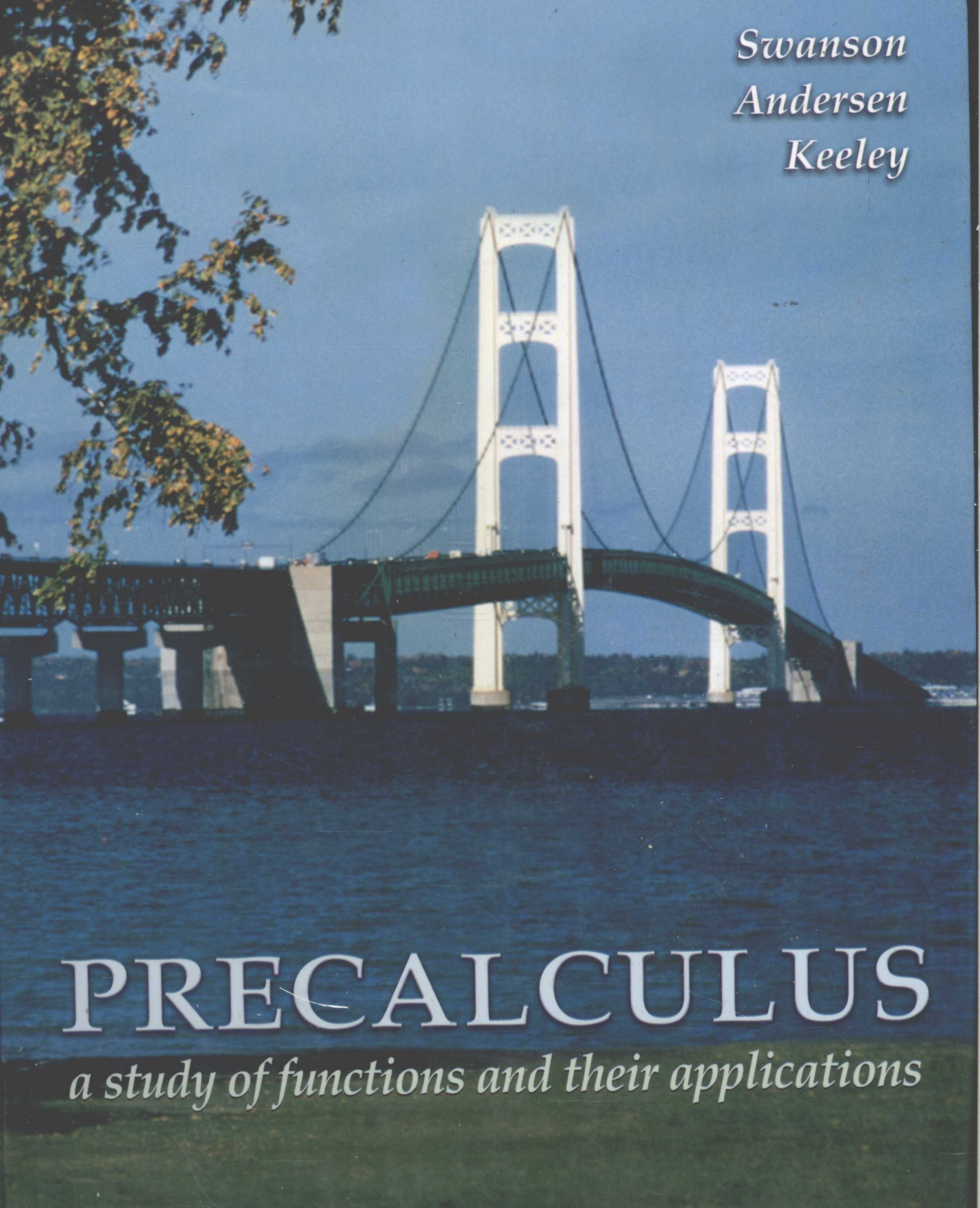

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PRECALCULUS

a study of functions and their applications

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PRECALCULUS

A Study of Functions and their Applications

To Debra, Emma, and Tyler

To Jim, John, and Joy

To Laura, Bethany, Meredith, Bryan, and Lynnae

Preface

PHILOSOPHY

This book treats functions as the object of study while focusing on important mathematical concepts. In contrast to the organization of most precalculus texts, we introduce students to linear, exponential, logarithmic, periodic, and power functions in the second chapter and develop the properties of these functions throughout the remainder of the text as mathematical concepts are explored. This allows us to emphasize the connections between mathematical ideas as we further the student's knowledge about a particular type of function.

Building the study of functions throughout the book, rather than isolating the study of a single type of function to one or two chapters, leads to a natural review of the properties of various functions throughout the text. This organization also addresses a common student misconception that mathematics consists of unrelated bits of information which can be forgotten once a particular topic is finished.

Another common student misconception is that mathematics is useless and dry. To address this, we have made the study of applications an integral part of our text. Applications occur in the readings in three different ways: the focus of a section, an example used to illustrate a concept, and the motivation for a mathematical concept. Applications also occur in homework exercises, investigations, and projects. Every application has been researched and care has been taken to insure that the data given is accurate.

Our underlying premise is that students learn best when they are engaged with the material. We have designed this text so that students will read, write, discuss, and explore mathematical concepts. It is written in a conversational format with reading questions interspersed throughout each section. Each homework set includes investigations where students are asked to extend and apply the material presented in that section. Each chapter includes a project, which is a larger scale exploration or application. All of these features invite students to be actively engaged in their study of precalculus.

This textbook is designed for a one-semester college course in precalculus (although we believe that it is equally well suited for a one-year high school precalculus course). It incorporates writing, projects, multiple representations of mathematical concepts, and the use of technology in the form of graphing calculators. This makes it an excellent preparation for a modern calculus course.

This text is also well suited for students going on to a traditional calculus course as well as various science courses. Throughout the text we have tried to illustrate how mathematicians think by asking the kinds of questions that are interesting to mathematicians. We feel that this text meets the recommendations of the Mathematical Association of America, and reflects the spirit of the standards set by the National Council of Teachers of Mathematics and the American Mathematical Association of Two-Year Colleges.

KEY FEATURES

Precalculus: A Study of Functions and their Applications has several important features:

- **Coverage of Functions.** Each of the basic types of functions (linear, exponential, logarithmic, periodic, and power) is introduced early in the book. Properties of these functions are developed throughout the remainder of the text. This leads to a natural review as students build on their understanding of these functions. It also emphasizes the connections of mathematical ideas by unifying the study of functions around mathematical concepts rather than around types of functions.
- **Writing Style.** Students are expected to read the text. It is written in a conversational format addressed to the student. Reading questions are incorporated throughout each section. In these questions, students are asked to write about key concepts and do simple problems.
- **Exercises & Investigations.** Each exercise set includes investigations. These are extended problems in which students explore either a mathematical concept or an application. Several topics covered in the reading first appear as an investigation in an earlier section. This foreshadows mathematical ideas and allows the students to gain some firsthand experience with a concept before encountering it formally. Investigations can be used as part of a regular homework assignment, group homework, a small group in-class activity, or as a class discussion.
- **Projects.** A project is included with each chapter. These incorporate a variety of topics and can be used to reinforce and extend the topics presented in that chapter. A typical project requires one or two class periods and approximately five additional hours of student work outside class. The projects are best completed by students working in groups. These projects were written with the partial support of a grant from the National Science Foundation. This material was previously published under the title *Projects for Precalculus* by Saunders College Publishing.
- **Applications.** The inclusion of applications is an integral part of the text. All of the applications included in the text involve real situations with actual data. These applications occur in the reading, the exercises, the investigations, and the projects. Every application has been

researched and care has been taken to insure that the data given is accurate.

- **Historical Anecdotes.** Included in the reading are historical anecdotes. We also include some recent developments in mathematics so students realize that mathematical knowledge continues to evolve and that interesting, applicable mathematical questions are still being explored. We want students to see mathematics as an ongoing human endeavor with roots in the past.
- **Approach.** Topics are presented using multiple representations (symbolic, numerical, graphical, and verbal).
- **Graphing Calculator.** It is assumed that students have daily access to graphing calculators. We emphasize ways in which the calculator can be used to gain mathematical understanding, but rarely give specific keystrokes.

ANNOTATED TABLE OF CONTENTS

CHAPTER 1: An Introduction to Functions. Various representations of functions as well as the language and notation associated with functions are introduced. This chapter also illustrates how graphing calculators can be used and misused in the study of functions.

CHAPTER 2: Families of Functions. Linear, exponential, logarithmic, periodic, and power functions are introduced. Students are shown how to recognize these functions in their various representations. Students are also shown how to obtain a formula when given a linear, exponential, or power function either numerically or graphically. This lays the groundwork for these functions and for their use throughout the remainder of the book. Examples of real world situations are included for each type of function.

CHAPTER 3: New Functions from Old. The basic functions from Chapter 2 are transformed to form new functions in a variety of ways. In particular, the relationship between a transformed function in its symbolic form is compared with its graphical form. Transformations include addition and multiplication as well as composition. The relationship between a function and its inverse is also explored.

CHAPTER 4: Polynomial and Rational Functions. Polynomials, introduced as transformations of particular power functions, are important enough to study as independent objects. We look at their properties as well as how they can be combined through division to form rational functions. Applications are given throughout the chapter.

CHAPTER 5: Trigonometric Functions. The periodic functions of sine and cosine, introduced in Chapter 2, are reviewed and other trigonometric functions are introduced in this chapter. These functions are introduced by using the unit circle definitions. The geometry of a circle, including arc length and area, is also explored. The transformations, introduced in Chapter 3, are applied to the trigonometric functions. Trigonometric identities are introduced throughout the chapter and are the focus of Section 5.4.

CHAPTER 6: Applications of Trigonometric Functions. Using trigonometric functions to model situations in the world is the focus of this chapter. It begins by looking at problems involving triangles. Then combinations of the periodic functions with other periodic functions as well as with non-period functions are explored. Doing this allows us to expand the areas in which we can use trigonometric functions to model applications.

CHAPTER 7: Solving Equations and Fitting Functions to Data. Different methods for solving equations and inequalities are introduced. This structure gives students a review of the functions first introduced in Chapter 2. The techniques of linear, exponential, and power regression are introduced as methods of fitting functions to data.

CHAPTER 8: Getting Ready for Calculus. This chapter serves as an introduction to calculus by exploring the concepts of limit, the derivative, and the integral. These topics are meant to help students prepare for the study of calculus and serve only as introductions.

CHAPTER 9: Additional Topics. The text concludes with a look at parametric equations, vectors, and multivariable functions. A property of a conic section is the focus of the project in this chapter. While these topics are not vital for a preparation for the study of calculus, they are of interest because many applications use these mathematical concepts.

SUPPLEMENTS TO THE TEXT

The following supplements are available for instructors and students from the publisher:

Instructor's Resource Manual. This manual contains solutions to all the reading questions, exercises, investigations, and projects. It also outlines methods for ways to effectively use the book.

Printed Test Bank and Prepared Tests. The test bank consists of multiple-choice and short-answer test items organized by chapter and section. The prepared tests include nine sets of ready-to-copy tests, one set for each chapter. Each set comprises of one multiple-choice and two show-your-work tests. Answers for every test item are provided.

ESATEST 2000 TM Computerized Test Bank. A flexible, powerful computerized testing system, the ESATEST 2000 TM Computerized Test Bank contains all the test bank questions and allows instructors to prepare quizzes and examinations quickly and easily. It offers teachers the ability to select, edit, and create not only test items but algorithms for test items as well. Teachers can tailor tests according to a variety of criteria, scramble the order of test items, and administer tests on-line, either over a network or via the Web. ESATEST 2000 TM also includes full-function grade book and graphing features. This software is available in Windows and Macintosh formats.

Student Solutions Manual. This manual contains complete solutions to every odd-numbered problem in the reading questions and exercises and to all chapter review exercises.

Graphing Calculator Manual. The Graphing Calculator Manual contains programming ideas for various Texas Instruments graphing calculators and investigates the application of those programming operations to solve problems.

Projects for Precalculus. The authors of the text previously published *Projects for Precalculus*, a popular and successful NSF-sponsored program in reform precalculus. This manual contains 26 carefully prepared and tested activities that promote conceptual understanding and active learning. The projects included at the end of each chapter in the text were originally published in this supplement.

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Todd Swanson
Janet Andersen
Robert Keeley

PRECALCULUS

A Study of Functions and their Applications

Contents

CHAPTER OUTLINE

CHAPTER OVERVIEW

- The idea of a function
- How functions are represented and evaluated
- How functions can be represented effectively on a calculator
- How functions can be used to mathematically model a physical situation

1

AN INTRODUCTION TO FUNCTIONS

Various representations of functions as well as the language and notation associated with functions are introduced in this chapter. Also included are illustrations of how graphing calculators can be used and misused in the study of functions. The chapter concludes with a section on mathematical modeling, which shows how functions can be developed in order to solve various problems.

1.1 FUNCTIONS

Functions are all around us. They are useful in modeling physical, financial, and even sociological situations in addition to being interesting to study from a mathematical perspective. By understanding the behavior of functions, we can better understand how to choose functions to describe real-world situations. In this section, we consider four ways to represent functions, how to determine if something is a function, and how to evaluate functions.

FOUR REPRESENTATIONS OF A FUNCTION

Functions can be represented in a variety of ways. The first thing that probably comes to your mind when you see the word *function* is a mathematical formula such as the formula for the area of a circle:

$$A = \pi r^2.$$

This is a *symbolic* representation of a function, but there are other ways of representing functions. Often you see functions represented graphically, especially in magazines and newspapers. Figure 1 is an example of a *graphical* representation of a function that describes the average salary of a National Basketball Association (NBA) player with respect to time.[1]

Another way that functions can be represented is as a table of numbers such as Table 1, which describes the U.S. population over time.[2] This table is known as a *numerical* representation of a function. Symbols, graphs, and numbers are all mathematical objects, so it should not surprise you that all three can be used in different ways to represent functions. There is, however, a fourth way of commonly representing functions, one that does

[1] USA Today, *15 November 1996, p. 16C.*

[2] *Mark S. Hoffman, ed.,* The World Almanac *(New York: Pharos Books, 1990), pp. 552–53.*

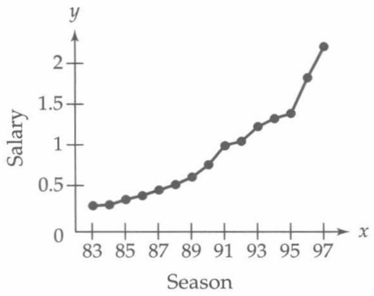

Figure 1 Graphical representation of the average salary of an NBA player (in millions of dollars).

TABLE 1 U.S. Population by Official Census

Year	Population
1800	5,308,483
1820	9,638,453
1840	17,063,353
1860	31,443,321
1880	50,189,209
1900	76,212,168
1920	106,021,537
1940	132,164,569
1960	179,323,175
1980	226,542,203
1990	248,709,873

not seem quite so mathematical: using words to describe a function. We call this a *verbal* representation of a function. For example, you might read the cost of tuition in a college catalog as

> Tuition for undergraduate students who are Michigan residents taking anywhere from 12 to 16 credit hours is the same, a total of $1,390 per semester. Tuition is $124 per credit hour for fewer than 12 credits and for each credit over 16.[3]

This description can be thought of as a function in which the input is the number of credit hours a student takes and the output is the total cost of tuition.

Understanding how functions can be represented in these four ways (symbolically, graphically, numerically, and verbally) and learning how these representations are related to one another is something you will see over and over again throughout this book. All four representations are equally valid, but each representation has its own advantages and disadvantages. A symbolic representation is best if you want to evaluate your function for a specific input. It is also the best representation if you want to manipulate or transform your function in some way. A graphical representation is best for quickly seeing the behavior of your function. It is easy to see where the function is changing when looking at a graph. A numerical representation is best if you want to convey information about your function at a few selected points, such as size of the U.S. population every 20 years from 1800 to 1980. This method is also the most common way to represent a function when you are collecting experimental data. Verbally is often the best way to describe a function to someone else and, in some sense, is the most natural way to represent many common functions. So, all four representations are equally important, and which one you want to use for a particular function depends on the function and what you are trying to do with it. For this reason, it is important to know how to change from one representation to another.

Definition of Function

Although we have talked about functions and looked at examples of functions, we have not yet defined exactly what we mean by the word *function*.[4] A **function** is a rule that assigns an input to at most one output. To see how this definition works, we look at each of the four examples we mentioned earlier.

The formula for the area of a circle is $A = \pi r^2$. Think about why this is a function. If you are given a radius, how many choices do you have for the area? Just one. This is the definition of a function: for each input, there is at most *one* output. For example, a circle of radius 9 centimeters has an area of 81π square centimeters. The area cannot possibly be anything else. Look at the function given in Figure 1, which describes the average salary for an NBA player. If you are given a season, there is at most *one* possible average salary. Again, the definition of a function is that there is at

[3]Grand Valley State University Undergraduate and Graduate Catalog *(Allendale, MI: Grand Valley State University, 1996–97), 1996, p. 42.*

[4]*In mathematics, words have very specific meanings, and one of the keys to doing well in mathematics is to learn and understand the definitions.*