable Calculus: Concepts and Contexts is supported by a complete set of ancillaries developed under my direction. Each piece has been designed to enhance student understanding and to facilitate creative instruction.

The following resources are available, free of charge, to adopters of the text.

Instructor's Guide with Test Items by Harvey B. Keynes, James Stewart, Douglas Shaw, Mike Lawler, and Daniel O'Loughlin

Offering suggestions on how to implement ideas about reform into your calculus course, the Guide serves as a practical roadmap to topics and projects in the text. Each section of the main text is discussed from several viewpoints and contains suggested time to allot, points to stress, text discussion topics, core materials for lecture, workshop/discussion suggestions, group work exercises in a form suitable for handout, and suggested homework problems.

Organized according to the main text, the complete set of Test Items contains both multiple-choice and open-ended questions, offering a range of model problems, including short-answer questions that focus narrowly on one basic concept; and items that integrate two or more concepts and require more detailed analysis and written response.

Complete Solutions Manual by Dan Clegg

Provides detailed solutions to all exercises in the text.

Transparencies by James Stewart

Full-color transparencies featuring 50 of the more complex diagrams from the text for use in the classroom.

Computerized Testing This computerized version of the printed Test Items allows instructors to insert their own questions and customize ones that are provided with some test items algorithmically generated. Available on the following platforms:

DOS (WESTEST 4.01)
Macintosh (Thomson World Class Testing Tools)
Windows (Thomson World Class Testing Tools)

A complete range of student ancillaries is also available:

Study Guide by Robert Burton and Dennis Garity

Offering additional explanations and worked-out examples, and formatted to provide guided practice, each section in this Study Guide corresponds to a section in the text. Every section contains a short list of key concepts; a short list of skills to master; a brief introduction to the ideas of the section; an elaboration of the concepts and skills, including extra worked-out examples; and links in the margin to earlier and later material in the text and Study Guide.

Student Solutions Manual by Dan Clegg

Contains detailed solutions to all odd-numbered exercises in the text.

Lab Manuals Each of these comprehensive lab manuals will help students learn to effectively use the technology tools available to them. Each lab contains clearly explained exercises and a variety of labs and projects to accompany the text.

CalcLabs with Maple®, Multivariable

by Art Belmonte and Philip Yasskin

CalcLabs with Mathematica®, Multivariable

by Selwyn Hollis

CalcLabs with Derive®, Multivariable

by Jeff Morgan

CalcLabs with the TI-92, Multivariable

by Jeff Morgan and Selwyn Hollis

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JAMES STEWART

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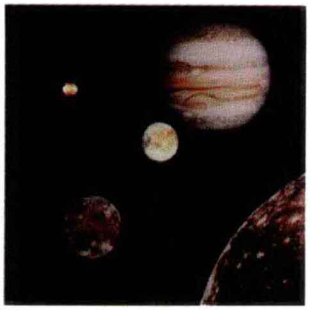

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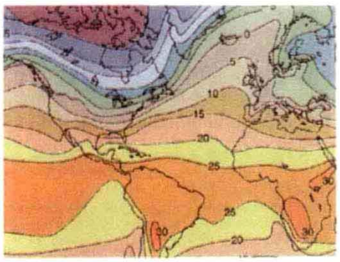

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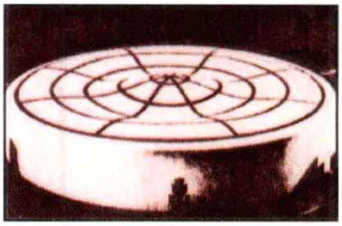
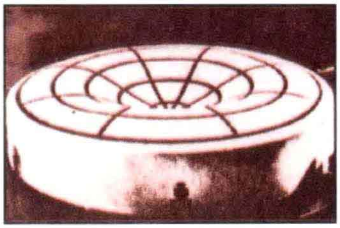
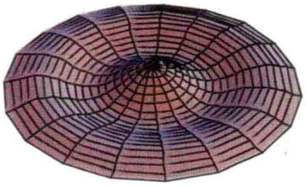
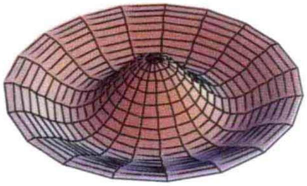
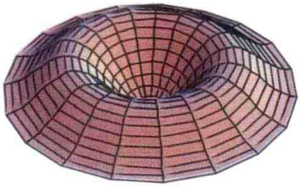
Bessel functions, which are used to model the vibrations of drumheads and cymbals, are defined as sums of infinite series in Section 8.5.











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Infinite Sequences and Series